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B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2025.

Third Semester

Electronics and Communication Engineering

MA 3355 – RANDOM PROCESSES AND LINEAR ALGEBRA

(Common to : Electronics and Telecommunication Engineering)

(Regulations 2021)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Two fair dice are tossed. Find the probability that the sum of the outcomes of the two dice is equal to 6 or 10.

2. Find moment generating function for geometric random variable.

3. Determine the value of the c such that the probability mass function $f(x, y) = c(x + y)$ for $x = 1, 2, 3$ and $y = 1, 2, 3$.

4. Let (X, Y) be a two-dimensional random variable. Define covariance of (X, Y). If X and Y are independent, what will be the covariance of (X, Y)?

5. Write classifications of random processes.

6. Write down the Chapman-Kolmogorov equation in terms of a Markov chain.

7. Consider the vectors $u = (1, 2, 3)$ and $v = (2, 3, 1)$ in R^3. For what value k, is the vector $w = (1, k, 4)$ a linear combination of u and v?

8. Show that a line through the origin is a subspace of R^3.

9. Let V and W be any two vector spaces and transformation $T : V \to W$, $T(v) = v$, for every v in V. What is kernel of T and range of T?

10. Determine whether the function $T : M_{nxn} \to R$, where $T(A) = tr(A)$ is a linear combination.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Engineers in charge of maintaining our nuclear fleet must continually check for corrosion inside the pipes that are part of the cooling systems. The inside condition of the pipes cannot be observed directly but a nondestructive test can give an indication of possible corrosion. This test is not infallible. The test has probability 0.7 of detecting corrosion when it is present but it also has probability 0.2 of falsely indicating internal corrosion. Suppose the probability that any section of pipe has internal corrosion is 0.1. (8)

(1) Determine the probability that a section of pipe has internal corrosion, given that the test indicates its presence.

(2) Determine the probability that a section of pipe has internal corrosion, given that the test is negative

(ii) A milk processing unit claims that, of the processed milk converted to powdered milk, 95% does not spoil. Find the probabilities that among 15 samples of powdered milk (8)

(1) all 15 will not spoil;

(2) at most 12 will not spoil;

(3) at least 9 will not spoil.

Or

(b) (i) The time for oil to percolate to all parts of an engine can be treated as a random variable having a normal distribution with mean 20 seconds. Find its standard deviation if the probability is 0.25 that it will take a value greater than 31.5 seconds. (6)

(ii) The time between calls to a plumbing supply business is exponentially distributed with a mean time between calls of 15 minutes. (10)

(1) What is the probability that there are no calls within a 30-minute interval?

(2) What is the probability that at least one call arrives within a 10-minute interval?

(3) What is the probability that the first call arrives within 5 and 10 minutes after opening?

(4) Determine the length of an interval of time such that the probability of at least one call in the interval is 0.90.

12. (a) Determine the value of the c such that the probability density function $f(x, y) = cxy$ for $0 < x < 3$ and $0 < y < 3$. Also determine the following (16)

(i) $P(X < Y)$

(ii) Marginal probability distribution functions of X and Y

(iii) $E(X)$ and $E(Y)$

(iv) Are X and Y independent variables?

Or

(b) (i) A fair coin is tossed three times. Let X be a random variable that takes the value 0 if the first toss is a tail and the value 1 if the first toss is a head. Also, let Y be a random variable that defines the total number of heads in the three tosses. (8)

(1) Determine the joint PMF of X and Y

(2) Are X and Y independent?

(ii) The two lines of regression are $8x - 10y + 66 = 0$, $40x - 18y - 214 = 0$. The variance of X is 9. (8)

Find :

(1) The mean values of X and Y

(2) Correlation coefficient between X and Y.

13. (a) (i) A random process $X(t)$ is defined by $X(t) = A\cos(t) + B\sin(t)$, $-\infty < t < \infty$ where A and B are independent random variables each of which has a value -2 with probability 1/3 and a value 1 with probability 2/3. Show that $X(t)$ is a wide sense stationary process. (8)

(ii) A machine goes out of order whenever a component fails. The failure of this part follows a Poisson process with a mean rate of 1 per week. Find the probability that 2 weeks have elapsed since last failure. If there are 5 spare parts of this component in an inventory and that the next supply is not due in 10 weeks, find the probability that the machine will not be out of order in the next 10 weeks. (8)

Or

(b) Consider the following social mobility problem. Studies indicate that people in a society can be classified as belonging to the upper class (state 1), middle class (state 2), and lower class (state 3). Membership in any class is inherited in the following probabilistic manner. Given that a person is raised in an upper-class family, he will have an upper-class family with probability 0.45, a middle-class family with probability 0.48, and a lower-class family with probability 0.07. Given that a person is raised in a middle-class family, he will have an upper-class family with probability 0.05, a middle-class family with probability 0.70, and a lower-class family with probability 0.25. Finally, given that a person is raised in a lower-class family, he will have an upper-class family with probability 0.01, a middle-class family with probability 0.50, and a lower-class family with probability 0.49. Determine the following:

(i) The state-transition diagram of the process.

(ii) The transition probability matrix of the process.

(iii) The limiting-state probabilities. (16)

14. (a) Check whether the set of all pairs of real numbers of the form (x, y) with the operations $(x, y)+(x', y')=(xx', yy')$ and $k(x, y)=(kx, ky)$ is vector space. If not, list the axioms that fails to hold. (16)

Or

(b) (i) Check whether the vectors $v_1=(1,-3,2)$, $v_2=(4,1,0)$, $v_3=(0,2,-1)$ form a basis for R^3. (8)

(ii) Find the dimension and a basis for the solution space W of the system of homogeneous equation given below. (8)

$$2x_1+2x_2-x_3+x_5=0;\ -x_1-x_2+2x_3-3x_4+x_5=0;$$

$$x_1+x_2-2x_3-5x_5=0;\ x_3+x_4+x_5=0.$$

15. (a) (i) State and prove dimension theorem for linear transformation (8)

(ii) Let $T: R^2 \to R^3$ be defined by $T(x, y)=(x+2y,-x,0)$. Find the matrix of the transformation from B' to B with respect to the bases $B=\{u_1, u_2\}$ and $B'=\{v_1, v_2, v_3\}$ where $u_1=(1,3), u_2=(-2,4)$, $v_1=(1,1,1)$, $v_2=(2,2,0)$ and $v_3(3,3,0)$. (8)

Or

(b) (i) Find an orthonormal basis of R^3, given that an arbitrary basis is $\{v_1=(1,1,0), v_2=(2,1,3), v_3=(1,1,1)\}$ using Gram Schmidt process. (8)

(ii) Find the least square solution of the linear system $AX=b$, where $A=\begin{bmatrix} 0 & 1 & 4 \\ 1 & 2 & -1 \\ 5 & 8 & 0 \end{bmatrix}$, $b=\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$. (8)