Reg. No. : E N G G T R E E . C O M

Question Paper Code : 21289

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2023.

Fourth Semester

Aeronautical Engineering

MA 3452 – VECTOR CALCULUS AND COMPLEX FUNCTIONS

(Common to : Aerospace Engineering)

(Regulations 2021)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the directional derivative of $f(x, y, z) = xy^2 + yz^3$ at the point (2, –1, 1) in the direction of the vector $\vec{i} + 2\vec{j} + 2\vec{k}$.

2. Prove that the force $\vec{F} = \left(e^x z - 2xy\right)\vec{i} - \left(x^2 - 1\right)\vec{j} + \left(e^x + z\right)\vec{k}$ is conservative.

3. Is the function $f(z) = 2xy + i\left(x^2 - y^2\right)$ analytic? Justify.

4. What is meant by bilinear transformation?

5. State Cauchy's integral theorem.

6. Identify and nature of the singular point of the function $f(z) = \tan \pi z$.

7. Find the Laplace transform of the unit step function.

8. State initial and final value theorem on Laplace transform.

9. Solve $\left(D^2 + 4D + 4\right)y = 0$.

10. Find the particular integral of $\left(D^2 - 6D + 8\right)y = e^{-4x}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) If $\vec{F} = (3x^2 + 6y)\vec{i} - 14yz\vec{j} + 20xz^2\vec{k}$, evaluate $\int_C \vec{F}.d\vec{r}$ from (0, 0, 0) to (1, 1, 1) along the path $x = t,\ y = t^2,\ z = t^3$. (8)

(ii) Use Green's theorem, evaluate $\int_C (xy + y^2)\,dx + (x^2)\,dy$, where C is the boundary of the common area between $y = x^2$ and $y = x$. (8)

Or

(b) (i) Verify Stoke's theorem for $\vec{F} = (2x - y)\vec{i} - yz^2\vec{j} - y^2z\vec{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is the circular boundary on $z = 0$ plane. (8)

(ii) State Gauss divergence theorem, and use it to evaluate $\iint_S x^3\,dy\,dz + x^2 y\,dz\,dx + x^2 z\,dx\,dy$ where S is the surface of the cube $x = 0$, $y = 0, z = 0, x = 1, y = 1$ and $z = 1$. (8)

12. (a) (i) Derive the Cauchy-Reimann equation for the complex function $f(z) = u(x, y) + iv(x, y)$. (8)

(ii) Find the bilinear transformation which maps the points $z = 1,\ i, -1$ onto the points $w = i, 0, -i$ respectively. (8)

Or

(b) (i) Find the analytic function $f(z) = u(x, y) + iv(x, y)$, if $u(x, y) = \dfrac{\sin 2x}{\cos 2x + \cosh 2y}$. (8)

(ii) Find the image of the upper semi-circle $|z| = 2$ by the mapping $f(z) = z^2$. (8)

13. (a) (i) Using Cauchy's integral formula, evaluate $\oint_C \dfrac{dz}{(z-1)(z-2)(z-3)}$, where $C : |z| = 5$. (8)

(ii) Obtain Taylor's or Laurent's series for $f(z) = \dfrac{1}{(z+2)(1+z^2)}$ in

(1) $|z| < 1$ (2) $1 < |z| < 2$ (3) $|z| < 2$ (8)

Or

(b) (i) State and prove Cauchy's residue theorem. (8)

(ii) Using contour integration technique, prove $\int_0^{\infty} \frac{x^2 dx}{(x^2+4)(x^2+9)} = \frac{\pi}{10}$. (8)

14. (a) (i) Find the value of $L\left(\frac{\cos 2t - \cos 3t}{t}\right)$. (8)

(ii) Find the Laplace transform of $f(t) = \begin{cases} t, & 0 \le t \le a \\ 2a - t, & a \le t \le 2a \end{cases}$ with $f(t+2a) = f(t)$ for all t. (8)

Or

(b) (i) Applying Laplace transform, solve $y'' + y' = t^2 + 2t$, given that $y(0) = 4$ and $y'(0) = -2$. (8)

(ii) Using the convolution theorem, find $L^{-1}\left(\frac{1}{(s+a)(s+b)}\right)$. (8)

15. (a) (i) Find the general solution of $(D^2 + 2D + 2)y = e^{-x}\sin x$. (8)

(ii) Solve : $\frac{dx}{dt} - y = t, \frac{dy}{dt} + x = t^2$. (8)

Or

(b) (i) Using the method of varitation of parameters, solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$. (8)

(ii) Solve $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos[\log(1+x)]$. (8)
