Reg. No. : E N G G T R E E . C O M

Question Paper Code : 51332

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2024.

Fourth Semester

Aeronautical Engineering

MA 3452 — VECTOR CALCULUS AND COMPLEX FUNCTIONS

(Common to Aerospace Engineering)

(Regulations 2021)

Time : Three hours　　　　　　　　　　　　Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. In what direction from (1, –1, 2) is the directional derivative of $\varphi = x^2y^2z^3$ a maximum? What is the magnitude of this maximum?

2. If $\overline{A}$ and $\overline{B}$ are irrotational, prove that $\overline{A} \times \overline{B}$ is solenoidal.

3. Is the function $f(z) = e^{\bar{z}}$ analytic? Justify.

4. Find the fixed points of the transformation $\omega = \frac{1}{z}$.

5. State Cauchy's integral theorem.

6. Identify the nature of the singular point of the function $f(z) = \frac{z - \sin z}{z^2}$.

7. Evaluate $L(t \sin t)$.

8. State initial and final value theorem on Laplace transform.

9. Solve $(D^2 + 3D - 40)y = 0$.

10. Find the particular integral of $(D^2 + 4)y = \sin x$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) If $\vec{F} = (3x^3 + 6y)\vec{i} - 14yz\,\vec{j} + 20xz^2\,\vec{k}$, evaluate $\int_C \vec{F} \cdot d\vec{r}$ from (0, 0, 0) to (1, 1, 1) along the curve $x = t,\ y = t^2, z = t^3$. (8)

(ii) Using Gauss divergence theorem, evaluate $\iint_S (2xy + z)dydz + y^2 dzdx - (x + 3y)dx\,dy$, where S is the surface of the tetrahedron bounded by $x = 0,\ y = 0,\ z = 0$, and $2x + 2y + z = 6$. (8)

Or

(b) (i) Find the common area between $y^2 = 4x$ and $x^2 = 4y$, using Green's theorem. (8)

(ii) Verify Stoke's theorem for $\vec{F} = (2x - y)\vec{i} - yz^2\,\vec{j} - y^2 z\,\vec{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is the circular boundary on $z = 0$ plane. (8)

12. (a) (i) Show that the function $u(x, y) = 3x^2 y + 2x^2 - y^3 - 2y^2$ is harmonic. Find the corresponding analytic function $f(z)$ and also find its harmonic conjugate function v. (8)

(ii) Find the bilinear transformation which maps the points $z = 1,\ i,\ -1$ of the z–plane onto the points $w = 0, 1, \infty$ of the $\omega-$ plane. (8)

Or

(b) (i) Find the analytic function $f(z) = u(x, y) + iv(x, y)$, if $u + v = \dfrac{\sin 2x}{\cosh 2y - \cos 2x}$. (8)

(ii) Under the transformation $\omega = \dfrac{1}{z}$, find the image of the line $2x + y = 0$. (8)

13. (a) (i) Using Cauchy's integral formula, evaluate $\oint_C \dfrac{(z+1)}{z^2 + 2z + 4} dz$, where $C : |z + 1 + i| = 2$. (8)

(ii) Find the Laurent's series about $z = 1$ for the function $f(z) = \dfrac{e^{2z}}{(z-1)^3}$. Also find its residue at the same. (8)

Or

(b) (i) Evaluate $\oint_C \frac{1}{(z^2+4)^2}\,dz$, where $C:|z-i|=2$, Using Cauchy's residue theorem. (8)

(ii) Using contour integration technique, prove that $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta} = \frac{2\pi}{\sqrt{3}}$. (8)

14. (a) (i) Find the value of $L^{-1}\left(\frac{s^2-s+2}{s(s+2)(s-3)}\right)$, using partial fraction method. (8)

(ii) Find the Laplace transform of $f(t)=\begin{cases} E, & 0\le t<\tau/2 \\ -E, & \tau/2\le t\le\tau \end{cases}$ with $f(t+\tau)=f(t)\,\forall t$. (8)

Or

(b) (i) Applying Laplace transform, solve $y''+y=2$, given that $y(0)=0$ and $y'(0)=1$. (8)

(ii) State and prove the convolution theorem on Laplace transform. (8)

15. (a) (i) Solve : $(D^2-3D+2)y = 2\cos(2x+3)+3e^{2x}$. (8)

(ii) Solve : $\frac{dx}{dt}+y=\sin t,\ \frac{dy}{dt}+x=\cos t$. (8)

Or

(b) (i) Using the method of variation of parameters, solve $\frac{d^2y}{dx^2}+y=\sec x$. (8)

(ii) Solve : $(2x+3)^2\frac{d^2y}{dx^2}-(2x+3)\frac{dy}{dx}-12y=6x$. (8)