Reg. No. : E N G G T R E E . C O M

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B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2024.

Third Semester

Mechanical Engineering

MA 3351 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to : Aeronautical Engineering/Aerospace Engineering/Automobile Engineering/Biomedical Engineering/Civil Engineering/Manufacturing Engineering/Marine Engineering/Material Science and Engineering/Mechanical Engineering(Sandwich)/Mechanical and Automation Engineering/Mechatronics Engineering/Medical Electronics/Petrochemical Engineering/Production Engineering/Robotics and Automation/Safety and Fire Engineering/Biotechnology/Biotechnology and Biochemical Engineering/Food Technology/Petrochemical Technology/Petroleum Engineering/Pharmaceutical Technology)

(Also common to : PTMA 3351 – Transforms and Partial Differential Equations for Regulations 2023)

(Regulations 2021)

Time : Three hours　　　　　　　　Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Formulate the differential equation of all spheres whose center is $(a,0,0)$ and radius b.

2. Find the complementary function of the partial differential equation $(9D^2+6DD'+D'^2)z=e^{2x+y}$.

3. Find the convergent value of Fourier series of $f(x)=\begin{cases}5 & in \quad -1<x<0\\ x & in \quad 0<x<1\end{cases}$ at $x=0$.

4. It is given that the Euler Fourier Co-efficient $a_n=\frac{4}{n^2\pi^2}$ and $a_0=\frac{2}{3}$ of the function

$f(x)=\begin{cases}(1+x)^2 & in \quad -1<x\le 0\\ (1-x)^2 & in \quad 0\le x<1\end{cases}$. Sum the series $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\dots$

5. Classify the following partial differential equation :

$$\frac{\partial^2 u}{\partial x^2} - 2x\frac{\partial^2 u}{\partial x \partial y} + x^2\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y} = 0$$

6. A uniform rod of length 10 cm through which heat flow is insulated at its sides. The ends are kept at zero temperature and the initial temperature at the interior points of the bar is given by $10x - x^2$, $0 < x < 10$. Write the governing differential equation with boundary conditions that associates to the heat produced.

7. Find the Fourier Transform of Unit step function.

8. Write the Parseval's identity for Fourier transform, Fourier Sine and Cosine Transforms.

9. Find the Z Transform of $\{a^n\}$.

10. Formulate the difference equation for the sequence 0, 1, 1, 2, 3, 5, 8, 13,..... and also write the initial conditions.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Solve the partial differential equation $p^2 + q^2 = 4pq$. (8)

(ii) Solve $x^2p + y^2q + z^2 = 0$. (8)

Or

(b) Solve $(D^2 + 2DD' + D'^2)z = x^2y + e^{x-y}$. (16)

12. (a) Obtain the Fourier Series expansion for (16)

$f(x) = \begin{cases} x & 0 \le x \le \pi \\ 2\pi - x & \pi \le x \le 2\pi \end{cases}$, $f(x + 2\pi) = f(x)$. Hence sum the series

$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots$

Or

(b) (i) Determine whether the function $f(x) = \begin{cases} x+1 & -\pi < x < 0 \\ x-1 & 0 < x < \pi \end{cases}$ is odd or even and hence find its Fourier series expansion. (8)

(ii) Obtain the Fourier series expansion of $f(x) = \begin{cases} 1+x & -\pi < x < 0 \\ 1-x & 0 < x < \pi \end{cases}$. (8)

13. (a) A lightly stretched string with the end points $x=0$ and $x=L$ is initially in a position given by $u(x,0)=kx(L-x)$. If it is released from rest from that position, find the displacement $u(x,t)$ at any point of the string.

Or

(b) A rectangular plate with insulated surfaces is 10 cm wide and so long compared to its width the it may be considered as infinite in length without introducing an appreciable error. If the temperature along the short edge $y=0$ is given by $u(x,0)=\begin{cases} 20x & 0<x\le 5 \\ 20(10-x) & 5<x\le 10 \end{cases}$ and the two long edges $x=0$ and $x=10$ as well as other edge are kept at 0°C. Find the steady state temperature distribution in the plate.

14. (a) (i) It is known that the Fourier transform of $f(x)=\begin{cases} 1-x^2 & for \quad |x|<1 \\ 0 & otherwise \end{cases}$ is $\frac{4}{\sqrt{2\pi}}\left(\frac{\sin s - s\cos s}{s^3}\right)$ and the inverse Fourier transform $\mathcal{F}^{-1}\,[F(s)]=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{ist}F(s)ds$. Then evaluate $\int_0^{\infty}\left(\frac{\sin t - t\cos t}{t^3}\right)\cos\left(\frac{t}{2}\right)dt$. (8)

(ii) It is known that the Fourier transform $f(x)=e^{-a^2x^2}, a>0$ is $\frac{e^{-\frac{s^2}{4a^2}}}{a\sqrt{2}}$, then prove that $e^{-\frac{x^2}{2}}$ is self-reciprocal under Fourier Transform. In addition, find the Fourier Cosine Transform of $e^{-a^2x^2}$ and Fourier Sine Transform $xe^{-a^2x^2}$. (8)

Or

(b) Find the Fourier Transform of $\frac{\sin ax}{x}$ and hence prove that $\int_0^{\infty}\left(\frac{\sin ax}{x}\right)^2 dx = \frac{a\pi}{2}$, $a>0$.

15. (a) (i) State and prove change of scale of property in Z transform. (8)

(ii) Find the $Z\{n\}$ and hence find the $Z\{2^n n\}$. (8)

Or

(b) Solve the difference equation $u_{n+2}+2u_{n+1}+u_n=n$, $u_0=0$, $u_1=0$ using Z transform.