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B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2023.

Third/Fourth Semester

Biomedical Engineering

MA 3355 – RANDOM PROCESSES AND LINEAR ALGEBRA

(Common to : Electronics and Communication Engineering/ Electronics and Telecommunication Engineering/ Medical Electronics)

(Regulations – 2021)

Time : Three hours | Maximum : 100 marks

(Codes/ Tables/ Charts to be permitted, if any may be indicated: Normal table to be permitted.)

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. If A, B and C are any 3 events such that $P(A) = P(B) = P(C) = 1/4$, $P(A \cap B) = P(B \cap C) = 0$, $P(A \cap C) = 1/8$. Find the probability that atleast 1 of the events A, B and C occurs.

2. A continuous random variable X that can assume any value between $x = 2$ and $x = 5$ has a density function given by $f(x) = (2/27)(1+x)$. Find $P(X < 4)$.

3. State the properties of the distribution function of a two dimensional random variable (X, Y).

4. The life time of a certain brand of an electric bulb may be considered a random variable with mean 1200h and standard deviation 250h. Find the probability, using central limit theorem, that the average lifetime of 60 bulbs exceeds 1250h.

5. A radioactive source emits particles at a rate of 5 per minute in accordance with Poisson process. Each particle emitted has probability 0.6 of being recorded. Find the probability that 10 particles are recorded in 4-minute period.

6. State the discrete random sequence. Give an example.

7. Determine whether the vectors $\upsilon_1 = (1, -2, 3)$, $\upsilon_2 = (5, 6, -1)$ and $\upsilon_3 = (3, 2, 1)$ form a linearly dependent or a linearly independent set.

8. Does a line passing through origin of $\mathbb{R}^3$ is a subspace of $\mathbb{R}^3$?

9. State the dimension theorem in linear algebra.

10. Find the angle between two vectors $u = (4, 3, 1, -2)$ and $\upsilon = (-2, 1, 2, 3)$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) For a certain binary communication channel, the probability that a transmitted '0' is received as a '0' is 0.95 and the probability that a transmitted '1' is received as '1' is 0.90. If the probability that a '0' is transmitted is 0.4, find the probability that (1) a '1' is received and (2) a '1' was transmitted given that '1' was received. (8)

(ii) If X represents the outcome, when a fair dice is tossed, find the moment generating function of X and hence find $E(X)$ and $Var(X)$. (8)

Or

(b) (i) An irregular 6-faced dice is such that the probability that it gives 3 even numbers in 5 throws is twice the probability that it gives 2 even numbers in 5 throws. How many sets of exactly 5 trials can be expected to give no even number out of 2500 sets? (8)

(ii) In an engineering examination, a student is considered to have failed, secured second class, first class and distinction, according as he scores less than 45% between 45% and 60%, between 60% and 72% and above 75% respectively. In particular year 10% of the students failed in the examination and 5% of the students got distinction. Find the percentages of students who have got first class and second class. (Assume normal distribution of marks). (8)

12. (a) (i) The joint probability mass function of (X, Y) is given by

$p(x,y) = k\,(2x + 3y)$, $x = 0,1,2$; $y = 1,2,3$. Find all the marginal probability distributions. (8)

(ii) The joint probability density function of a two-dimensional random variable (X,Y) is given by $f(x,y) = xy^2 + \frac{x^2}{8}$, *if* $0 \le x \le 2, 0 \le y \le 1$.

Compute $P(X > 1)$, $P(Y < \frac{1}{2})$, and $P(X > 1 / Y < \frac{1}{2})$ (8)

Or

(b) If X and Y each follow an exponential distribution with parameter 1 and are independent, find the pdf of $U = X - Y$. (16)

13. (a) A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair dice and drove to work if and only if a 6 appeared. Find

(i) the probability that he takes a train on the third day

(ii) the probability that he drives to work in the long run. (16)

Or

(b) The transition probability matrix of a Markov chain $\{X_n\}$, n=1,2,3... having 3 states 1, 2, and 3 is $P = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$ and the initial distribution is $p^{(0)} = (0.7, 0.2, 0.1)$. Find

(i) $P(X_2 = 3)$ and

(ii) $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$. (16)

14. (a) Show that the set V of all 2×2 matrices with real entries is a vector space if addition is defined to be matrix addition and scalar multiplication is defined to be matrix scalar multiplication. (16)

Or

(b) (i) Find a basis for the space spanned by the vectors $v_1 = (1,-2,0,0,3)$, $v_2 = (2,-5,-3,-2,6)$, $v_3 = (0,5,15,10,0)$ and $v_4 = (2,6,18,8,6)$. (8)

(ii) Let $v_1 = (1,2,1)$, $v_2 = (2,9,0)$ and $v_3 = (3,3,4)$. Show that the set $S = \{v_1, v_2, v_3\}$ is basis for $\mathbb{R}^3$. (8)

15. (a) (i) Find the basis for the nullspace of $\begin{pmatrix} 2 & 2 & -1 & 0 & 1 \\ -1 & -1 & 2 & -3 & 1 \\ 1 & 1 & -2 & 0 & -1 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix}$. (8)

(ii) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation defined by $T\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ -5x_1 + 13x_2 \\ -7x_1 + 16x_2 \end{pmatrix}$. Find the matrix for the transformation T with respect to the basis $B = \{u_1, u_2\}$ for $\mathbb{R}^2$ and $B' = \{v_1, v_2, v_3\}$ for $\mathbb{R}^3$, where $u_1 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$, $u_2 = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$, $v_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$, $v_2 = \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$, $v_3 = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$. (8)

Or

(b) (i) Apply the Gram-Schmidt process to transform the basis vector $u_1 = (1,1,1)$, $u_2 = (0,1,1)$ and $u_3 = (0,0,1)$ into an orthonormal basis. (8)

(ii) Find the least squares solution of the linear system $Ax = b$ given by $x_1 - x_2 = 4$; $3x_1 + 2x_2 = 1$; $-2x_1 + 4x_2 = 3$. (8)