Reg. No. : E N G G T R E E . C O M

Question Paper Code : 21272

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2023.

First Semester

Civil Engineering

MA 3151 — MATRICES AND CALCULUS

(Common to : All Branches (Except Marine Engineering))

(Regulations 2021)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Find the eigenvalues of A^{-1} and A^2 if $A = \begin{pmatrix} 3 & 0 & 0 \\ 8 & 4 & 0 \\ 6 & 2 & 5 \end{pmatrix}$.

2. State Cayley-Hamilton theorem.

3. Sketch the graph of the function $f(x) = \begin{cases} x^2 & if\ -2 \le x \le 0 \\ 2 - x & if\ 0 < x \le 2 \end{cases}$.

4. The equation of motion of a particle is given by $s = 2t^3 - 5t^2 + 3t + 4$ where s is measured in meters and t in seconds. Find the velocity and acceleration as functions of time.

5. If $u = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$, find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$.

6. Write any two properties of Jacobians.

7. Evaluate $\int_0^{\frac{\pi}{2}} \sin^9 x\, dx$.

8. Prove that the integral $\int_1^{\infty} \frac{1}{x} dx$ is divergent.

9. Evaluate $\int_1^2 \int_1^3 xy^2 \, dx \, dy$.

10. Find the area of a circle $x^2 + y^2 = a^2$ using polar coordinates in double integrals.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigenvalues and eigenvectors of $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{pmatrix}$. (8)

(ii) Using Cayley-Hamilton theorem, find A^{-1} if $A = \begin{pmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$. (8)

Or

(b) Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy - 8yz + 4zx$ into the canonical form and hence find its rank, index, signature and nature. (16)

12. (a) (i) Let $f(x) = \begin{cases} \sqrt{-x} & if \ x < 0 \\ 3 - x & if \ 0 \le x \le 3 \\ (x-3)^2 & if \ x > 3 \end{cases}$. Evaluate each of the following limits, if they exist.

(1) $\lim_{x \to 0^-} f(x)$

(2) $\lim_{x \to 0^+} f(x)$

(3) $\lim_{x \to 3^-} f(x)$

(4) $\lim_{x \to 3^+} f(x)$

(5) $\lim_{x \to 0} f(x)$

(6) $\lim_{x \to 3} f(x)$

Also, find where $f(x)$ is continuous. (8)

(ii) Find the n^{th} derivative of $f(x) = xe^x$. (4)

(iii) Differentiate $F(t) = \frac{t^2}{\sqrt{t^3 + 1}}$. (4)

Or

(b) (i) Use logarithmic differentiation to differentiate $y = \dfrac{x^{3/2}\sqrt{x^2+1}}{(3x+2)^5}$. (8)

(ii) Discuss the curve $f(x) = x^4 - 4x^3$ for points of inflection, and local maxima and minima. (8)

13. (a) (i) Given the transformations $u = e^x \cos y$ and $v = e^x \sin y$ and that f is a function of u and v and also of x and y, prove that

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \left(u^2 + v^2\right)\left(\frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial v^2}\right) \quad (8)$$

(ii) Expand $e^x \cos y$ in a series of powers of x and y as far as the terms of the third degree. (8)

Or

(b) (i) Examine for extreme values of $f(x, y) = x^3 + y^3 - 12x - 3y + 20$. (8)

(ii) A rectangular box, open at the top is constructed so as to have a volume of 108 cubic meters. Find the dimensions of the box that requires the least material for its construction. (8)

14. (a) (i) Find a reduction formula for $\int e^{ax} \sin^n x \, dx$. (8)

(ii) Integrate the following : $\int \dfrac{x^4 - 2x^2 + 4x + 1}{x^3 - x^2 - x + 1} dx$. (8)

Or

(b) (i) Evaluate $\int \sqrt{\dfrac{1-x}{1+x}}\, dx$. (8)

(ii) Find the centre of mass of a semicircular plate of radius r. (8)

15. (a) (i) Change the order of integration in $\int_0^4 \int_{x^2/4}^{2\sqrt{x}} xy \, dy\, dx$ and then evaluate it. (8)

(ii) Find the area enclosed by the curves $y = 2x - x^2$ and $x - y = 0$. (8)

Or

(b) (i) Find the volume of the tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$. (8)

(ii) Find the moment of inertia of a hollow sphere about a diameter, given that its internal and external radii are 4 meters and 5 meters respectively. (8)
