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B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2023.

Fourth Semester

Aeronautical Engineering

MA 3452 — VECTOR CALCULUS AND COMPLEX FUNCTIONS

(Common to : Aerospace Engineering)

(Regulations 2021)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. The force field $\vec{F} = y\vec{i} + x\vec{j} + cz\vec{k}$ is solenoidal. Find the value of c.

2. Let $\vec{v} = e^x \cos y\vec{i} + e^x \sin y\vec{j} + v_3\vec{k}$ be the velocity of the fluid flow. For what value of v_3 the fluid flow will be incompressible?

3. Find the fixed points of the mapping $w = \frac{z-1}{z+1}$.

4. Find the points at which the mapping $w = z(z^4 - 5)$ is not conformal.

5. Is $\oint \bar{z}dz$ over the unit circle is zero? Why?

6. Write the singularities of the following function and classify it

   (a) $f(z) = e^{\frac{1}{z}}$ (b) $\frac{\sin z}{z}$

7. If $L\{f(t)\} = \frac{1}{s(s+a)}$, find the $\lim_{t\to\infty} f(t)$.

8. Derive Laplace Transform of Unit step function $u(t-a)$.

9. Find the general solution of the differential equation $(D^3 + 6D^2 - 11D + 6)y = 0$.

10. Reduce the differential equation $((1+x)^3 D^3 + 2(1+x)^2 D^2 - (1+x)D + I)y = (1+x)^{-2}$ into a linear differential equation with constant coefficients.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Use Stoke's theorem to evaluate $\int_C \vec{F}.d\vec{r}$ for $\vec{F} = xz\vec{i} + xy\vec{j} + 3xz\vec{k}$ where C is the boundary of the portions of the plane $2x + y + z = 2$ in the first octant traversed counterclockwise as viewed from above. (8)

(ii) Show that the function $\vec{F} = (x^2 + y)\vec{i} + (y^2 + z)\vec{j} + ze^z\vec{k}$ is conservative. Also find the corresponding potential function f such that $\vec{F} = \nabla f$. (8)

Or

(b) (i) Verify Divergence theorem for $\vec{F} = 4xy\vec{i} - y^2\vec{j} + yz\vec{k}$, taken over the cube bounded by the planes $x = 0$, $y = 0$, $z = 0$, $x = 1$, $y = 1$, $z = 1$. (12)

(ii) Use Green's theorem to find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (4)

12. (a) (i) Prove that an analytic function with constant modulus is constant. (8)

(ii) Verify $u = x^2 - y^2 - y$ is harmonic in the whole complex plane and find a harmonic conjugate function v of u. (8)

Or

(b) Find the linear fractional transformation that maps $z_1 = -1$, $z_2 = i$, $z_3 = 1$ onto $w_1 = 0$, $w_2 = i$, $w_3 = \infty$ respectively. Also show that unit disk is mapped onto right half plane by this transformation. (16)

13. (a) (i) Find all Taylor and Laurent series expansion of the function $f(z) = \frac{-2z + 3}{z^2 - 3z + 2}$ with center 0 over the regions (1) $|z| < 1$ (2) $1 < |z| < 2$ (3) $|z| > 2$. (8)

(ii) Integrate $\frac{\tan z}{z^2 - 1}$ counterclockwise around the circle $|z| = 3/2$. (8)

Or

(b) (i) Evaluate $\int_{0}^{2\pi} \frac{d\theta}{\sqrt{2} - \cos\theta}$. (8)

(ii) Using contour integration method, show that $\int_{0}^{\infty} \frac{dx}{1+x^4}$. (8)

14. (a) (i) Find the Laplace Transform of the half wave rectifier $f(t) = \begin{cases} \sin\omega t, & 0 \le t \le \frac{\pi}{\omega} \\ 0, & \frac{\pi}{\omega} \le t \le \frac{2\pi}{\omega} \end{cases}$. (8)

(ii) Find the inverse Laplace transform of the function $\ln\left(1 + \frac{\omega^2}{s^2}\right)$. (8)

Or

(b) (i) Using Laplace transform solve the differential equation $y'' - 3y' + 2y = e^{-t}$, $y(0) = 1$, $y'(0) = 0$. (8)

(ii) Find the Laplace inverse of the function $\frac{se^{-s}}{s^2 + \omega^2}$. (8)

15. (a) (i) Solve by method of variation of parameters the following differential equation $y'' + y = \sec x$. (8)

(ii) Find the general solution of differential equation $x^2 y'' + y = 3x^2$. (8)

Or

(b) (i) Solve the initial value problem $y'' + 5y' + 6y = 2x + 1$ with initial conditions $y(0) = 0$ and $y'(0) = 1/3$. (8)

(ii) Solve the simultaneous equations $\frac{dx}{dt} - 7x + y = 0$, $\frac{dy}{dt} - 2x - 5y = 0$. (8)
