Subject Title   : Power system analysis   **Subject Code** : EE3501

UNIT I - INTRODUCTION

Need for system planning and operational studies – basic components of a power system.- Introduction to restructuring - Single line diagram – per phase and per unit analysis – Generator - transformer – transmission line and load representation for different power system studies.- Primitive network - construction of Y-bus using inspection and singular transformation methods – z-bus.

PART – A

1. **Mention the requirements of planning the operation of power system**

   To monitor the voltage at various buses, real and reactive power flow between buses.

   To design the circuit breakers.

   To plan future expansion of the existing system

   To analyze the system under different fault conditions

   To study the ability of the system for small and large disturbances (Stability studies)

2. **What is the need for base values?**

   The components or various sections of power system may operate at different voltage and power levels. It will be convenient for analysis of power system if the voltage power, current and impedance ratings of components of power system are expressed with reference to a common value called base value. Hence for analysis purpose a base value is chosen for voltage, power, current and impedance. Then all the voltage, power, current and impedance ratings of the components are expressed as a percent or per unit of the base value.

3. **Define per unit value of an electrical quantity and write the equations for base impedance for a three phase power system.**   ,

   The per unit value of any quantity is defined as the ratio of the actual value of the quantity to the base value expressed as a decimal. The base value is an arbitrary chosen value of the quantity.

   The per unit value of base impedance for a three phase power system is as,

   $$Z_b = \frac{kV_b \times 1000}{\sqrt{3} I_b}$$

4. **Write the equation for per unit impedance if change of base occurs.**

   The equation for converting per unit impedance form one base to another can be given as follows.

   $$Z_{p.u.\ new} = Z_{pu,old} \times \frac{kV_{b,old}^2}{kV_{b,new}^2} \times \frac{MVA_{b,new}}{MVA_{b,old}}$$

5. **What are the advantages of per unit computation?**

   The advantages of per unit method over percent method is that the product of two Quantities expressed in per unit are expressed in per unit itself, but the product of two quantities

expressed in percent must be divided by100 to obtain the result in percent.

6. **A Y connected generator rated at 300 MVA, 33 KV has a reactance of 1.24 p.u. Find the Ohmic value of the reactance.**

$$\text{Per unit Value} = \frac{Actual\ value}{Base\ value}$$

   Actual Value = Pu value * base value

   $$= 1.24 * (33^2/300)$$
   $$= 4.5012\ \Omega$$

7. **State the advantages of per unit analysis.**

   The advantages of per unit representation are
   1. Per unit data representation yields valuable relative magnitude information.
   2. Circuit analysis of system containing transformers of various transformation ratio is greatly simplified.
   3. Circuit parameters tend to fall in relatively narrow numerical ranges making erroneous data east to spot.

8. **How are the loads represented in the reactance and impedance diagram?   (NOV/DEC 2016)**

   The loads are represented in reactance diagram with an internal emf in series with reactance and resistance.

   The same load is represented in impedance diagram as internal emf in series with reactance without resistance.

9. **What is single line diagram**

   Single line diagram is diagrammatic representation of power system in which the components are represented by their symbols and the interconnections between them are shown by a single straight line (even though the system is 3- phase system). The ratings and the impedances of the components are also marked on the single line diagram.

10. **Define per unit value.**

   The per unit value of any quantity is defined as the ratio of the actual value of the quantity to the base value expressed as a decimal. The base value is an arbitrary chosen value of the quantity.

   $$\text{Per unit Value} = \frac{Actual\ value}{Base\ value}$$

11. **Define Power System, Power System Analysis and Per Phase Analysis.**

   Power system

   The conveyance of electrical power from a power station to consumer premises is known as electrical power system.

   Power System Analysis

   The evaluation of power system is called as power system analysis.

   Per Phase Analysis.

   A balanced three phase system is always analyses on per phase basis by considering one of the three phase lines and neutral.

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12. **What are the components of power system?**

The various components of power system includes

Generators,

Power Transformers,

Transmission lines,

Substation Transformers,

Distribution Transformers and

Loads.

13. **What are the main divisions of power system?**

If a sudden change or sequence of changes occurs in one or more of the system

parameters or one or more of its operating quantities, the system is said to have undergone a disturbance from its steady state operating condition.

The two types of disturbances in a power system are,

Large disturbance

Small disturbance

14. **What is a small disturbance? Give example.**

If the power system is operating in a steady state condition and it undergoes

Change, which can be properly analyzed by linearized versions of its dynamic and algebraic equations, a small disturbance is said to have occurred.

Example of small disturbance is a change in the gain of the automatic voltage regulator in the excitation system of a large generating unit.

15. **What is a large disturbance? Give some examples.**

A large disturbance is one for which the nonlinear equations describing the dynamics of the power system cannot be validly linearized for the purpose of analysis.

Examples of large disturbances are transmission system faults, sudden load changes, loss of generating units and line switching.

16. **What are the assumptions for transient stability?**

The assumptions to be followed for the transient stability are as follows.

Generators are represented by the constant internal voltage behind transient reactance.

The turbine mechanical power outputs are assumed to be constant and the governor corrective action is ignored.

All resistance is neglected.

Damping is neglected.

17. **When is a power system said to be transiently stable?**

Transient stability is defined as the ability of the power system to remain in synchronism under large disturbance conditions, such as fault and switching operations. The maximum power transfer limit is less than that of the steady state condition.

If the machines of the system are found to remain essentially in synchronism within the first second following a system fault or other large disturbance, the system is considered to be transiently stable.

18. **What is the objective of short circuit study?**

The objective of the short circuit analysis is to precisely determine the currents and voltages at the

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different locations of the power system corresponding to the different types of faults, such as three phase to ground fault, line to ground fault, line to line fault, double line to ground fault and open conductor fault. The data is used to select fuses, protective relays and circuit breakers to rescue the system from the abnormal condition. The symmetrical components and sequence networks are used in the analysis of unsymmetrical faults.

19. **What is a bus?**

    The meeting point of various components in a power system is called a bus. The bus is a conductor made of copper or aluminum having an eligible resistance. The buses are considered as points of constant voltage in a power system.

    Types of bus includes

    Slack bus,

    Generator Bus,

    Load Bus,

20. **State the need for per unit value. (or) What is the need of per unit**

    The needs of per unit value are stated as follows.

    The per unit impedance referred to either side of a single phase transformer is the same.

    The chance of confusion between line and phase quantities in a three phase balanced system is greatly reduced.

21. **What are the advantages of per-unit computations?**

    1. Manufacturers usually specify the impedance of a device or machine in per unit on the base of the name plate rating.
    2. The p.u values of widely different rating machines lie within a narrow range, even though the ohmic value has a very large range.
    3. The p.u. impedance of circuit element connected by transformers expressed on a proper base will be same if it is referred to either side of a transformer.

    The p.u. impedance of a 3-phase transformer is independent of the type of winding connection (Y or Δ).

22. **A generator rated at 30 MVA, 11 kV has a reactance of 20%. Calculate its p.u. Reactance's for a base of 50 MVA and 10kV.**
    Solution.

    New p.u. reactance if generator = $X_{pu,old} \times \dfrac{kV_{b,old}^2}{kV_{b,new}^2} \times \dfrac{MVA_{b,new}}{MVA_{b,old}}$

    Here, $X_{pu,old}$=20%=0.2 p.u.; $kV_{b,old}$= 11kV, $MVA_{b,\,old}$=30MVA, $kV_{b,new}$=10kV; $MVA_{b,\,new}$=50MVA

    New p.u. reactance of generator=$0.2 \times (\dfrac{11}{10})^2 \times \dfrac{50}{30} = 0.403$ p.u

23. **What is impedance and reactance diagram?**

    The impedance diagram is the equivalent circuit of power system in which the various components of power system are represented by the approximate or simplified equivalent circuits.

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The impedance diagram is used for load flow studies. The reactance diagram is the simplified equivalent circuit of power system in which the various components are represented by their reactance. The reactance diagram can be obtained for impedance diagram fall the resistive components are neglected. The reactance diagram is used for fault calculations.

24. **What are the approximations made in impedance diagram?**

The following approximations are made in impedance diagram.

The neutral reactance's are neglected.

Shunt branches in the equivalent circuits of transformer are neglected

The resistances are neglected.

All static loads and induction motors are neglected.

The capacitances of the transmission lines are neglected.

25. **Whatis busadmittancematrix?**

The matrix consisting of the self and mutual admittances of the network of a power system is called bus admittance matrix. It is given by the admittance matrix in the node basis matrix equation of a power system and it is denoted as Y_{bus}.

The bus admittance matrix is symmetrical. Inverse of bus impedance matrix is the bus admittance matrix.

26. **What is bus impedance matrix?**

The matrix consisting of driving point impedances and impedances of the network of a power system is called bus impedance matrix. It is given by the inverse of bus admittance matrix and it is denoted as Z_{bus}.

The bus impedance matrix is symmetrical matrix.

27. **How the Z_{bus} is modified when a branch of impedance Z_b is added from a new bus-p to the reference bus?**

When a branch of impedance Z_b is added from a new bus-p to the reference bus, the order of the bus impedance matrix increases by one.

Let the original bus impedance matrix have an order of n and so the new bus impedance matrix has an order of (n+1).The first nx n sub matrix of new bus impedance matrix is the original bus impedance matrix. The elements of $(n+1)^{th}$ column and row are all zeros except the diagonal. The (n+1) diagonal element is the added branch impedance Z_b.

28. **What are symmetrical components?**

An unbalanced system of N related vectors can be resolved in to N systems of balanced vectors. The N–sets of balanced vectors are called symmetrical components. Each set consists of N–vectors which are equal in length and having equal phase angles between adjacent vectors.

29. **What are sequence impedance and sequence networks?**

The sequence impedances are impedances offered by the devices or components for the like sequence component of the current. The single phase equivalent circuit of a power system consisting of impedances to the current of any one sequence only is called sequence network.

30. **List out the major stages in a single line diagram of a power system.**

The different stages of power in power system are

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Primary transmission

Secondary transmission

Primary distribution

Secondary distribution

31. **Give the formula to calculate base current, I_b and base impedance of a three- phase system.**

The equation for base current I_b is,

$$I_b = \frac{KVA_b}{\sqrt{3}\ KV_b}$$

The equation for base impedance is,

$$Z_b = \frac{kV_b \times 1000}{\sqrt{3}I_b}$$

Where

I_b=Line value of base current.

kVA_b=3-phasebaseKVA

kV_b=line to line base kV

Z_b=Base impedance per phase.

32. **What is the advantage of per unit method over percent method?**

Theadvantageofperunitmethodoverpercentmethodisthattheproductoftwo

Quantities expressed in per unit are expressed i n per unit itself, but the product of two quantities expressed in percent must be divided by100 to obtain theresult inpercent.

33. **What is the need for base values?**

Thecomponentsofvarioussectionsofpowersystemmayoperateatdifferent

Voltage and power levels. It will be convenient for analysis of power system if the voltage, power, current and impedance ratings of power system components are expressed with reference to a common value called base value. Then all the voltages, power, current and impedance ratings of the components are expressed as a percent or per unit of the base value.

34. **Why the three phase kVA is directly used for per unit calculation in three phase systems?**

The per unit value of a 3-phase kVA on the 3-phase kVA base is identical to the per unit value of kVA per phase on the kVA per phase base.

$$\frac{3\ phase\ KVA}{3\ phase\ base\ KVA} = \frac{KVA\ per\ phase}{base\ KVA\ per\ phase}$$,

Thereforein3phasesystems,thelinevalueofvoltageand3phasekVAare directly used for per unit calculations.

35. **Give the equation for transforming base kV on LV side to HV side of a transformer and vice versa.**

$$\text{Base kV on HVside} = \text{Base kV on LV side} \times \frac{HV\ rating}{LV\ rating}$$

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$$\text{Base kV on LV side} = \text{Base kV on HV side} \times \frac{\text{LV rating}}{\text{HV rating}}$$

36. **List the methods of improving the transient stability limit of a power system.**

    The methods of improving the transient stability limit of a power system are listed as follows.

    (1) Increase of system voltage, use of AVR.

    (2) Use of high speed excitation systems.

    (3) Reduction in system transfer reactance.

    (4) Use of high speed reclosing breakers.

37. **Give the equation for load impedance and load admittance per phase of a balanced star connected load.**

    Load impedance per phase, $Z = \dfrac{|V_L|}{P - jQ}$ Load admittance per phase, $= \dfrac{P - jQ}{|V_L|^2}$

    Where,

    P = Three phase active power of star connected load in watts.

    Q = Three phase reactive power of star connected load in VARs.

    V_L = Line voltage of load.

38. **What are the methods available for forming bus impedance matrix?**

    Form the bus impedance matrix and then take its inverse to get bus

    1. Impedance Matrix.

    2. Directly from the bus impedance matrix from the reactance diagram. This Method utilizes the techniques of modifications of existing bus impedance Matrix due to addition of new bus.

39. **Name the diagonal and off diagonal elements of Bus Impedance Matrix.**

    Bus Admittance Matrix.

    The diagonal elements of bus admittance matrix are called self-admittances of the Buses and off diagonal elements are called mutual admittances of the buses.

    Bus Impedance Matrix.

    The diagonal elements of bus impedance matrix are called driving point impedances of the buses and off diagonal elements of bus impedance matrix are called transfer Impedances of the buses.

40. **Mention the advantages of bus admittance matrix, Y$_{bus}$.**

    The advantages if bus admittance matrix is listed as follows.

    i) Data preparation is simple.

    ii) Formation and modification is easy.

    iii) Since the bus admittance matrix is sparse matrix(i.e., most of its elements are zero), the computer memory requirements are less.

41. **What are the considerations used to select base values?**

    Selection of Base MVA.

    First a base value is chosen for the network.

    The same MVA will be used in all parts of the system.

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It may be the largest MVA of a section, or total MVA of the system or any value like 10,100,1000 MVA etc.

Selection of Base KVA.

The rated voltage of the largest section may be taken as base Selection of Base MVA.

The base voltages of remaining sections assigned, depends on the turns ratio of the transformer.

42. Prove the per unit impedance of the transformer referred to the primary side is equal to the per unit impedance referred to secondary side.

Let the impedance of the transformer referred to primary side be Z_P and that on secondary side be Z_S then,

$Z_{P=} Z_S (V_P / V_S)^2$

Where, V_P and V_S are the primary and secondary voltage of the transformer.

Z_P p.u $= (I_P \ Z_P / V_P)$

$= Z_S (V_P / V_S)^2 (I_P / V_P)$

$^{(i)} = Z_S \ I_P \ V_P / V_S^2$

i) $= Z_{S.} (Is \ Vs / V_S^2)$

ii) $= Z_S \ I_S / V_S = Z_S$ p.u

Therefore Z_P p.u $= Z_S$ p.u

43. A generator rated at 30MVA, 11KV has a reactance of 20%.Calculate its per unit reactance for a base of 50 MVA and 10KV.

MVA $_{new}$ = 50; KV $_{new}$ = 10 ; MVA $_{old}$ = 30; KV $_{old}$ = 11

X $_{p.u}$ = 20% = 20/100 = 0.2 p.u

$$X_{p.u,new} = X_{p.u,old} \times \left[\frac{Base \ KV_{old}}{Base \ KV_{new}} \right]^2 \times \left[\frac{Base \ MVA_{new}}{Base \ MVA_{old}} \right]$$

$$X_{p.u,new} = j0.2 \times \left[\frac{11}{10} \right]^2 \times \left[\frac{50}{30} \right] = j0.4033 \ p.u$$

44. What is the new p.u impedance if the new base MVA is twice the old base MVA?

$$MVA_{new} = 2 \ MVA_{old}$$

$$Z_{p.u,new} = Z_{p.u,old} \times \left[\frac{Base \ KV_{old}}{Base \ KV_{new}} \right]^2 \times \left[\frac{Base \ MVA_{new}}{Base \ MVA_{old}} \right]$$

$$Z_{p.u,new} = Z_{p.u,old} \times \left[\frac{Base \ KV_{old}}{Base \ KV_{new}} \right]^2 \times \left[\frac{2 \ Base \ MVA_{old}}{Base \ MVA_{old}} \right]$$

45. Write the equation for base impedance and per unit impedance if change of base occurs.

Base Impedance

The equation for base impedance is given as follows

$$Base \ Impedance = \frac{(Base \ KV)^2}{Base \ MVA}$$

Per unit impedance if change of base occurs.

The equation for per unit impedance if change of base occurs.

$$Z_{p.u,new} = Z_{p.u,old} \times \left[\frac{Base \ KV_{old}}{Base \ KV_{new}} \right]^2 \times \left[\frac{Base \ MVA_{new}}{Base \ MVA_{old}} \right]$$

46. Why bus admittance matrix is preferred in load flow?

Bus admittance matrix is preferred in load flow problem because,

It is easy to formulate.

No need of taking inverse.

Computation time is less.

Matrix is symmetric, so calculation of upper or lower triangular matrix is enough.

Each bus is connected to only a few nearby buses.so many diagonal elements are zero.

47. Distinguish between impedance and reactance diagram

The resistive and reactive loads can be represented by any one of the following representation.

Constant power representation.

Load power, $S = P + JQ$.

Constant current representation.

Load current. , $I = \dfrac{\sqrt{P^2 + Q^2}}{|V|} \angle \delta - \theta$

Constant impedance representation.

Load current, $Z = \dfrac{|V|^2}{P - jQ}$

48. Give the methods available for forming bus impedance matrix.

The three main methods available in forming bus impedance matrix are as follows.

Form the bus admittance matrix and take the inverse to get bus impedance matrix.

Using bus building algorithm.

Using L-U factorization of Y-bus matrix.

49. List out the application of Y-Bus.

The application of Y-bus matrix is as follows.

Y-bus is used in solving load flow problems.

It has gained applications owing to the simplicity in data preparation.

It can be easily formed and modified for changes in the network.

It reduces computer memory and time requirements because of sparse matrix.

50. What are the various methods to form Y-Bus matrix by singular transformation?

Various methods to form Y-Bus matrix by singular transformation are

Formation of network.

Formation of Graph.

Formation of oriented Graph.

Formation of Loop.

Formation of Tree.

Formation of Link or chord.

PART B

1. **In the single line diagramshown in fig. 1 each three phase generator G is rated at 200 MVA, 13.8 Kv and has a reactance of 0.85 pu and are generating 1.15 pu. Transformer T1 is rated at 500 MVA, 13.5 KV/220 KVA and has a reactance of 8%. The transmission line has a reactance of 7.8 ohm. T=Transformer T2 has a rating of 4010 MVA, 22 KV/33 KV and a reactance of 11%. The load is 250 MVA at a power factor of 0.85 lag. Convert all quantities to a common base of 500MVA and 220 KV on the line and draw the circuit diagram with values expressed in pu.** (APR/MAY 2018)
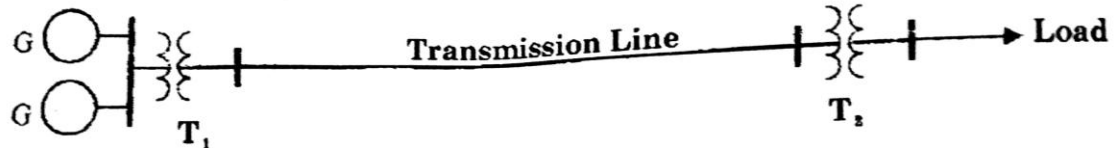
2. **A 200 MVA, 13.8 KV generator has a reactance of 0.85pu. and is generating 1.15 pu voltage. Determine the actual value of the line voltage, phase voltage and reactance.**

(APR/MAY 2018)

3. **Determine Z-bus for system whose reactance diagram is shown in given fig. where the impedance is given in p.u, preserve all the nodes.** **(APR/MAY 2018)**
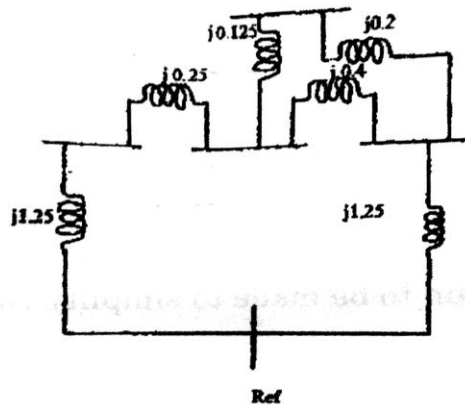
4. **Draw the reactance diagram for the power system shown infig. Neglect resistance and use a base value of 50 MVA and 13.8 KV on generator G1.** **(Nov/Dec 2015, 2017)**
   Generator, G_1=20 MVA, 13.8 kV, X"= 20%
   Generator, G_2=30 MVA, 18.0 kV, X"= 20%
   Generator, G_3=30 MVA, 20.0 kV, X"= 20%
   Transformer, T_1 = 25 MVA, 220/13.8 kV, X=10%
   Transformer, T_2 = 3 single phase unit rated 10 MVA, 127/18 kV, X=10%
   Transformer, T_3 = 35 MVA, 220/22 kV, X=10%
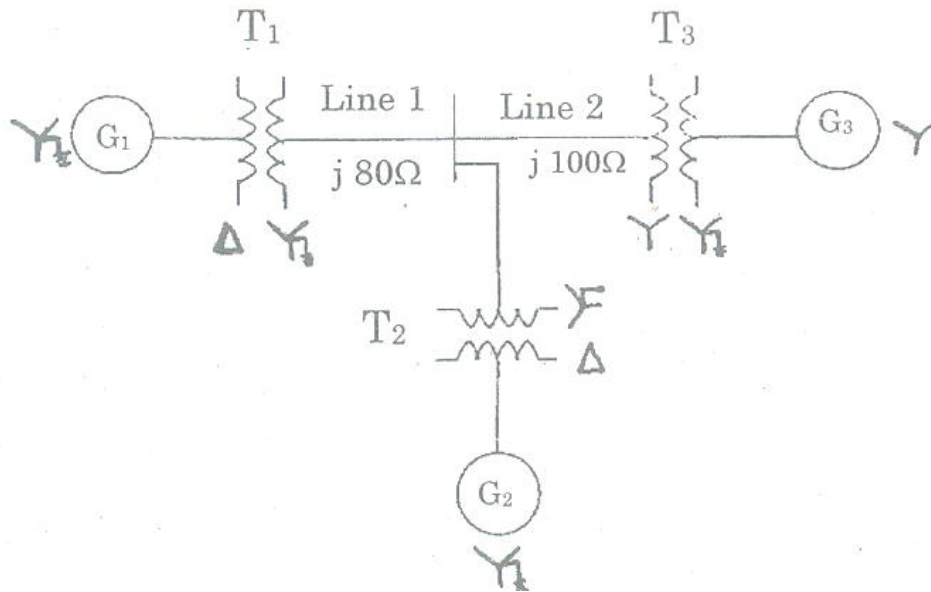
Determine the new values of per unit reactance of G1, T1, transmission line Transmission line 2, G2, T2, G3 and T3.

Solution.

Choose

$KV_{b,new}$

$MVA_{b,new}$

Generator 1,2 & 3:

$$Z_{p.u.\ new} = Z_{pu,given} \times \frac{kV_{b,given}^2}{kV_{b,new}^2} \times \frac{MVA_{b,new}}{MVA_{b,given}}$$

$$=$$

Transformer, T_1(Py):

$$Z_{p.u.\ new} = Z_{pu,given} \times \frac{kV_{b,given}^2}{kV_{b,new}^2} \times \frac{MVA_{b,new}}{MVA_{b,given}} = j0.2\ p.u$$

Transmission Line:

Transformer secondary side change occurs, so calculate $KV_{b,new}$ as

$$KV_{b,new} = KV_{b,old} * \left(\frac{H.T\ side\ rating\ of\ T_1}{L.T\ side\ rating\ of\ T_1} \right)$$

$$Z_{p.u} = \left(\frac{Z_{actual}}{kV_b^2} \right) \times MVA_b$$

Transformer, T_2(Sy):

$$Z_{p.u.\ new} = Z_{pu,given} \times \frac{kV_{b,given}^2}{kV_{b,new}^2} \times \frac{MVA_{b,new}}{MVA_{b,given}} = j0.2\ p.u$$

Transformer, T_3(Sy):

$$Z_{p.u.\ new} = Z_{pu,given} \times \frac{kV_{b,given}^2}{kV_{b,new}^2} \times \frac{MVA_{b,new}}{MVA_{b,given}} = j0.2\ p.u$$

Load, M : Transformer secondary side change occurs, so calculate $KV_{b,new}$ as

$$KV_{b,new} = KV_{b,old} * \left(\frac{L.T \ side \ rating \ of \ T_2}{H.T \ side \ rating \ of \ T_2} \right)$$

$$Z_{p.u. \ new} = Z_{pu,given} \ x \ \frac{kV_{b,given}^2}{kV_{b,new}^2} \ x \ \frac{MVA_{b,new}}{MVA_{b,given}}$$

5. **Describe Z bus building algorithm in details by using a three bus system.   (NOV/DEC 2017)**

6. **300 MVA, 20 kv, 3 Φ generator has sub transient reactance of 20%. The generator supplies 2 synchronous motors through a 64 KM transmission line having transformer at both ends as shown. In this , T1 is a 3 Φ transformer 350MVA, 20/230 KV, 10% reactance & T2 is made of 3 single phase transformer of rating 100 MVA, 127/13.2 KV, 10% reactance.**

    (MAY/JUNE 2017)

    Series reactance of the transmission line is 0.5Ω/KM. The rating of 2 motors are M1 = 200 MVA, 13.2 KV, & M2 = 100 MVA, 13.2 KV, 20%. Draw the reactance diagram with all the reactance marked in p.u. select the generator rating as a base value.
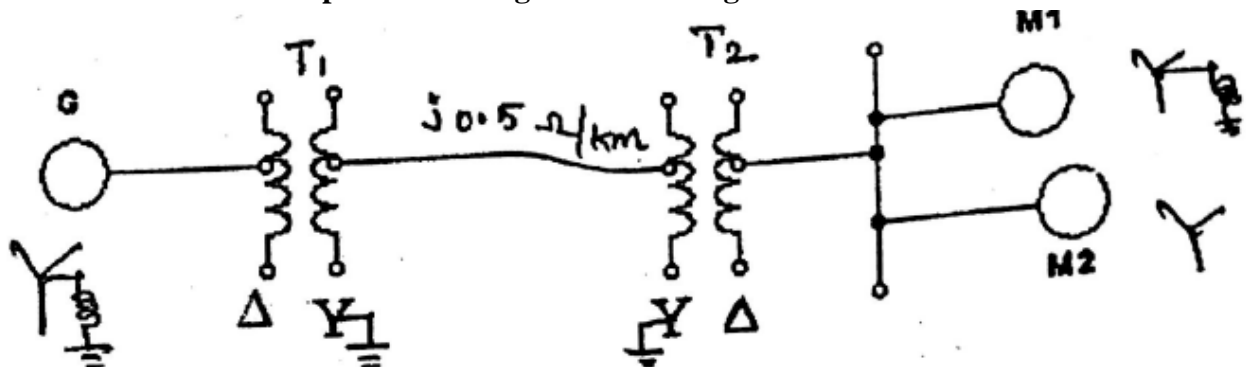
Solution:

Choose $MVA_{b,new}$

Choose $kV_{b,new}$

Generator:

$$Z_{p.u.\ new} = Z_{pu,given} \times \frac{kV_{b,given}^2}{kV_{b,new}^2} \times \frac{MVA_{b,new}}{MVA_{b,given}}$$

Transformer T_1 referred to Primary side:

Transformer T_1 Primary side change occurs, so calculate $KV_{b,new}$ as

$$Z_{p.u.\ new} = Z_{pu,given} \times \frac{kV_{b,given}^2}{kV_{b,new}^2} \times \frac{MVA_{b,new}}{MVA_{b,given}}$$

Transformer T_1 referred to Primary side:

$KV_{b,new} = 11\ kV$

$$Z_{p.u.\ new} = Z_{pu,given} \times \frac{kV_{b,given}^2}{kV_{b,new}^2} \times \frac{MVA_{b,new}}{MVA_{b,given}}$$

j 0.5Ω/KM line :

Transformer T_3 Secondary side change occurs, so calculate $KV_{b,new}$ as

$$KV_{b,new} = KV_{b,old} * \left(\frac{H.T\ side\ rating\ of\ T_3}{L.T\ side\ rating\ of\ T_3} \right)$$

$$Z_{p.u\ new} = \frac{Z_{actual}}{Z_{base}} = \frac{Z_{actual}}{(kV_b^2)} \times MVA_b$$

Transformer T_5 referred to Primary side:

$KV_{b,} = 66\ kV$

$$Z_{p.u.\ new} = Z_{pu,given} \times \frac{kV_{b,given}^2}{kV_{b,new}^2} \times \frac{MVA_{b,new}}{MVA_{b,given}}$$

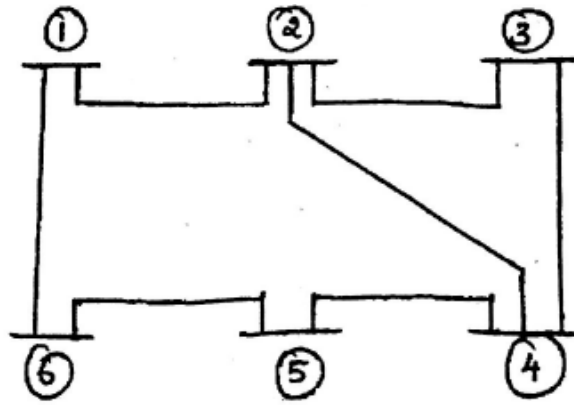**Motor :** $\quad KV_{b,new} = KV_{b,old} * \left(\frac{L.T\ side\ rating\ of\ T_5}{H.T\ side\ rating\ of\ T_5} \right)$

$$Z_{p.u.\ new} = Z_{pu,given} \times \frac{kV_{b,given}^2}{kV_{b,new}^2} \times \frac{MVA_{b,new}}{MVA_{b,given}} = j\ 0.225\ p.u$$

7. **Form bus admittance matrix for the data given below using singular transformation method. Take node 6 as reference node.** **(MAY/JUNE 2017)**

ELEMENTS	BUS CODE	X (p.u)
1	1-2	0.04
2	1-6	0.06
3	2-4	0.03
4	2-3	0.02
5	3-4	0.08
6	4-5	0.06
7	5-6	0.05

Solution:

Solution:

The Y_{bus} Matrix of the network is

The elements of new bus matrix after eliminating

$$Y_{jknew} = Y_{jk} - \left(\frac{Y_{jn}.Y_{nk}}{Y_{nn}}\right), \text{ where, } n=4, j=1,2,3 , k=1,2,3.$$

The bus admittance matrix is symmetrical. $\therefore Y_{kjnew} = Y_{jknew}$

$$Y_{11new} = Y_{11} - \frac{Y_{14}.Y_{41}}{Y_{44}} = -j1.3 - \frac{(j0.4).(j0.4)}{-j0.9} = -j1.12$$

$$Y_{12new} = Y_{12} - \frac{Y_{14}.Y_{42}}{Y_{44}} = j0.5 - \frac{(j0.4).(0)}{-j0.9} = j0.5$$

$$Y_{13new} = Y_{13} - \frac{Y_{14}.Y_{43}}{Y_{44}} = j0.4 - \frac{(j0.4).(j0.5)}{-j0.9} = j0.622$$

$$Y_{21new} = Y_{12new} = j0.5$$

$$Y_{22new} = Y_{22} - \frac{Y_{24}.Y_{42}}{Y_{44}} = -j1.1 - \frac{(0).(0)}{-j0.9} = -j1.1$$

$$Y_{23new} = Y_{23} - \frac{Y_{24}.Y_{43}}{Y_{44}} = 0.6 - \frac{(0).(0.5)}{-0.9} = j0.6$$

$$Y_{31new} = Y_{13new} = j0.622$$

$$Y_{32new} = Y_{23new} = j0.6$$

$$Y_{33new} = Y_{33} - \frac{Y_{34}.Y_{43}}{Y_{44}} = -j1.5 - \frac{(j0.5).(j0.5)}{-j0.9} = -j1.222$$

The reduced bus admittance matrix after eliminating $n+1^{th}$ row is in order $5*5$

8. **Prepare a per phase schematic of the system shown and show all the impedance in per unit on a 100 MVA, 132 KV, base in the transmission line circuit. The necessary data are given as follows:**
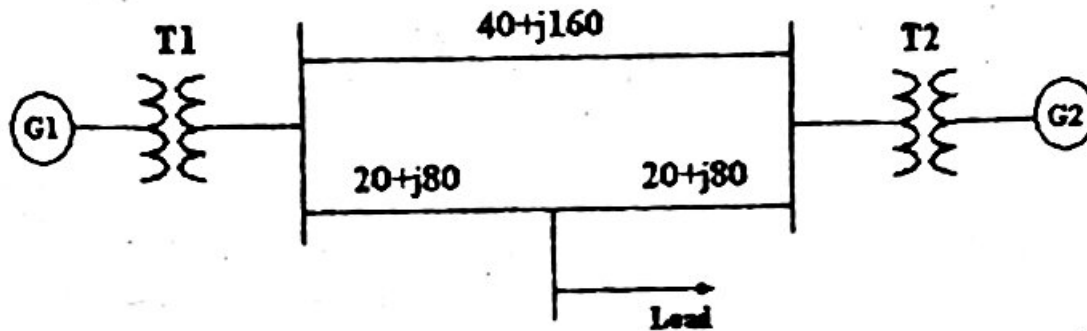
   G1 : 50 MVA, 12.2 KV, X = 0.15 pu

   G2 : 20 MVA, 13.8 KV, X = 0.15 ohms

T1 : 80 MVA, 12.2/161 KV, X = 0.1 pu
T2 : 40 MVA, 13.8/161 KV, X" = 16.0 pu
Load : 50 MVA, 0.8 pf lag operating at 154 KV
Determine the pu impedance of the load.

Solution.

Choose

$KV_{b,new}$

$MVA_{b,new}$

Generator:

$$Z_{p.u.\ new} = Z_{pu,given} \times \frac{kV_{b,given}^2}{kV_{b,new}^2} \times \frac{MVA_{b,new}}{MVA_{b,given}}$$

$$=$$

Transformer, T_1(Py):

$$Z_{p.u.\ new} = Z_{pu,given} \times \frac{kV_{b,given}^2}{kV_{b,new}^2} \times \frac{MVA_{b,new}}{MVA_{b,given}} = j0.2 \text{ p.u}$$

Transmission Line:

Transformer secondary side change occurs, so calculate $KV_{b,new}$ as

$$KV_{b,new} = KV_{b,old} * (\frac{H.T \text{ side rating of } T_1}{L.T \text{ side rating of } T_1})$$

$$Z_{p.u} = (\frac{Z_{actual}}{kV_b^2}) \times MVA_b$$

Transformer, T_2(Sy):

$$Z_{p.u.\ new} = Z_{pu,given} \times \frac{kV_{b,given}^2}{kV_{b,new}^2} \times \frac{MVA_{b,new}}{MVA_{b,given}} = j0.2 \text{ p.u}$$

Load, M : Transformer secondary side change occurs, so calculate $KV_{b,new}$ as

$$KV_{b,new} = KV_{b,old} * (\frac{L.T \text{ side rating of } T_2}{H.T \text{ side rating of } T_2})$$

$$Z_{p.u.\ new} = Z_{pu,given} \times \frac{kV_{b,given}^2}{kV_{b,new}^2} \times \frac{MVA_{b,new}}{MVA_{b,given}}$$

9. **The parameters of a 4-bus system are as under:**

Line starting bus	Line ending bus	Line Impedance	Line charging admittance
1	2	0.2+j0.8	j0.02
2	3	0.3+j0.9	J0.03
2	4	0.25+j1.0	J0.04
3	4	0.2+j0.8	J0.02
1	3	0.1+j0.4	J0.01

Draw the network and find bus admittance matrix.

Solution:

The Y_{bus} Matrix of the network is

$$Y_{jknew=}Y_{jk} - \left(\frac{Y_{jn}.Y_{nk}}{Y_{nn}} \right), \text{ where, n=4, j=1,2,3 , k=1,2,3.}$$

The bus admittance matrix is symmetrical. $\therefore Y_{kjnew} = Y_{jknew}$

$$Y_{11new} = Y_{11} - \frac{Y_{14}.Y_{41}}{Y_{44}} = -j1.3 - \frac{(j0.4).(j0.4)}{-j0.9} = -j1.12$$

$$Y_{12new} = Y_{12} - \frac{Y_{14}.Y_{42}}{Y_{44}} = j0.5 - \frac{(j0.4).(0)}{-j0.9} = j0.5$$

$$Y_{13new} = Y_{13} - \frac{Y_{14}.Y_{43}}{Y_{44}} = j0.4 - \frac{(j0.4).(j0.5)}{-j0.9} = j0.622$$

$$Y_{21new} = Y_{12new} = j0.5$$

$$Y_{22new} = Y_{22} - \frac{Y_{24}.Y_{42}}{Y_{44}} = -j1.1 - \frac{(0).(0)}{-j0.9} = -j1.1$$

$$Y_{23new} = Y_{23} - \frac{Y_{24}.Y_{43}}{Y_{44}} = 0.6 - \frac{(0).(0.5)}{-0.9} = j0.6$$

$$Y_{31new} = Y_{13new} = j0.622$$

$$Y_{32new} = Y_{23new} = j0.6$$

$$Y_{33new} = Y_{33} - \frac{Y_{34}.Y_{43}}{Y_{44}} = -j1.5 - \frac{(j0.5).(j0.5)}{-j0.9} = -j1.222$$

The reduced bus admittance matrix after eliminating 4th row is shown below

10. The data for the system whose single line diagram shown

G1 : 30 MVA, 10.5 KV, X" = 1.6 ohms

G2 : 15 MVA, 6.6 KV, X" = 1.2 ohms

G3 : 25 MVA, 6.6 KV, X" = 0.56 ohms

T1 : 15 MVA, 33/11 KV, X" = 15.62 ohms/phase on H.T side

T2 : 15 MVA, 33/6.2 KV, X" = 16.0 ohms/phase on H.T side

Transmission line : X = 20.5 ohms/phase

Loads : A : 40 MW, 11 KV, 0.9 pf lagging

B : 40 MW, 6.6 KV, 0.85 pf lagging

Choose the base power as 30 MVA and approximate base voltages for different parts. Draw the reactance diagram, indicate pu reactance on the diagram.

Solution.

Choose Base values

$KV_{b,new}$, $MVA_{b,new}$

For Generator, transformer and load:

$$Z_{p.u.\ new} = Z_{pu,given} \times \frac{kV_{b,given}^2}{kV_{b,new}^2} \times \frac{MVA_{b,new}}{MVA_{b,given}}$$

$$=$$

Transmission Line:

Transformer secondary side change occurs, so calculate $KV_{b,new}$ as

$$KV_{b,new} = KV_{b,old} * \left(\frac{H.T\ side\ rating\ of\ T_1}{L.T\ side\ rating\ of\ T_1} \right)$$

$$Z_{p.u} = \left(\frac{Z_{actual}}{kV_b^2} \right) \times MVA_b$$

Load, M : Transformer secondary side change occurs, so calculate $KV_{b,new}$ as

$$KV_{b,new} = KV_{b,old} * \left(\frac{L.T\ side\ rating\ of\ T_2}{H.T\ side\ rating\ of\ T_2} \right)$$

$$Z_{p.u.\ new} = Z_{pu,given} \times \frac{kV_{b,given}^2}{kV_{b,new}^2} \times \frac{MVA_{b,new}}{MVA_{b,given}}$$

9. Form Y_{bus} of the test system shown in fig using singular transformation method. The impedance data is given in Table Take (1) as reference node.
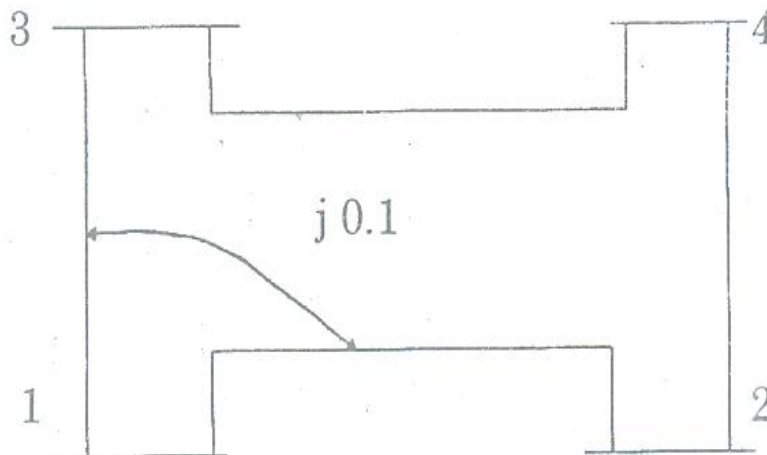
Table

Element No	self		Mutual	
	Bus code	**Impedance**	**Bus code**	**Impedance**
1	**1-2**	**0.5**	**1-2**	**0.1**
2	**1-3**	**0.6**		
3	**3-4**	**0.4**		
4	**2-4**	**0.3**		

Solution:

Let us first eliminate 4th bus. $\therefore Y_{nn} = Y_{44} = -j18.0$ **(2)**

The elements of new bus admittance after eliminating 4th row and 4th column is given by,

$$Y_{jknew} = Y_{jk} = \frac{Y_{jn}.Y_{nk}}{Y_{nn}} \text{, Where, n=4, j=1,2,3 , k=1,2,3.} \qquad \textbf{(6)}$$

$$Y_{11new} = Y_{11} - \frac{Y_{14}.Y_{41}}{Y_{44}} = -j9.8 - \frac{(j5.0).(j5.0)}{-j18.0} = -j8.411$$

$$Y_{12new} = Y_{12} - \frac{Y_{14}.Y_{42}}{Y_{44}} = 0 - \frac{(j5.0).(j5.0)}{-j18.0} = j1.388$$

$$Y_{13new} = Y_{13} - \frac{Y_{14}.Y_{43}}{Y_{44}} = j4.0 - \frac{(j5.0).(j8.0)}{-j18.0} = j6.222$$

$$Y_{21new} = Y_{12new} = j1.3888$$

$$Y_{22new} = Y_{22} - \frac{Y_{24} - Y_{42}}{Y_{44}} = -j8.3 - \frac{(j5.0).(j5.0)}{-j18.0} = -j6.911$$

$$Y_{23new} = Y_{23} - \frac{Y_{24}.Y_{43}}{Y_{44}} = j2.5 - \frac{(j5.0).(j8.0)}{-j18.0} = j4.722$$

$$Y_{31new} = Y_{13new} = j6.222$$

$$Y_{32new} = Y_{23new} = j4.722$$

$$Y_{33new} = Y_{33} - \frac{Y_{34}.Y_{43}}{Y_{44}} = -j14 - \frac{(j8.0).(j8.0)}{-j18.0} = -j10.444$$

The reduced bus admittance matrix after eliminating 4th node is given by,

$$Y_{bus} = \begin{bmatrix} -j8.411 & j1.388 & j6.222 \\ j1.388 & -j6.911 & j4.722 \\ j6.222 & j4.722 & -j10.444 \end{bmatrix}$$

Elimination of node 3: $Y_{nn} = Y_{33} = -j10.444$

The other elements of reduced bus admittance matrix can be formed from the equation

$$Y_{jknew} = Y_{jk} - \frac{Y_{jn}.Y_{nk}}{Y_{nn}} \text{, Where, n=3, j=1,2, k=1,2}$$

$$Y_{11new} = Y_{11} - \frac{Y_{13}.Y_{31}}{Y_{33}} = -j8.411 - \frac{(j6.222).(j6.222)}{-j10.444} = -j4.7043$$

$$Y_{12new} = Y_{12} - \frac{Y_{13}.Y_{32}}{Y_{33}} = j1.388 - \frac{(j6.222).(j4.722)}{-j10.444} = j4.2011$$

$$Y_{21new} = Y_{12new} = j4.2011$$

$$Y_{22new} = Y_{22} - \frac{Y_{23}.Y_{32}}{Y_{33}} = -j6.911 - \frac{(j4.722).(j4.722)}{-j10.444} = -j4.7761$$

The reduced bus admittance matrix after eliminating node 3 and 4 is

$$Y_{bus} = \begin{bmatrix} -j4.7043 & j4.2011 \\ j4.2011 & -j4.7761 \end{bmatrix}$$

10. **Draw the structure of an electrical power system and describe the components of the system with typical values.**

Single line diagram

Single line diagram is a simplified representation of power system components along with their interconnections with each other. Each component is represented by its symbol.

Limitations

The only limitation of single line diagram is that it cannot represent the conditions during unbalanced operation of a power system. Under the unbalanced operation of a power system, all three phases are to be shown for currents and voltages and single line diagram proves to be insufficient.

Power system Components - Generator, Transformer, Transmission lines & Distribution

Generator –Generates electrical energy

Transformer – transfer power from one circuit to another without change in frequency.

Transmission line - Power transfer from one location to other location

Purpose of control equipment's - Protection from lightning and prevent damage

Tolerance level - +5 to 10%. Difference in voltages caused due to variation in loads

Primary transmission - First stage of transmission, 110kV,132 kV or 220 kV or400 kV or 765 kV, high voltage transmission, 3ϕ , 3 wire system.

Secondary transmission - 3∅ , 3 wire system,33kV high voltage line 66kv to factory supply

Primary distribution - 3∅ , 3 wire system,11kv or 6.6 kV, 3∅ , 3 wire system

Secondary distribution - 400V, 3phase, 230V, 1phase, 3 phase 4 wire

Components of secondary distribution - Substation, feeders, service mains

Interconnection diagram - Feeders, service mains, distributors

Feeder - Conductors that take power from receiving station to substation

Distributor - Conductor that transfer power to consumers by tapping

Service mains - Connects distributor and consumer premises

    i.   3 phase 3 wire circuits - Instantaneous sum of three line current is zero

    ii.   3 phase circuit advantages - Economical , carry three times more power than single phase

    iii.   3 phase 4 wire circuits - 4th wire is Neutral wire and acts as return conductor

11. **The three phase power and line to line voltage rating of the electric power system is shown in figure**
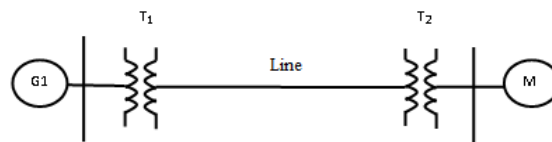
    Generator, G= 60MVA, 20 kV, X"= 9 %

    Transformer, T_1 = 50 MVA, 20/200 kV, X=10 %

    Transformer, T_2 = 50 MVA, 200/20 kV, X=10 %

    Motor, M = 43.2 MVA, 18 kV, X"= 8 %

    Line, 200kV, Z = 120 + j200 Ω

    Draw an impedance diagram showing all impedance in p.u on a 100 MVA base. Choose 20 kV as base voltage for generator.

  Solution.

      $KV_{b,new}$ = 20

      $MVA_{b,new}$ = 100

Generator: $KV_{b,given}$ = 20, $MVA_{b,given}$ = 60 MVA, $Z_{pu,given}$ = 9% = 0.09

$$Z_{p.u.\ new} = Z_{pu,given}\ x\ \frac{kV_{b,given}^{2}}{kV_{b,new}^{2}}\ x\ \frac{MVA_{b,new}}{MVA_{b,given}}$$

$$= 0.09\ x\ \frac{20^2}{20^2}\ x\ \frac{100}{60}\ = j0.15\ p.u$$

Transformer, T_1(Py): $KV_{b,new}$ = 20, $Z_{pu,given}$ = 10% = 0.1

$$Z_{p.u.\ new} = Z_{pu,given}\ x\ \frac{kV_{b,given}^{2}}{kV_{b,new}^{2}}\ x\ \frac{MVA_{b,new}}{MVA_{b,given}}\ = j0.2\ p.u$$

Transmission Line: 200kV, Z = 120 + j200 Ω,

Transformer secondary side change occurs, so calculate $KV_{b,new}$ as

$$KV_{b,new} = KV_{b,old} * \left(\frac{H.T\ side\ rating\ of\ T_1}{L.T\ side\ rating\ of\ T_1} \right)$$

$$= 20 * \left(\frac{200}{20} \right) = 200\ kV$$

$$Z_{p.u} = \left(\frac{Z_{actual}}{kV_b^2}\right) \times MVA_b = \left(\frac{120 + j200}{200^2}\right) \times 100 = 0.3 + j\,0.5 \text{ p.u}$$

Transformer, T_2(Sy): $\qquad KV_{b,new} = 200, Z_{pu,given} = 10\% = 0.1$

$$Z_{p.u.\ new} = Z_{pu,given} \times \frac{kV_{b,given}^2}{kV_{b,new}^2} \times \frac{MVA_{b,new}}{MVA_{b,given}} = j0.2 \text{ p.u}$$
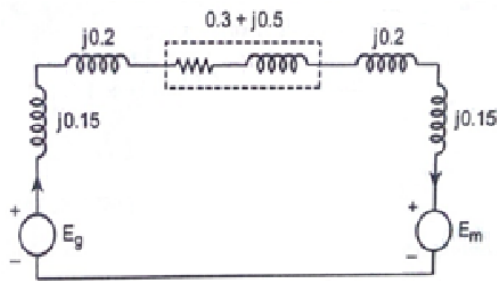
Motor, M : Transformer secondary side change occurs, so calculate $KV_{b,new}$ as

$$KV_{b,new} = KV_{b,old} * \left(\frac{L.T \text{ side rating of } T_2}{H.T \text{ side rating of } T_2}\right)$$

$$= 20 * \left(\frac{20}{200}\right) = 20 \text{ kV}$$

$$Z_{p.u.\ new} = Z_{pu,given} \times \frac{kV_{b,given}^2}{kV_{b,new}^2} \times \frac{MVA_{b,new}}{MVA_{b,given}} = j\,0.15 \text{ p.u}$$

Impedance diagram

12. **Draw the reactance diagram for the power system shown in fig. The ratings of generator, motor and transformers are given below. Assume 50MVA as base in the j50Ω line**

      Generator G1:50MVA, 11kV, X"=14%

      Generator G2:50MVA, 11 kV, X"=16%

      Transformer, T_1, T_2, T_3, T_4 : 30MVA, 66/11 kV, X=12%

Synchronous motor: 20MVA, 11 kV, X"=15%
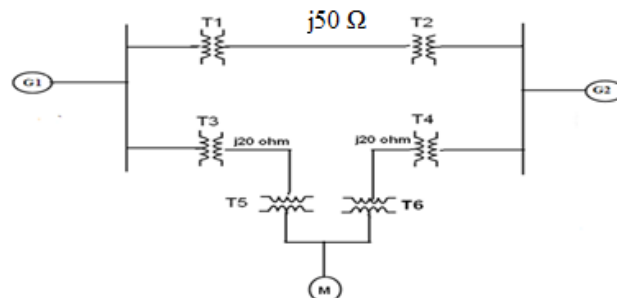
Solution:

   $MVA_{b,new} = 30$ in transmission line (j 40Ω)

   $kV_{b,new} = 66$ kV (voltage in the j 40Ω)

j 50Ω line :

$$Z_{\text{p.u new}} = \frac{Z_{actual}}{Z_{base}} = \frac{Z_{actual}}{\left(\frac{kV_b{}^2}{}\right)} \times MVA_b$$

$$= \frac{j50}{66^2} \times 30 = j0.344 \text{ p.u}$$

Transformer T$_1$ referred to Primary side:

Transformer T$_1$ Primary side change occurs, so calculate KV$_{b,new}$ as

$$KV_{b,new} = 66 * \left(\frac{11}{66}\right) = 11 \text{ kV}$$

$$Z_{\text{p.u. new}} = Z_{pu,given} \times \frac{kV_{b,given}{}^2}{kV_{b,new}{}^2} \times \frac{MVA_{b,new}}{MVA_{b,given}}$$

$$\boxed{Z_{\text{p.u. new}} = j0.12 \times \frac{11^2}{11^2} \times \frac{30}{30} = j\ 0.12 \text{ p.u}}$$

Generator, G$_1$: KV$_{b,new}$ = 11 kV

$$KV_{b,new} = j0.14 \times \frac{11^2}{11^2} \times \frac{30}{50} = j\ 0.084 \text{ p.u}$$

Transformer T$_1$ referred to Primary side:

KV$_{b,new}$ = 11 kV

$$Z_{\text{p.u. new}} = Z_{pu,given} \times \frac{kV_{b,given}{}^2}{kV_{b,new}{}^2} \times \frac{MVA_{b,new}}{MVA_{b,given}}$$

$$\boxed{Z_{\text{p.u. new}} = j0.12 \times \frac{11^2}{11^2} \times \frac{30}{30} = j\ 0.12 \text{ p.u}}$$

j 50Ω line :

Transformer T$_3$ Secondary side change occurs, so calculate KV$_{b,new}$ as

$$KV_{b,new} = KV_{b,old} * \left(\frac{H.T \text{ side rating of } T_3}{L.T \text{ side rating of } T_3}\right)$$

$$KV_{b,new} = 11 * \left(\frac{66}{11}\right) = 66 \text{ kV}$$

$$Z_{\text{p.u new}} = \frac{Z_{actual}}{Z_{base}} = \frac{Z_{actual}}{\left(\frac{kV_b{}^2}{}\right)} \times MVA_b{}_{,\ new}$$

$$\boxed{Z_{\text{p.u. new}} = \frac{j20}{66^2} \times 30 = j\ 0.138 \text{ p.u}}$$

Transformer T$_5$ referred to Primary side:

KV$_{b,}$ = 66 kV

$$Z_{\text{p.u. new}} = Z_{pu,given} \times \frac{kV_{b,given}{}^2}{kV_{b,new}{}^2} \times \frac{MVA_{b,new}}{MVA_{b,given}}$$

$$= j0.1 \times \frac{66^2}{66^2} \times \frac{30}{15} = j\ 0.2 \text{ p.u}$$

Motor : $KV_{b,new} = KV_{b,old} * \left(\frac{L.T \text{ side rating of } T_5}{H.T \text{ side rating of } T_5}\right)$

$$= 66 * \left(\frac{11}{66}\right) = 11 \text{ kV}$$

$$Z_{p.u. new} = Z_{pu,given} \text{ x } \frac{kV_{b,given}^2}{kV_{b,new}^2} \text{ x } \frac{MVA_{b,new}}{MVA_{b,given}} = j\ 0.225\ p.u$$

Transformer T₆ referred to Secondary side:

$KV_{b,new} = 11\ kV$

$$Z_{p.u. new} = Z_{pu,given} \text{ x } \frac{kV_{b,given}^2}{kV_{b,new}^2} \text{ x } \frac{MVA_{b,new}}{MVA_{b,given}}$$

$$Z_{p.u. new} = j0.1 \text{ x } \frac{11^2}{11^2} \text{ x} \frac{30}{15} = j\ 0.2\ p.u$$

j 20Ω line :

Transformer T₆ Primary side change occurs, so calculate $KV_{b,new}$ as

$KV_{b,new} = 11 * (\frac{66}{11}) = 66\ kV$

$$Z_{p.u\ new} = \frac{Z_{actual}}{Z_{base}} = \frac{Z_{actual}}{(kV_b^2)} \text{ x } MVA_{b,\ new}$$

$$= \frac{j20}{66^2} \text{ x } 30 = j\ 0.138\ p.u$$

Transformer T₄ : $Z_{p.u. new} = j\ 0.12\ p.u$ (Because transformer T₄ is identical with transformer)

Transformer T₂ : $Z_{p.u. new} = j\ 0.12\ p.u$ (Because transformer T₂ is identical with transformer)

Generator, G₁ :   Transformer T₂ Secondary side change occurs, so calculate$KV_{b,new\ as}$

$= 66 * (\frac{11}{66}) = 11\ kV$

$$Z_{p.u. new} = j0.16 \text{ x } \frac{11^2}{11^2} \text{ x} \frac{30}{50} = j\ 0.096\ p.u$$

Reactance diagram
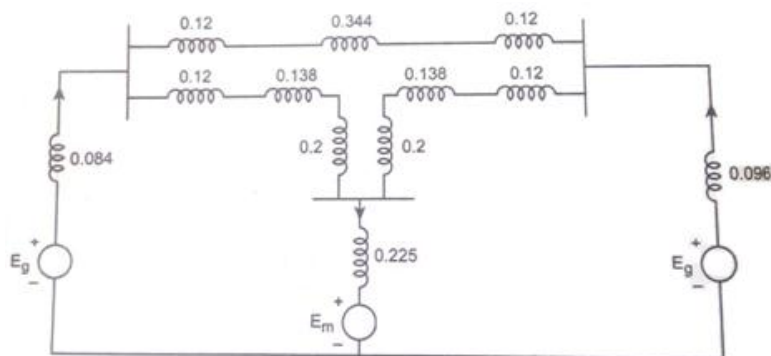
13. **The single line diagram of an unloaded power system is shown in fig. The generator and transformers are rated as follows.**

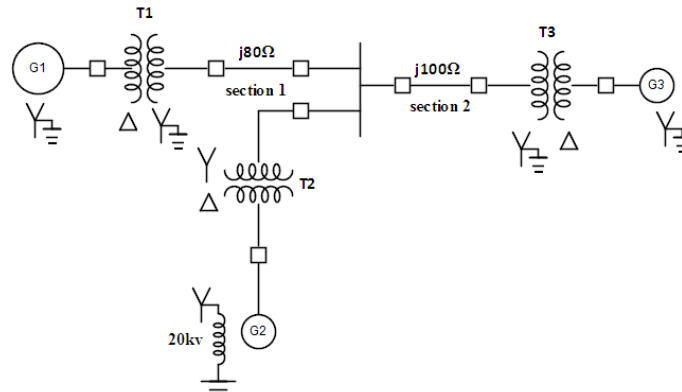**Generator, G1=20MVA, 13.8 kV, X"=20 %**

Generator, G2=30 MVA, 18 kV, X"=20 %

Generator, G3=30 MVA, 20 kV, X"=20 %

Transformer, T1 = 25 MVA, 220/13.8 kV, X=10 %

Transformer, T2 = 3 single phase units each rated at 10 MVA, 127/18 kV, X=10 %

Transformer, T3 = 35 MVA, 220/22 kV, X=10 %

Draw the reactance diagram using a base of 50 MVA and 13.8 kV on the generator G1.

Solution :

$MVA_{b,new} = 50$

$kV_{b,new} = 13.8$ kV

Generator, G_1 : $Z_{p.u.\ new} = Z_{pu,given}$ x $\dfrac{kV_{b,given}^2}{kV_{b,new}^2}$ x $\dfrac{MVA_{b,new}}{MVA_{b,given}}$

$= j0.2$ x $\dfrac{13.8^2}{13.8^2}$ x $\dfrac{50}{20}$ $= j\ 0.5$ p.u

Transformer T_1 referred to Primary side:

$kV_{b,new} = 13.8$ kV

$Z_{p.u.\ new} = Z_{pu,given}$ x $\dfrac{kV_{b,given}^2}{kV_{b,new}^2}$ x $\dfrac{MVA_{b,new}}{MVA_{b,given}}$

$= j0.1$ x $\dfrac{13.8^2}{13.8^2}$ x $\dfrac{50}{25}$ $= j\ 0.2$ p.u

Transmission line j 80Ω :

Transformer T_1 Secondary side change occurs, so calculate $KV_{b,new}$ as

$KV_{b,new} = KV_{b,old} * (\dfrac{H.T\ side\ rating\ of\ T_1}{L.T\ side\ rating\ of\ T_1})$

$KV_{b,new} = 13.8 * (\dfrac{220}{13.8}) = 220$ kV

$Z_{actual} = j\ 80Ω$

$Z_{p.u\ new} = \dfrac{Z_{actual}}{Z_{base}} = \dfrac{Z_{actual}}{(kV_b^2)}$ x $MVA_{b,\ new}$

$= \dfrac{j80}{220^2}$ x $50 = j\ 0.0826$ p.u

Transmission line j 100Ω :

$KV_{b,new} = 220$ kV

$Z_{actual} = j\ 100Ω$

$Z_{p.u} = \dfrac{j100}{220^2}$ x $50 = j\ 0.1033$ p.u

Transformer T_2 referred to Primary side: 3 single phase units are used.

Voltage rating: 3 x 127/18 kV = 220/18 kV

Note: **Star side** , $V_L = \sqrt{3}\ V_p$; **Delta side** , $V_L = V_p$; Power $= 3\ V_L\ I_L$

$MVA_{b,given} = 3\text{x}10 = 30\ MVA$

$KV_{b,new} = 220$ kV

$$Z_{p.u. \; new} = Z_{pu,given} \; x \; \frac{kV_{b,given}^2}{kV_{b,new}^2} \; x \; \frac{MVA_{b,new}}{MVA_{b,given}}$$

$= j0.1 \; x \; \frac{220^2}{220^2} \; x \frac{50}{30} = j \; 0.1667$ p.u

Generator, G_2 :

Transformer T_2 Primary side change occurs, so calculate $KV_{b,new}$ as

$$KV_{b,new} = KV_{b,old} * (\frac{L.T \; side \; rating \; of \; T_3}{H.T \; side \; rating \; of \; T_3})$$

$KV_{b,new} = 20 * (\frac{18}{220}) = 18$ kV

$$Z_{p.u. \; new} = Z_{pu,given} \; x \; \frac{kV_{b,given}^2}{kV_{b,new}^2} \; x \; \frac{MVA_{b,new}}{MVA_{b,given}}$$

$= j0.2 \; x \; \frac{18^2}{18^2} \; x \frac{50}{30} = j \; 0.333$ p.u

Transformer T_3 referred to Secondary side:

$KV_{b,new} = KV_{b,given} = 220$ kV

$$Z_{p.u. \; new} = Z_{pu,given} \; x \; \frac{kV_{b,given}^2}{kV_{b,new}^2} \; x \; \frac{MVA_{b,new}}{MVA_{b,given}}$$

$Z_{p.u. \; new} = j0.1 \; x \; \frac{220^2}{220^2} \; x \frac{50}{35} = j \; 0.1429$ p.u

Generator, G_3 :

Transformer T_3 Primary side change occurs, so calculate $KV_{b,new}$ as

$$KV_{b,new} = KV_{b,old} * (\frac{L.T \; side \; rating \; of \; T_3}{H.T \; side \; rating \; of \; T_3})$$

$KV_{b,new} = 220 * (\frac{22}{220}) = 22$ kV

$KV_{b,given} = 20$ kV

$$Z_{p.u. \; new} = Z_{pu,given} \; x \; \frac{kV_{b,given}^2}{kV_{b,new}^2} \; x \; \frac{MVA_{b,new}}{MVA_{b,given}}$$

$Z_{p.u. \; new} = j0.2 \; x \; \frac{20^2}{22^2} \; x \frac{50}{30} = j \; 0.2755$ p.u

Reactance diagram
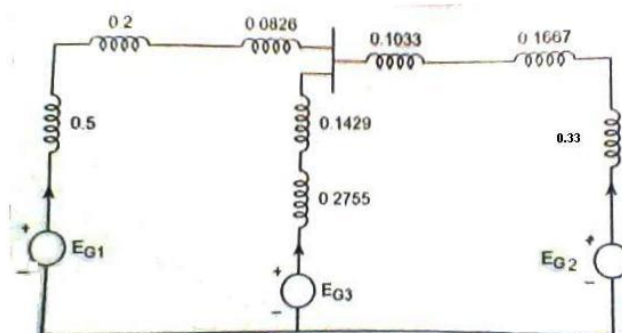
14. Draw the reactance diagram for the power system shown in fig. Neglect resistance and use a base of 100 MVA, 220 kV in 50Ω line. The ratings of the generator, motor and transformer are given below.

Generator: 40MVA, 25 kV, X"=20 %

Synchronous motor: 50 MVA, 11 kV, X"=30 %

Y- Y Transformer: 40MVA, 33/220 kV, X=15 %

Y -Δ Transformer: 30 MVA, 11/220 kV (Δ/Y), X=15 %

Solution :

Assume base MVA_{new} = 100 MVA (Highest rating of the Machine)
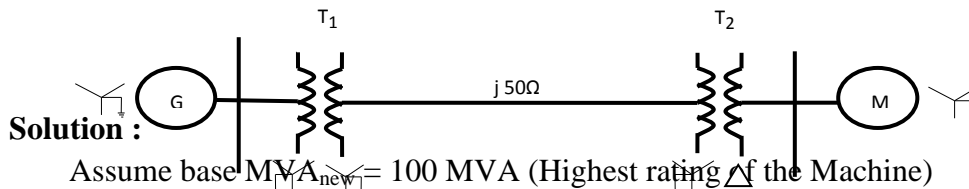
Base kV_{new} = 11 KV

Generator, G_1 : $KV_{b,given}$ = 11 kV, $MVA_{b,given}$ = 100 MVA

$$Z_{p.u.\ new} = Z_{pu,given} \times \frac{kV_{b,given}^2}{kV_{b,new}^2} \times \frac{MVA_{b,new}}{MVA_{b,given}}$$

$$= j0.2 \times \frac{11^2}{11^2} \times \frac{100}{100} = j\ 0.2\ p.u$$

Transformer Primary : $KV_{b,given}$ = 11 kV, $MVA_{b,given}$ = 50 MVA

$$Z_{p.u.\ new} = Z_{pu,given} \times \frac{kV_{b,given}^2}{kV_{b,new}^2} \times \frac{MVA_{b,new}}{MVA_{b,given}}$$

$$\boxed{Z_{p.u.\ new} = j0.1 \times \frac{11^2}{11^2} \times \frac{100}{50} = j\ 0.2\ p.u}$$

Transmission line : $KV_{b,\ new}$ = KV_b of secondary side of transformer.

$$= KV_{b,old} \times \left(\frac{H.T\ side\ rating\ of\ Transformer}{L.T\ side\ rating\ of\ Transformer} \right)$$

$$= 11 \times \frac{220}{11} = 220\ kV$$

$$Z_{p.u} = \left(\frac{Z_{actual}}{kV_b^2} \right) \times MVA_b$$

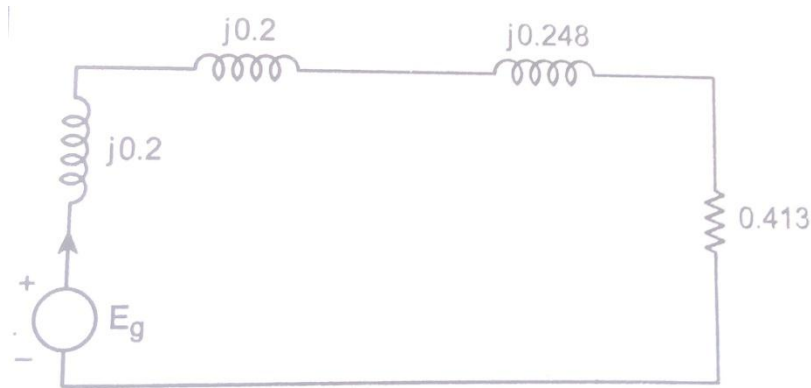$$= \frac{j120}{220^2} \times 100$$

$$\boxed{Z_{p.u} = j\ 0.248\ p.u}$$

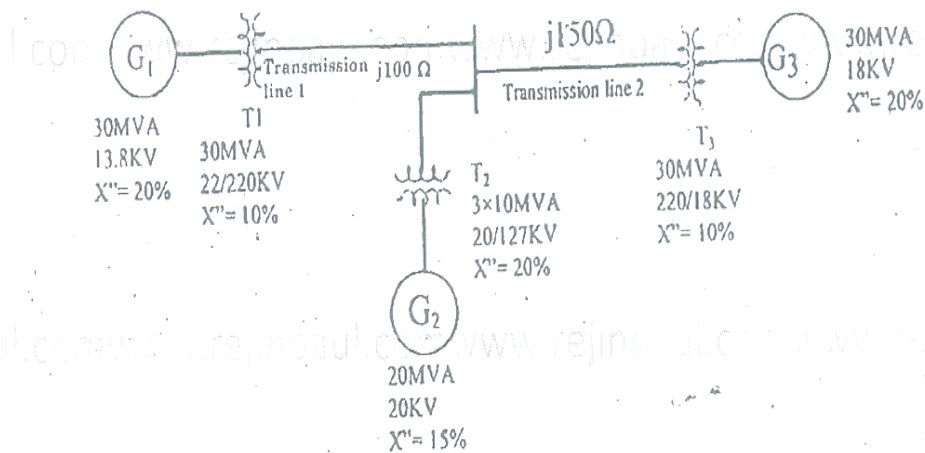**Resistive load :**

$KV_{b,\ new}$ = 220 kV

$$R_{p.u} = \left(\frac{R_{actual}}{kV_b^2} \right) \times MVA_b$$

$$\boxed{R_{p.u} = \frac{200}{220^2} \times 100 = 0.413\ p.u}$$

Impedance diagram

15. **The single line diagram of a power system is shown in fig. . Determine the new per unit values and draw the reactance diagram. Assume 25 MVA and 20 KV as new base on Genretor G_1**

Solution:

Base $MVA_{b,new}$ = 100 MVA

Base $kV_{b,new}$ = 220 kV

Transmission line j 50Ω:

Z_{actual} = j50

$Z_{p.u\ new} = \dfrac{Z_{actual}}{Z_{base}} = \dfrac{Z_{actual}}{(kV_{b,new}^{2})} \times MVA_{b,new}$

$Z_{p.u\ new} = \dfrac{j50}{220^{2}} \times 100 = j0.1033$ p.u

Transformer T_1:

$KV_{b,old}$ = 220 kV

$KV_{b,new} = KV_{b,old} * (\dfrac{L.T\ side\ rating\ of\ T_1}{H.T\ side\ rating\ of\ T_1})$

$KV_{b,new} = 220 * (\dfrac{33}{220}) = 33$ kV

$Z_{p.u.\ new} = Z_{pu,given} \times \dfrac{kV_{b,given}^{2}}{kV_{b,new}^{2}} \times \dfrac{MVA_{b,new}}{MVA_{b,given}}$

$$Z_{p.u\ new} = j0.15\ x\ \frac{33^2}{33^2}\ x\ \frac{100}{40}\ = j\ 0.375\ p.u$$

Generator, G$_1$: $Z_{p.u.\ new} = Z_{pu,given}\ x\ \frac{kV_{b,given}^2}{kV_{b,new}^2}\ x\ \frac{MVA_{b,new}}{MVA_{b,given}}$

$$Z_{p.u\ new} = j0.2\ x\ \frac{33^2}{33^2}\ x\ \frac{100}{40}\ = j\ 0.287\ p.u$$

Transformer T$_2$:

$\quad$ KV$_{b,old}$ = 220 kV(H.T side voltage)

KV$_{b,new}$ = KV$_{b,old}$ * ($\frac{L.T\ side\ rating\ of\ T_2}{H.T\ side\ rating\ of\ T_2}$)

$\quad$ = 220 * ($\frac{11}{220}$) = 11 kV

$Z_{p.u.\ new} = Z_{pu,given}\ x\ \frac{kV_{b,given}^2}{kV_{b,new}^2}\ x\ \frac{MVA_{b,new}}{MVA_{b,given}}$

$$Z_{p.u\ new} = j0.15\ x\ \frac{11^2}{11^2}\ x\ \frac{100}{30}\ = j\ 0.5\ p.u$$

Synchronous Motor :

$Z_{p.u.\ new} = Z_{pu,given}\ x\ \frac{kV_{b,given}^2}{kV_{b,new}^2}\ x\ \frac{MVA_{b,new}}{MVA_{b,given}}$

$$Z_{p.u.\ new} = j0.3\ x\ \frac{11^2}{11^2}\ x\ \frac{100}{50}\ = j\ 0.6\ p.u$$
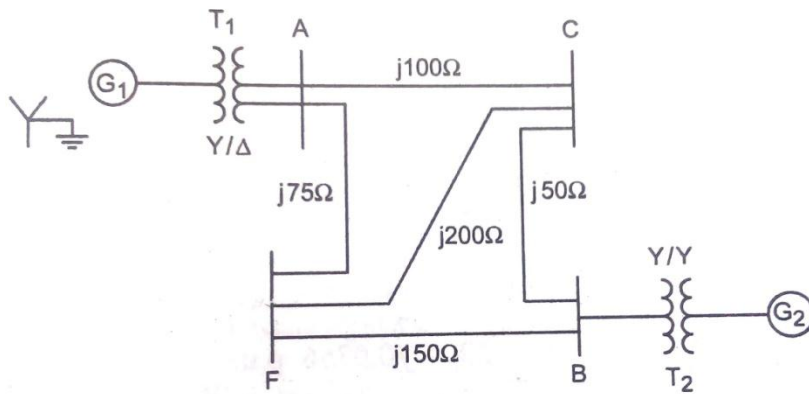
Reactance diagram.
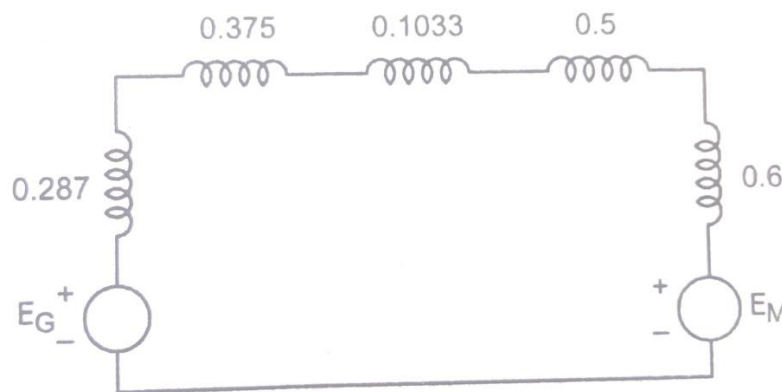
16. **Draw the reactance diagram for the system is shown in fig. and mark all reactance in p.u on 20 MVA and 6.6 kV basis .**

Generator, G1=10MVA, 65.6 kV, X"=10 %

Generator, G2=20 MVA, 11.5 kV, X"=10 %

Transformer, T1 = 10 MVA, 3∅, 6.6/115 kV, X=15 %

Transformer, T2 = 3 single phase units each rated at 10 MVA, 7.5/75 kV, X=10 %

Solution:

$MVA_{b,new} = 20$

$kV_{b,new} = 6.6\ kV$

Therefore first consider G_1 which has 6.6kV as base.

Generator, G_1 : $kV_{b,new} = 6.6\ kV$

$Z_{p.u.\ new} = Z_{pu,given}\ x\ \dfrac{kV_{b,given}^2}{kV_{b,new}^2}\ x\ \dfrac{MVA_{b,new}}{MVA_{b,given}}$

$\boxed{Z_{p.u.\ new} = j0.1\ x\ \dfrac{6.6^2}{6.6^2}\ x\dfrac{20}{10} = j\ 0.2\ p.u}$

Transformer T_1 referred to Primary side:

$kV_{b,new} = 6.6\ kV$

$Z_{p.u.\ new} = Z_{pu,given}\ x\ \dfrac{kV_{b,given}^2}{kV_{b,new}^2}\ x\ \dfrac{MVA_{b,new}}{MVA_{b,given}}$

$\boxed{Z_{p.u.\ new} = j0.15\ x\ \dfrac{6.6^2}{6.6^2}\ x\dfrac{20}{10} = j\ 0.3\ p.u}$

Transmission line $j\ 100\Omega$:

Transformer T_1 Secondary side change occurs, so calculate $KV_{b,new}$ as

$KV_{b,new} = KV_{b,old} * (\dfrac{H.T\ side\ rating\ of\ T_1}{L.T\ side\ rating\ of\ T_1})$

$KV_{b,new} = 6.6 * (\dfrac{115}{6.6}) = 115\ kV$

$Z_{actual} = j\ 100\Omega$

$Z_{p.u\ new} = \dfrac{Z_{actual}}{Z_{base}} = \dfrac{Z_{actual}}{(kV_b^2)}\ x\ MVA_{b,\ new}$

$\boxed{Z_{p.u} = \dfrac{j100}{115^2}\ x\ 20 = j\ 0.1512\ p.u}$

Transmission line $j\ 75\Omega$:

$KV_{b,new} = 115\ kV$

$Z_{actual} = j\ 75\Omega$

$\boxed{Z_{p.u} = \dfrac{j75}{115^2}\ x\ 20 = j\ 0.1134\ p.u}$

Transmission line j 50Ω :

$KV_{b,new}$ = 115 kV

Z_{actual} = j 50Ω

$$Z_{p.u} = \frac{j50}{115^2} \times 20 = j\ 0.0756\ p.u$$

Transmission line j 200Ω :

$KV_{b,new}$ = 115 kV

Z_{actual} = j 200Ω

$$Z_{p.u} = \frac{j200}{115^2} \times 20 = j\ 0.0302\ p.u$$

Transmission line j 150Ω :

$KV_{b,new}$ = 115 kV

Z_{actual} = j 150Ω

$$Z_{p.u} = \frac{j150}{115^2} \times 20 = j\ 0.2268\ p.u$$

Transformer T$_2$ referred to Primary 3 single phase transformer units.

   Note: **Star side** , $V_L = \sqrt{3}\ V_p$;**Delta side ,** $V_L = V_p$; Power = 3 $V_L\ I_L$

$MVA_{b,given}$ = 3x10 = 30 MVA

$KV_{b,new}$ = 115 kV, $KV_{b,given}$ = 75 x $\sqrt{3}$ = 130 kV

$$Z_{p.u.\ new} = Z_{pu,given} \times \frac{kV_{b,given}^2}{kV_{b,new}^2} \times \frac{MVA_{b,new}}{MVA_{b,given}}$$

$$Z_{p.u.\ new} = j0.1 \times \frac{130^2}{115^2} \times \frac{20}{30} = j\ 0.085\ p.u$$

Generator, G$_2$:

Transformer T$_2$ Secondary side change occurs, so calculate $KV_{b,new}$ as
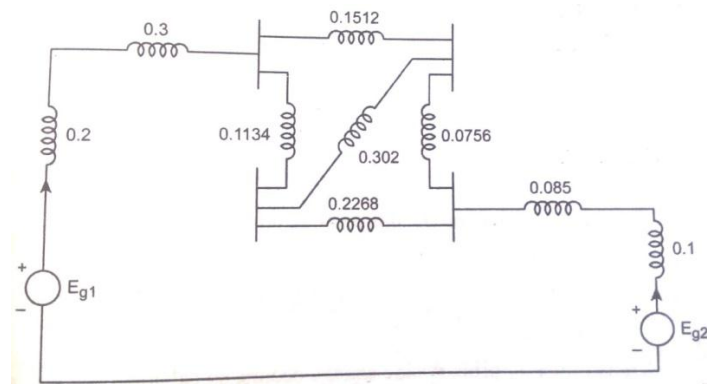
$$KV_{b,new} = KV_{b,old} * \left(\frac{H.T\ side\ rating\ of\ T_2}{L.T\ side\ rating\ of\ T_2} \right)$$

$$KV_{b,new} = 115 * \left(\frac{13}{130} \right) = 11.5\ kV$$

$$Z_{p.u.\ new} = Z_{pu,given} \times \frac{kV_{b,given}^2}{kV_{b,new}^2} \times \frac{MVA_{b,new}}{MVA_{b,given}}$$

$$Z_{p.u.\ new} = j0.1 \times \frac{11.5^2}{11.5^2} \times \frac{20}{20} = j\ 0.1\ p.u$$

Reactance diagram.
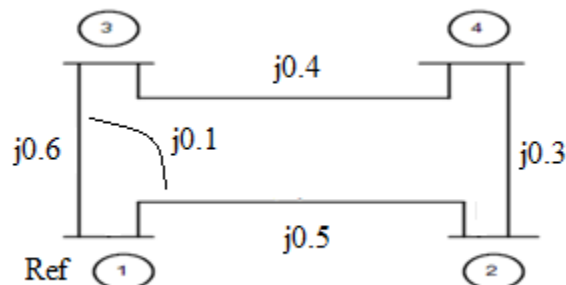
17. **Form Y-bus for the network shown in fig. The impedance data is given in table. Select node (1) as reference node.**

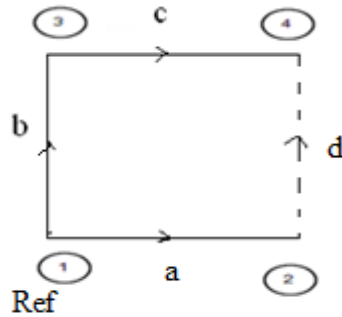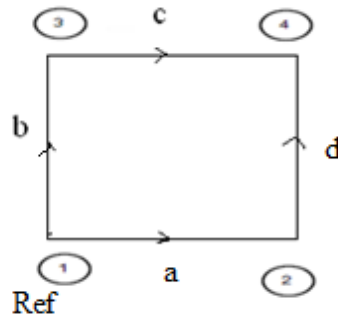| Element No.	Self		Mutual	
	Bus code	Impedance	Bus code	Impedance
1	1-2	0.6		
2	1-3	0.5	1-2	0.1
3	3-4	0.5		
4	2-4	0.2		

Solution:

Oriented graph.

Take (1) as reference. Draw Tree

$$\text{Incidence matrix } [A] = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & -1 \end{bmatrix}$$

$$[A]^T = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\text{Primitive impedance matrix } [Z_{Primitive}] = \begin{bmatrix} j0.5 & j0.1 & 0 & 0 \\ j0.1 & j0.6 & 0 & 0 \\ 0 & 0 & j0.4 & 0 \\ 0 & 0 & 0 & j0.3 \end{bmatrix}$$

Primitive admitance matrix $[Y_{Primitive}] = [Z_{Primitive}]^{-1}$

$$\text{Consider the matrix } \begin{bmatrix} j0.5 & j0.1 \\ j0.1 & j0.6 \end{bmatrix}^{-1} = \frac{1}{-0.29}\begin{bmatrix} j0.6 & -j0.1 \\ -j0.1 & j0.5 \end{bmatrix}$$

$$= \begin{bmatrix} -j2.0689 & j0.3448 \\ j0.3448 & -j1.724 \end{bmatrix}$$

$$[Y_{Primitive}] = \begin{bmatrix} -j2.0689 & j0.3448 & 0 & 0 \\ j0.3448 & -j1.724 & 0 & 0 \\ 0 & 0 & -j2.5 & 0 \\ 0 & 0 & 0 & -j3.333 \end{bmatrix}$$

Bus admittance matrix $[Y_{bus}] = [A][Y_{Primitive}][A]^T$

$$[Y_{Primitive}][A]^T = \begin{bmatrix} -j2.0689 & j0.3448 & 0 & 0 \\ j0.3448 & -j1.724 & 0 & 0 \\ 0 & 0 & -j2.5 & 0 \\ 0 & 0 & 0 & -j3.333 \end{bmatrix}\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -j2.0689 & j0.3448 & 0 \\ -j0.3448 & j1.724 & 0 \\ 0 & -j2.5 & j2.5 \\ -j3.333 & 0 & j3.333 \end{bmatrix}$$

Bus admittance matrix

$$[Y_{bus}] = [A][Y_{Primitive}][A]^T = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} -j2.0689 & j0.3448 & 0 \\ -j0.3448 & j1.724 & 0 \\ 0 & -j2.5 & j2.5 \\ -j3.333 & 0 & j3.333 \end{bmatrix}$$

Bus admittance matrix $[Y_{bus}] = \begin{bmatrix} -j5.4019 & j03448 & j3.333 \\ j0.3448 & -j4.224 & j2.5 \\ j3.333 & j2.5 & -j5.8333 \end{bmatrix}$

18. **Determine Z_{bus} for the system whose reactance diagram is shown in fig. where the impedance are given in p.u. preserve all the three nodes.**
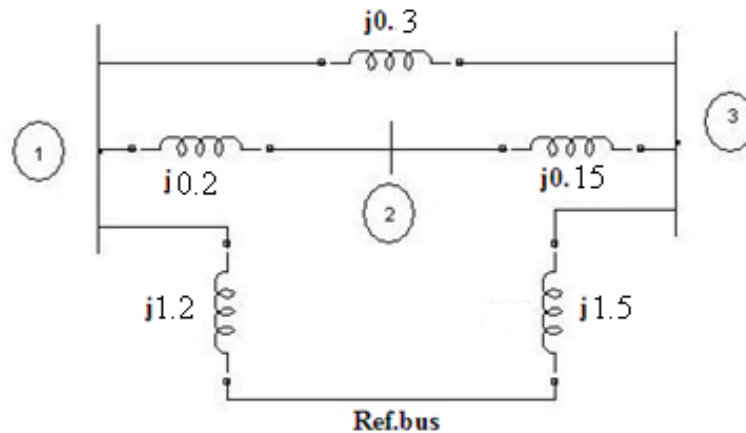
Solution

Step 1:

Consider the branch with impedance j 1.2 p.u connected between bus-1 and reference bus.

- The system having a single bus and so the order of bus impedance matrix is one.

  $Z_{bus} = [j \ 1.2]$
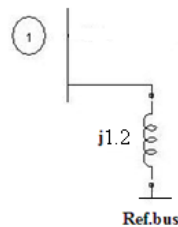
Step 2:

- Connect bus 2 to bus 1 through am impedance j 0.2.
- This is case 2 modification and so the order of matrix is increased by one.
- In this new bus impedance matrix the elements of 1st row and column is copied as element of 2nd row and 2nd column. The diagonal matrix is given by $Z_{11} + Z_b$ where $Z_b = j \ 0.2$
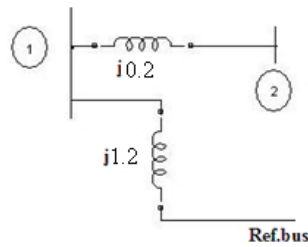
$$Z_{bus} = \begin{bmatrix} j\ 1.2 & j\ 1.2 \\ j\ 1.2 & j\ 1.2 + j\ 0.2 \end{bmatrix} = \begin{bmatrix} j\ 1.2 & j\ 1.2 \\ j\ 1.2 & j\ 1.4 \end{bmatrix}$$

Step 3:

- Connect bus 3 to bus 2 through am impedance j 0.15.
- This is case 2 modification and so the order of bus impedance matrix is increased by one.
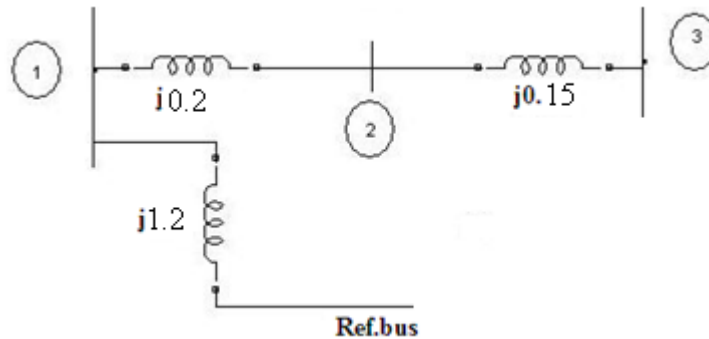- In this new bus impedance matrix the elements of 2nd row and column is copied as element of 3rd row and column. The diagonal matrix is given by $Z_{22} + Z_b$ where $Z_b = j\ 0.15$

$$Z_{bus} = \begin{bmatrix} j\ 1.2 & j\ 1.2 & j\ 1.2 \\ j\ 1.2 & j\ 1.4 & j\ 1.4 \\ j\ 1.2 & j\ 1.4 & j\ 1.4 + j\ 0.15 \end{bmatrix} = \begin{bmatrix} j\ 1.2 & j\ 1.2 & j\ 1.2 \\ j\ 1.2 & j\ 1.4 & j\ 1.4 \\ j\ 1.2 & j\ 1.4 & j\ 1.55 \end{bmatrix}$$

Step 4 :

- Connect impedance j1.5 from bus 3 to reference bus.
- This is case 3 modification. In this case new bus impedance matrix is framed as that of the last row and column are eliminated by node elimination techniques.
- In new bus impedance matrix the elements of 3rd row and column are copied for the 4th row and column.
- The diagonal matrix is given by $Z_{33} + Z_b$ where $Z_b = j\ 1.5$
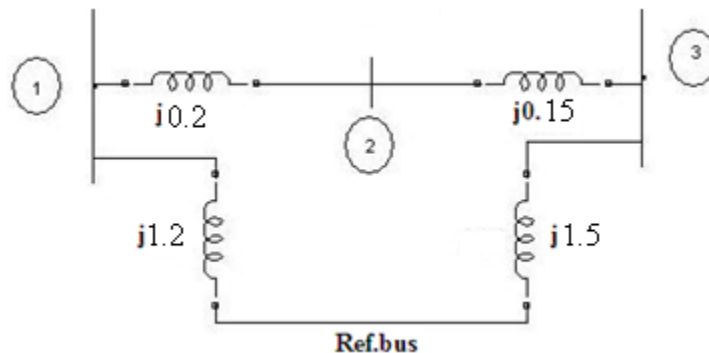
$$Z_{bus} = \begin{bmatrix} j\,1.2 & j\,1.2 & j\,1.2 & j\,1.2 \\ j\,1.2 & j\,1.4 & j\,1.4 & j\,1.4 \\ j\,1.2 & j\,1.4 & j\,1.55 & j\,1.55 \\ j\,1.2 & j\,1.4 & j\,1.55 & j\,1.55 + j\,1.5 \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} j\,1.2 & j\,1.2 & j\,1.2 & j\,1.2 \\ j\,1.2 & j\,1.4 & j\,1.4 & j\,1.4 \\ j\,1.2 & j\,1.4 & j\,1.55 & j\,1.55 \\ j\,1.2 & j\,1.4 & j\,1.55 & j\,3.05 \end{bmatrix}$$

Actual new bus impedance matrix is obtained by eliminating the 3rd row and 3rd column. The element Z_{jk} of the new bus impedance matrix is given by,

$$Z_{jk, \, act} = Z_{jk} - \frac{Z_{j(n+1)}\, Z_{(n+1)k}}{Z_{(n+1)(n+1)}} \qquad \text{where } n=3 \text{ ; } j=1,2,3 \text{ and } k=1,2,3$$

$Z_{11, \, act} = Z_{11} - \dfrac{Z_{14}\, Z_{41}}{Z_{44}} = j1.2 - \dfrac{j1.2 \times j1.2}{j\,3.05} = j0.728$

$Z_{12, \, act} = Z_{12} - \dfrac{Z_{14}\, Z_{42}}{Z_{44}} = j1.2 - \dfrac{j1.2 \times j1.4}{j\,3.05} = j0.649$

$Z_{13, \, act} = Z_{13} - \dfrac{Z_{14}\, Z_{43}}{Z_{44}} = j1.2 - \dfrac{j1.2 \times j1.55}{j\,3.05} = j0.590$

$Z_{21, \, act} = Z_{12, \, act} = j0.649$

$Z_{22, \, act} = Z_{22} - \dfrac{Z_{24}\, Z_{42}}{Z_{44}} = j1.4 - \dfrac{j1.4 \times j1.4}{j\,3.05} = j0.757$

$Z_{23, \, act} = Z_{23} - \dfrac{Z_{24}\, Z_{43}}{Z_{44}} = j1.4 - \dfrac{j1.4 \times j1.55}{j\,3.05} = j0.689$

$Z_{31, \, act} = Z_{13, \, act} = j0.590$

$Z_{32, \, act} = Z_{23, \, act} = j0.689$

$Z_{33, \, act} = Z_{33} - \dfrac{Z_{34}\, Z_{43}}{Z_{44}} = j1.55 - \dfrac{j1.55 \times j1.55}{j\,3.05} = j0.762$

$$Z_{bus} = \begin{bmatrix} j\,0.728 & j\,0.649 & j\,0.590 \\ j\,0.649 & j\,0.757 & j\,0.689 \\ j\,0.590 & j\,0.689 & j\,0.762 \end{bmatrix}$$

Step 5 : Connect impedance j0.3 between bus 1 and bus 3.

- In new bus impedance matrix, the elements of 4th row and column are obtained by subtracting the elements of 1st row and column.
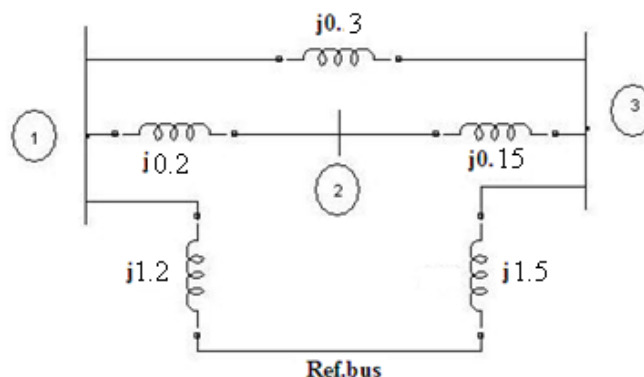
- The element of Z_{44} is given by $Z_{44} = Z_b + Z_{11} + Z_{33} - 2Z_{13}$

Where $Z_b = j0.3$

Therefore $Z_{44} = j0.3 + j0.728 + j0.762 - 2(j0.59) = j0.61$

$$Z_{bus} = \begin{bmatrix} j\,0.728 & j\,0.649 & j\,0.59 & j\,0.728 - j\,0.590 \\ j\,0.649 & j\,0.757 & j\,0.689 & j\,0.649 - j\,0.689 \\ j\,0.590 & j\,0.689 & j\,0.762 & j\,0.59 - j\,0.762 \\ j\,0.728 - j\,0.590 & j\,0.649 - j\,0.689 & j\,0.59 - j\,0.762 & j\,0.61 \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} j\,0.728 & j\,0.649 & j\,0.59 & j\,0.138 \\ j\,0.649 & j\,0.757 & j\,0.689 & -0.04 \\ j\,0.590 & j\,0.689 & j\,0.762 & -j\,0.172 \\ j\,0.138 & -0.04 & -j\,0.172 & j\,0.61 \end{bmatrix}$$

Since this modification does not add a new, the 4th row and column has to be eliminated using node elimination technique, to determine the actual new bus impedance matrix. The element Z_{jk} of actual new bus impedance matrix is given by.

$$Z_{jk,\,act} = Z_{jk} - \frac{Z_{j(n+1)}\,Z_{(n+1)k}}{Z_{(n+1)(n+1)}} \qquad \text{where } n=3 \text{ ; } j=1,2,3 \text{ and } k=1,2,3$$

$Z_{11,\,act} = Z_{11} - \dfrac{Z_{14}\,Z_{41}}{Z_{44}} = j0.728 - \dfrac{j\,0.138 \times j\,0.138}{j\,0.61} = j0.697$

$Z_{12,\,act} = Z_{12} - \dfrac{Z_{14}\,Z_{42}}{Z_{44}} = j\,0.649 - \dfrac{j\,0.138 \times (-j\,0.04)}{j\,0.61} = j0.658$

$Z_{13,\,act} = Z_{13} - \dfrac{Z_{14}\,Z_{43}}{Z_{44}} = j\,0.59 - \dfrac{j\,0.138 \times (-j0.172)}{j\,0.61} = j0.629$

$Z_{21,\,act} = Z_{12,\,act} = j0.658$

$Z_{22,\,act} = Z_{22} - \dfrac{Z_{24}\,Z_{42}}{Z_{44}} = j0.757 - \dfrac{(-j\,0.04) x (-j\,0.04)}{j\,0.61} = j0.754$

$Z_{23,\,act} = Z_{23} - \dfrac{Z_{24}\,Z_{43}}{Z_{44}} = j0.689 - \dfrac{(-j\,0.04) x (-j\,0.172)}{j\,0.61} = j0.678$

$Z_{31,\,act} = Z_{13,\,act} = j0.629$

$Z_{32,\,act} = Z_{23,\,act} = j0.678$

$Z_{33,\,act} = Z_{33} - \dfrac{Z_{34}\,Z_{43}}{Z_{44}} = j0.762 - \dfrac{(-j\,0.172) x (-j\,0.172)}{j\,0.61} = j0.714$
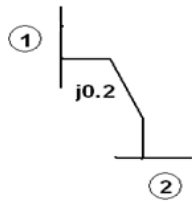
$$Z_{bus} = \begin{bmatrix} j\,0.697 & j\,0.658 & j\,0.629 \\ j\,0.658 & j\,0.754 & j\,0.678 \\ j\,0.629 & j\,0.678 & j\,0.714 \end{bmatrix}$$

19. For the system shown in fig form the bus impedance matrix using building algorithm. Consider node 2 as reference node.

Solution :

Step 1: Add an element between reference and node (1).

$Z_{bus} = [j\,0.2]$

Step 2 : Add element between existing node (1) and the new node (3).

$$Z_{bus} = \begin{bmatrix} j\,0.2 & j\,0.2 \\ j\,0.2 & j\,0.6 \end{bmatrix}$$
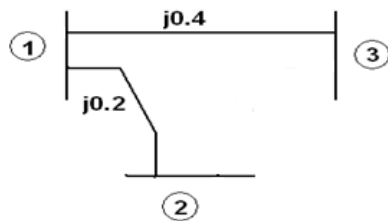
Step 3 : Add element between existing node (3) and the reference node.

$$Z_{bus} = \begin{bmatrix} j\,0.2 & j\,0.2 & j\,0.2 \\ j\,0.2 & j\,0.6 & j\,0.6 \\ j\,0.2 & j\,0.6 & j\,0.8 \end{bmatrix}$$
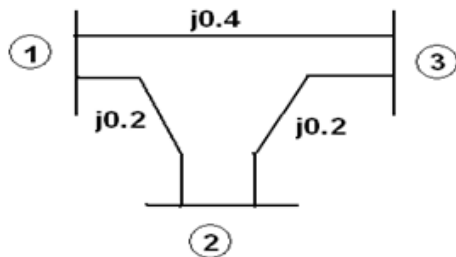
Using Kron's reduction technique

$$Z_{11} = Z_{11} - \frac{Z_{13}\,Z_{31}}{Z_{33}} = j0.2 - \frac{j\,0.2 \times j\,0.2}{j\,0.8} = j0.15$$

$$Z_{12} = Z_{21} = Z_{12} - \frac{Z_{13}\,Z_{32}}{Z_{33}} = j0.2 - \frac{j\,0.2 \times j\,0.6}{j\,0.8} = j0.05$$

$$Z_{22} = Z_{22} - \frac{Z_{23}\,Z_{32}}{Z_{33}} = j0.6 - \frac{j\,0.6 \times j\,0.6}{j\,0.8} = j0.15$$

$$Z_{bus} = \begin{bmatrix} j\,0.15 & j\,0.05 \\ j\,0.05 & j\,0.15 \end{bmatrix}$$

20. Form Y-bus by singular transformation for the network shown in fig. The impedance data is given in table. Take (1) as reference node.
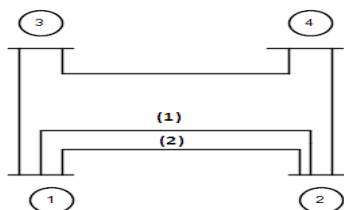
Element No.	Self	
	Bus code	Impedance
1	1-2 (1)	0.6
2	1-3	0.5
3	3-4	0.5
4	1-2 (2)	0.4
5	2-4	0.2

Solution:

Oriented graph.
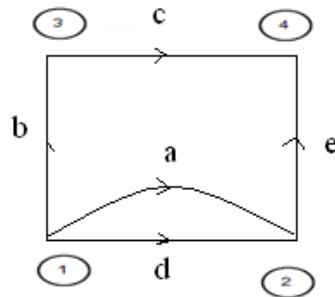
Take (1) as reference. Draw Tree
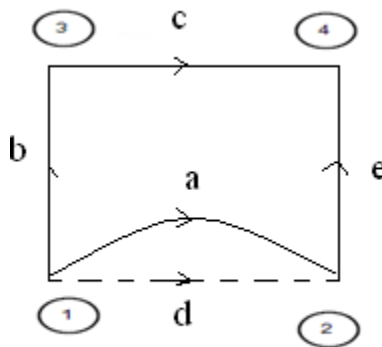
Incidence matrix $[A] = \begin{bmatrix} -1 & 0 & 0 & -1 & 1 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 \end{bmatrix}$

$[A]^T = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}$

Primitive impedance matrix $[Z_{Primitive}] = \begin{bmatrix} j0.6 & 0 & 0 & 0 & 0 \\ 0 & j0.5 & 0 & 0 & 0 \\ 0 & 0 & j0.5 & 0 & 0 \\ 0 & 0 & 0 & j0.4 & 0 \\ 0 & 0 & 0 & 0 & j0.2 \end{bmatrix}$

Primitive admitance matrix $[Y_{Primitive}] = [Z_{Primitive}]^{-1}$

$$\begin{bmatrix} -j1.667 & 0 & 0 & 0 & 0 \\ 0 & -j2.0 & 0 & 0 & 0 \\ 0 & 0 & -j2 & 0 & 0 \\ 0 & 0 & 0 & -j2.5 & 0 \\ 0 & 0 & 0 & 0 & -j5 \end{bmatrix}$$

Bus admittance matrix $[Y_{bus}] = [A][Y_{Primitive}][A]^T$

$$[Y_{Primitive}][A]^T = \begin{bmatrix} j1.667 & 0 & 0 \\ 0 & j2 & 0 \\ 0 & -j2 & j2 \\ j2.5 & 0 & 0 \\ -j5 & 0 & j5 \end{bmatrix}$$

Bus admittance matrix $[Y_{bus}] = [A][Y_{Primitive}][A]^T = \begin{bmatrix} -j1.667 - j2.5\ j5 & 0 & j5 \\ 0 & -j2 - j2 & j2 \\ j5 & j2 & -j2 - j5 \end{bmatrix}$

Bus admittance matrix $[Y_{bus}] = \begin{bmatrix} -j9.167 & 0 & j5 \\ 0 & -j4 & j2 \\ j5 & j2 & -j7 \end{bmatrix}$

PART-C

21. **For the network shown in fig. form the bus admittance matrix. Determine the reduced admittance matrix by eliminating node 4. The values are marked in p.u.**
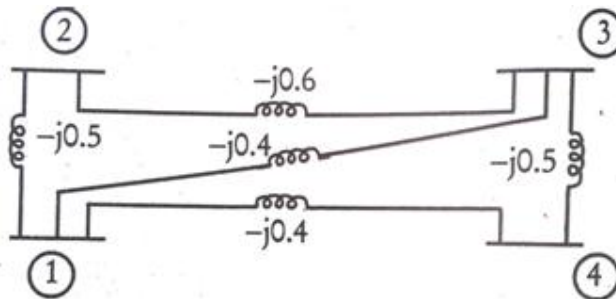
Solution:

The Y_{bus} Matrix of the network is

$$Y_{bus} = \begin{bmatrix} -(j0.5 + j0.4 + j0.4) & j0.5 & j0.4 & j0.4 \\ j0.5 & -(j0.5 + j0.6) & j0.6 & 0 \\ j0.4 & j0.6 & -(j0.6 + j0.5 + j0.4) & j0.5 \\ j0.4 & 0 & j0.5 & -(j0.5 + j0.4) \end{bmatrix}$$

$$Y_{bus} = \begin{bmatrix} -j1.3 & j0.5 & j0.4 & j0.4 \\ j0.5 & -j1.1 & j0.6 & 0 \\ j0.4 & j0.6 & -j1.5 & j0.5 \\ j0.4 & 0 & j0.5 & -j0.9 \end{bmatrix}$$

The elements of new bus matrix after eliminating 4th row and 4th column is given by

$$Y_{jknew=}Y_{jk} - \left(\frac{Y_{jn}.Y_{nk}}{Y_{nn}} \right), \text{ where, n=4, j=1,2,3 , k=1,2,3.}$$

The bus admittance matrix is symmetrical. $\therefore Y_{kjnew} = Y_{jknew}$

$$Y_{11new} = Y_{11} - \frac{Y_{14}.Y_{41}}{Y_{44}} = -j1.3 - \frac{(j0.4).(j0.4)}{-j0.9} = -j1.12$$

$$Y_{12new} = Y_{12} - \frac{Y_{14}.Y_{42}}{Y_{44}} = j0.5 - \frac{(j0.4).(0)}{-j0.9} = j0.5$$

$$Y_{13new} = Y_{13} - \frac{Y_{14}.Y_{43}}{Y_{44}} = j0.4 - \frac{(j0.4).(j0.5)}{-j0.9} = j0.622$$

$$Y_{21new} = Y_{12new} = j0.5$$

$$Y_{22new} = Y_{22} - \frac{Y_{24}.Y_{42}}{Y_{44}} = -j1.1 - \frac{(0).(0)}{-j0.9} = -j1.1$$

$$Y_{23new} = Y_{23} - \frac{Y_{24}.Y_{43}}{Y_{44}} = 0.6 - \frac{(0).(0.5)}{-0.9} = j0.6$$

$$Y_{31new} = Y_{13new} = j0.622$$

$$Y_{32new} = Y_{23new} = j0.6$$

$$Y_{33new} = Y_{33} - \frac{Y_{34}.Y_{43}}{Y_{44}} = -j1.5 - \frac{(j0.5).(j0.5)}{-j0.9} = -j1.222$$

The reduced bus admittance matrix after eliminating 4th row is shown below

$$Y_{bus} = \begin{bmatrix} -j1.12 & j0.5 & j0.622 \\ j0.5 & -j1.1 & j0.6 \\ j0.622 & j0.6 & -j1.222 \end{bmatrix}$$

22. **A 90 MVA 11 KV 3 phase generator has a reactance of 25%. The generator supplies two motors through transformer and transmission line as shoen in fig. The transformer T1 is a 3-phase transformer 100 MVA 10/132 KV, 6% reactance. The transformer T2 is composed of 3 single phase units each rated 300 MVA: 66/10 KV with 5% reactance. The connection of T1 & T2 are shown. The motors are rated at 50 MVA and 400 MVA both 10 KV and 20% reactance. Taking the generator rating as base, Draw reactance diagram and indicate the reactance in per unit. The reactance of line is 100 ohms.**
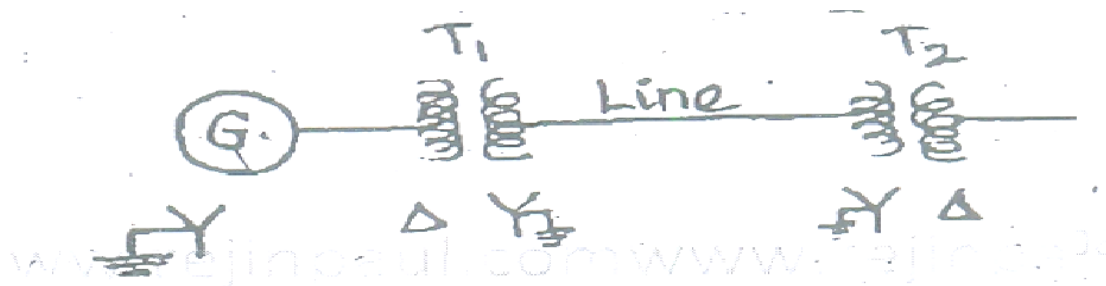
Solution:

$MVA_{b,new} = 90$

$kV_{b,new} = 11$ kV (Generator,G_1)

Generator, G_1 : $Z_{p.u. new} = Z_{pu,given} \times \dfrac{kV_{b,given}^2}{kV_{b,new}^2} \times \dfrac{MVA_{b,new}}{MVA_{b,given}}$

$Z_{p.u. new} = j0.2 \times \dfrac{6.6^2}{6.6^2} \times \dfrac{30}{25} = j\,0.24 \text{ p.u}$

Transformer T_1 referred to Primary side:

$kV_{b,new} = 6.6 \text{ kV}$

$Z_{p.u. new} = Z_{pu,given} \times \dfrac{kV_{b,given}^2}{kV_{b,new}^2} \times \dfrac{MVA_{b,new}}{MVA_{b,given}}$

$Z_{p.u. new} = j0.1 \times \dfrac{6.9^2}{6.6^2} \times \dfrac{30}{30} = j\,0.109 \text{ p.u}$

Transmission line j 120Ω :

$KV_{b,new} = KV_{b,old} \times \left(\dfrac{H.T \text{ side rating of } T_1}{L.T \text{ side rating of } T_1} \right)$

$KV_{b,new} = 6.6 \times \left(\dfrac{115}{6.9} \right) = 110 \text{ kV}$

$Z_{actual} = j\,120Ω$

$Z_{p.u\ new} = \dfrac{Z_{actual}}{Z_{base}} = \dfrac{Z_{actual}}{\left(kV_b^2 \right)} \times MVA_{b,\ new}$

$Z_{p.u. new} = \dfrac{j120}{110^2} \times 30 = j\,0.298 \text{ p.u}$

Transmission line j 90Ω :

$KV_{b,new} = 110 \text{ kV}$

$Z_{actual} = j\,90Ω$

$Z_{p.u} = \dfrac{j90}{110^2} \times 30 = j\,0.223 \text{ p.u}$

Transformer T_3 referred to Primary side: 3 single phase units are used.

Voltage rating: $\dfrac{\sqrt{3} \times 69}{\sqrt{3} \times 6.9} = 119.5/11.95 \text{ kV}$

$MVA_{b,given} = 3 \times 10 = 30 \ MVA$

$KV_{b,new} = 119.5 \text{ kV}$

$Z_{p.u. new} = Z_{pu,given} \times \dfrac{kV_{b,given}^2}{kV_{b,new}^2} \times \dfrac{MVA_{b,new}}{MVA_{b,given}}$

$Z_{p.u. new} = j0.1 \times \dfrac{119.5^2}{119.5^2} \times \dfrac{30}{30} = j\,0.1 \text{ p.u}$

Generator, G_3 :

$KV_{b,new} = 119.5 \times \left(\dfrac{11.95}{119.5} \right) = 11.95 \text{ kV}$

$Z_{p.u. new} = Z_{pu,given} \times \dfrac{kV_{b,given}^2}{kV_{b,new}^2} \times \dfrac{MVA_{b,new}}{MVA_{b,given}}$

$Z_{p.u. new} = j0.15 \times \dfrac{13.2^2}{11.95^2} \times \dfrac{30}{30} = j\,0.183 \text{ p.u}$

Transformer T_2 referred to Secondary side:

$KV_{b,new} = 110$ kV

$$Z_{p.u.\ new} = Z_{pu,given} \times \frac{kV_{b,given}^2}{kV_{b,new}^2} \times \frac{MVA_{b,new}}{MVA_{b,given}}$$

$$Z_{p.u.\ new} = j0.1 \times \frac{115^2}{110^2} \times \frac{30}{15} = j\ 0.218\ \text{p.u}$$

Generator, G_2 :

Transformer T_3 Primary side change occurs, so calculate $KV_{b,new}$ as

$$KV_{b,new} = KV_{b,old} \times (\frac{L.T\ side\ rating\ of\ T_2}{H.T\ side\ rating\ of\ T_2})$$

$$KV_{b,new} = 110 \times (\frac{6.9}{115}) = 6.6\ \text{kV}$$

$$Z_{p.u.\ new} = Z_{pu,given} \times \frac{kV_{b,given}^2}{kV_{b,new}^2} \times \frac{MVA_{b,new}}{MVA_{b,given}}$$

$$Z_{p.u.\ new} = j0.15 \times \frac{6.6^2}{6.6^2} \times \frac{30}{15} = j\ 0.3\ \text{p.u}$$

Impedance diagram.
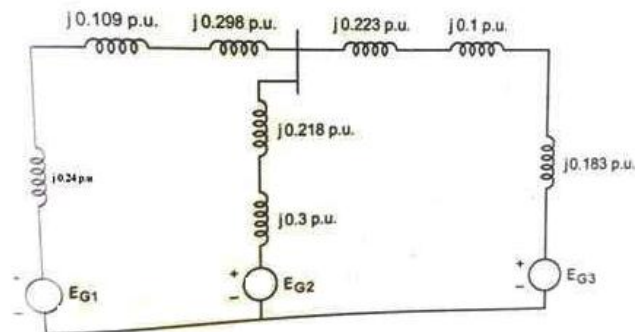
23. **Determine Ybus for the 3-bus system shown in fig. the line series impedance as follows**

Line (bus to bus)	Impedance (pu)
1-2	0.06+j0.18
1-3	0.03+j0.09
2-3	0.08+j0.24

Neglect the shunt capacitance of the lines.
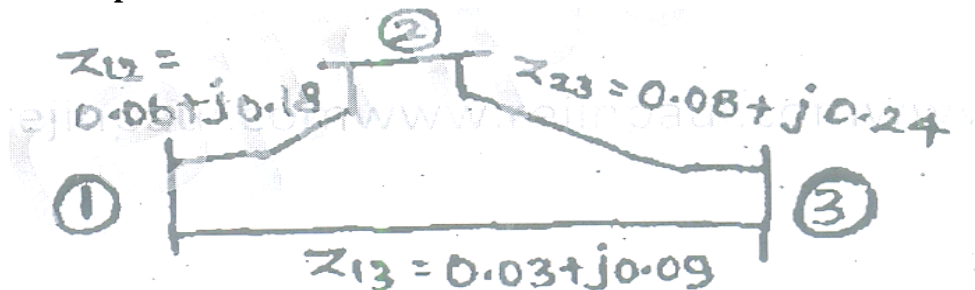
Solution:

Solution :

Step 1: Add an element between reference and node (1).

$Y_{bus} = [j\ 1.0]$
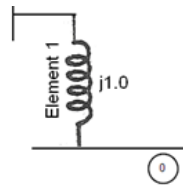
Step 2 : Add element between existing node (1) and the new node (2).

$$Y_{bus} = \begin{bmatrix} j\ 1.0 & j\ 1.0 \\ j\ 1.0 & j\ 1.0 + j\ 0.25 \end{bmatrix} = Z_{bus} = \begin{bmatrix} j\ 1.0 & j\ 1.0 \\ j\ 1.0 & j\ 1.25 \end{bmatrix}$$
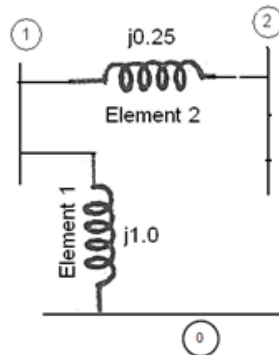
Step 3 : Add element between existing node (3) and the reference node.

$$Y_{bus} = \begin{bmatrix} j\ 1.0 & j\ 1.0 & j\ 1.0 \\ j\ 1.0 & j\ 1.25 & j\ 1.25 \\ j\ 1.0 & j\ 1.25 & j\ 1.25 + j\ 1.25 \end{bmatrix} = \begin{bmatrix} j\ 1.0 & j\ 1.0 & j\ 1.0 \\ j\ 1.0 & j\ 1.25 & j\ 1.25 \\ j\ 1.0 & j\ 1.25 & j\ 2.5 \end{bmatrix}$$
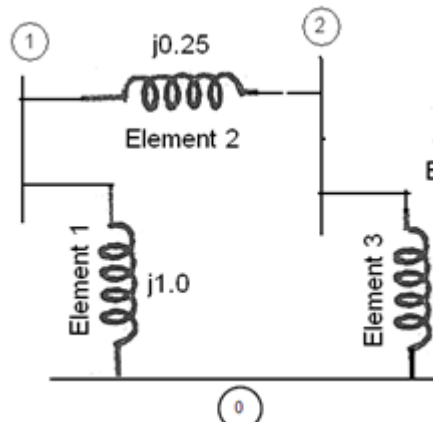
Fictitious node can be eliminated by

$$Z_{ij}^{new} = Z_{ij}^{old} - \frac{Z_{i(n+1)}\ Z_{(n+1)j}}{Z_{(n+1)(n+1)}}$$

$$Z_{11}^{new} = Z_{11}^{old} - \frac{Z_{13}\ Z_{31}}{Z_{33}} = j1.0 - \frac{j\ 1.0\ x\ j\ 1.0}{j\ 2.5} = j0.6$$

$$Z_{12}^{new} = Z_{21}^{old} = Z_{12}^{new} - \frac{Z_{13}\ Z_{32}}{Z_{33}} = j1.0 - \frac{j\ 1.0\ x\ j\ 1.25}{j\ 2.5} = j0.5$$

$$Z_{22}^{new} = Z_{22}^{old} - \frac{Z_{23}\ Z_{32}}{Z_{33}} = j1.25 - \frac{j\ 1.25\ x\ j\ 1.25}{j\ 2.5} = j0.625$$

$$Y_{bus} = \begin{bmatrix} j\ 0.6 & j\ 0.6 \\ j\ 0.6 & j\ 0.625 \end{bmatrix}$$

Step 4 : Add element between existing node (2) and the new node (3).
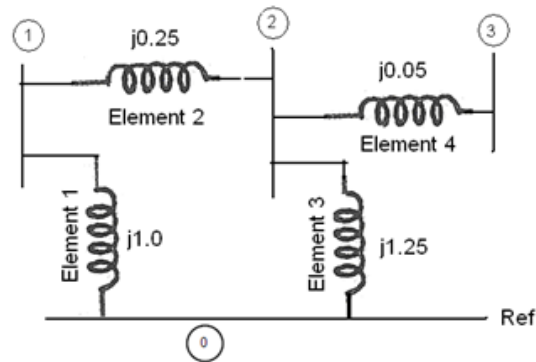
$$Y_{bus} = \begin{bmatrix} j\,0.6 & j\,0.6 & j\,0.6 \\ j\,0.6 & j\,0.625 & j\,0.625 \\ j\,0.6 & j\,0.625 & j\,0.625 + j\,0.05 \end{bmatrix}$$

$$Y_{bus} = \begin{bmatrix} j\,0.6 & j\,0.6 & j\,0.6 \\ j\,0.6 & j\,0.625 & j\,0.625 \\ j\,0.6 & j\,0.625 & j\,0.675 \end{bmatrix}$$

24. Obtain the per unit impedance diagram of the power system of fig. shown below
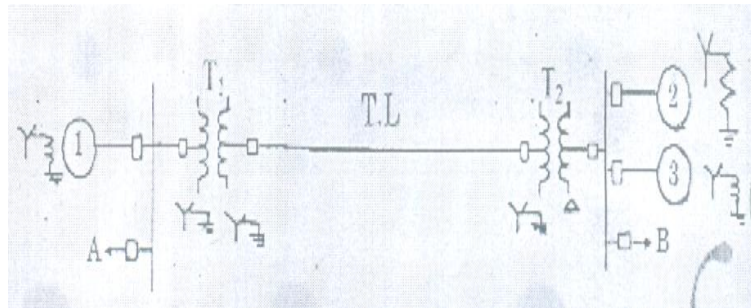
Fig one line diagram representation of a simple power system
Generator, G_1=1:30 MVA, 10.5 kV, X''=1.6 ohms
Generator, G_2=2:15 MVA, 6.6 kV, X''=1.2 ohms
Generator, G_3=3:25 MVA, 6.6 kV, X''=0.56 ohms
Transformer, T_1 = 15 MVA, 33/11 kV, X=15.2 ohms/phase on hgh tension side
Transformer, T_1 = 15 MVA, 33/6.2 kV, X=16 ohms/phase on hgh tension side
Transmission line: 20.5 ohms/phase
Load A: 15 MW, 11 KV, 0.9 lagging power factor
Load B: 40 MW, 6.6 KV, 0.85 lagging power factor
Solution:

$$MVA_{b,new} = 50$$
$$kV_{b,new} = 11 \text{ kV}$$

Generator, G_1 : $Z_{p.u.\ new} = Z_{pu,given} \times \dfrac{kV_{b,given}^{2}}{kV_{b,new}^{2}} \times \dfrac{MVA_{b,new}}{MVA_{b,given}}$

$$Z_{p.u.\ new} = j0.25 \times \frac{11^2}{11^2} \times \frac{50}{20} = j\,0.625 \text{ p.u}$$

Transformer T_1 referred to Primary side:
$$kV_{b,new} = 11 \text{ kV}$$

$$Z_{p.u.\ new} = Z_{pu,given} \times \frac{kV_{b,given}^2}{kV_{b,new}^2} \times \frac{MVA_{b,new}}{MVA_{b,given}}$$

$$\boxed{Z_{p.u.\ new} = j0.15 \times \frac{13.8^2}{11^2} \times \frac{50}{25} = j\ 0.472\ p.u}$$

Transmission line j 80Ω :

Transformer T_1 Secondary side change occurs, so calculate $KV_{b,new}$ as

$$KV_{b,new} = KV_{b,old} \times \left(\frac{H.T\ side\ rating\ of\ T_1}{L.T\ side\ rating\ of\ T_1} \right)$$

$$KV_{b,new} = 11 \times \left(\frac{220}{13.8} \right) = 175.36\ kV$$

Z_{actual} = j 80Ω

$$Z_{p.u\ new} = \frac{Z_{actual}}{Z_{base}} = \frac{Z_{actual}}{\left(kV_b^2 \right)} \times MVA_{b,\ new}$$

$$\boxed{Z_{p.u} = \frac{j80}{175.36^2} \times 50 = j\ 0.163\ p.u}$$

Transmission line j 100Ω :

$KV_{b,new}$ = 175.36 kV

Z_{actual} = j 100Ω

$$\boxed{Z_{p.u} = \frac{j100}{175.36^2} \times 50 = j\ 0.163\ p.u}$$

Transformer T_2 referred to line side: 3 single phase units are used.

     Voltage rating: 3 x 127/18 kV = 220/18 kV

Note: **Star side** , $V_L = \sqrt{3}\ V_p$; **Delta side** , $V_L = V_p$; Power = 3 $V_L\ I_L$

$MVA_{b,given}$ = 3x10 = 30 MVA

$$Z_{p.u.\ new} = Z_{pu,given} \times \frac{kV_{b,given}^2}{kV_{b,new}^2} \times \frac{MVA_{b,new}}{MVA_{b,given}}$$

$$\boxed{Z_{p.u.\ new} = j0.15 \times \frac{220^2}{175.36^2} \times \frac{50}{30} = j\ 0.393\ p.u}$$

Generator, G_2 :

Transformer T_2 Primary side change occurs, so calculate $KV_{b,new}$ as

$$KV_{b,new} = KV_{b,old} \times \left(\frac{L.T\ side\ rating\ of\ T_2}{H.T\ side\ rating\ of\ T_2} \right)$$

$$KV_{b,new} = 175.36 \times \left(\frac{18}{220} \right) = 14.348\ kV$$

$$Z_{p.u.\ new} = Z_{pu,given} \times \frac{kV_{b,given}^2}{kV_{b,new}^2} \times \frac{MVA_{b,new}}{MVA_{b,given}}$$

$$\boxed{Z_{p.u.\ new} = j0.25 \times \frac{18^2}{14.348^2} \times \frac{50}{30} = j\ 0.656\ p.u}$$

Transformer T_3 referred to line side:

$KV_{b,new}$ = 175.36 kV

$$Z_{p.u.\ new} = Z_{pu,given} \times \frac{kV_{b,given}^2}{kV_{b,new}^2} \times \frac{MVA_{b,new}}{MVA_{b,given}}$$

$$Z_{p.u.\ new} = j0.15 \times \frac{220^2}{175.36^2} \times \frac{50}{35} = j\ 0.337\ p.u$$

Generator, G_3 :

Transformer T_3 Primary side change occurs, so calculate $KV_{b,new}$ as

$$KV_{b,new} = KV_{b,old} \times \left(\frac{L.T\ side\ rating\ of\ T_3}{H.T\ side\ rating\ of\ T_3} \right)$$

$$KV_{b,new} = 175.36 \times \left(\frac{22}{220} \right) = 17.536\ kV$$

$$KV_{b,given} = 20\ kV$$

$$Z_{p.u.\ new} = Z_{pu,given} \times \frac{kV_{b,given}^2}{kV_{b,new}^2} \times \frac{MVA_{b,new}}{MVA_{b,given}}$$

$$Z_{p.u.\ new} = j0.21 \times \frac{20^2}{17.536^2} \times \frac{50}{30} = j\ 0.455\ p.u$$

Impedance diagram.
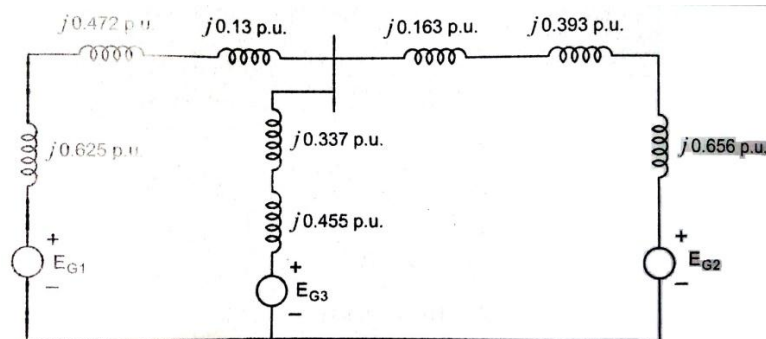
25. Explain the structure of modern power system with a neat sketch.

    i.   3 phase 3 wire circuits - Instantaneous sum of three line current is zero

        a.   3 phase circuit advantages - Economical , carry three times more power than single phase

    ii.   3 phase 4 wire circuits - 4th wire is Neutral wire and acts as return conductor

Single line diagram

    Single line diagram is a simplified representation of power system components along with their interconnections with each other. Each component is represented by its symbol.

Power system Components - Generator, Transformer, Transmission lines & Distribution

Tolerance level - +5 to 10%. Difference in voltages caused due to variation in loads

Primary transmission - First stage of transmission, 110kV,132 kV or 220 kV or400 kV or 765 kV, high voltage transmission, 3ϕ , 3 wire system.

Secondary transmission - 3ϕ , 3 wire system,33kV high voltage line 66kv to factory supply

Primary distribution - 3ϕ , 3 wire system,11kv or 6.6 kV, 3ϕ , 3 wire system

Secondary distribution - 400V, 3phase, 230V, 1phase, 3 phase 4 wire

Components of secondary distribution - Substation, feeders, service mains

Interconnection diagram - Feeders, service mains, distributors

Feeder - Conductors that take power from receiving station to substation

Distributor - Conductor that transfer power to consumers by tapping

Service mains - Connects distributor and consumer premises

Unit : II - POWER FLOW ANALYSIS

Importance of power flow analysis in planning and operation of power systems. Statement of power flow problem - classification of buses into P-Q buses, P-V (voltage-controlled) buses and slack bus. Development of Power flow model in complex variables form and polar variables form. Iterative solution using Gauss-Seidel method including Q-limit check for voltage-controlled buses – algorithm and flow chart. Iterative solution using Newton-Raphson (N-R) method (polar form) including Q-limit check and bus switching for voltage-controlled buses - Jacobian matrix elements – algorithm and flow chart. Development of Fast Decoupled Power Flow (FDPF) model and iterative solution – algorithm and flowchart; Comparison of the three methods.

PART – A

1. **Write the need for slack bus in load flow analysis.   (APR/MAY 18, NOV/DEC 16)**

    The slack bus is needed to account for transmission line losses. In a power system the total power generated will be equal to sum of power consumed by loads and losses. In a power system only the generated power and load power are specified for buses. The slack bus is assumed to generate the power required for losses. Since the losses are unknown the real and reactive power are not specified for slack bus. They are estimated through the solution of load flow equations.

2. **Discuss the effect of acceleration factor in load flow study.   (APR/MAY 18)**
    In load flow solution by iterative methods, the number of iterations can be reduced if the correction voltage at each bus is multiplied by some constant. The multiplication of the constant will increase the amount of correction to bring the voltage closer to the value it is approaching. The multipliers that accomplish this improved convergence are called acceleration factors. An acceleration factor of 1.6 is normally used in load flow problems. Studies may be made to determine the best choice for a particular system

3. **What is need for load flow analysis?**

    (MAY/JUNE 2016 & NOV/DEC 2015 & 2017)

    Power flow analysis or load flow analysis is one of the basic tools used in power systems studies. It is concerned with the steady state analysis of the system when it is working under a normal balanced operating condition. Load flow or power flow analysis is the determination of the voltage, current, real power and reactive power at points in electrical network

4. **Mention the various types of buses in power system with specified quantities for each bus.   (MAY/JUNE 2016, NOV/DEC 2017)**
    The following table shows the quantities specified and to be obtained for various types of buses.

Bus type	Quantities specified	Quantities to be obtained
Load bus	P,Q	$\lvert V \rvert$, δ
Generator bus	P, $\lvert V \rvert$	Q, δ
Slack bus	$\lvert V \rvert$, δ	P,Q

5. **Compare the Newton Raphson and Gauss Seidal methods of load flow solutions.**

    (MAY/JUNE 2017)

Gauss Seidal method	Newton Raphson method
Variable is expressed in rectangular	Variables are expressed in polar

coordinates.	coordinates.
Computation time per iteration is less	Computation time per iteration is more.
It has linear convergence characteristics.	It has quadratic convergence characteristics.
The number of iterations required for convergence increases with size of the system.	The numbers of iterations are independent of the size of the system.
The choice of slack bus is critical.	The choice of slack bus is arbitrary.

6. Write are the quantities that are associated with each bus in a system. (MAY/JUNE 2017)

Each bus in a power system is associated with four quantities and they are

i. real power
ii. reactive power
iii. magnitude of voltage,
iv. phase angle of voltage.

7. What is jacobian matrix? (NOV/DEC 2016)

The matrix formed from the first derivatives of load flow equation is called jacobian matrix and it is denoted by J.

The elements of jacobian matrix will change in every iteration. The elements of the jacobian matrix are obtain matrix are obtained by partially differentiating the load flow equation with respect to a unknown variable and then evaluating the first derivative as using the solution of previous iteration

8. When is generator buses treated as load bus? (NOV/DEC 2015)

If the reactive power constraints of a generator bus violates the specified limits then the generator is treated as load bus.

If $Q_i > Q_{i(max)}$, substitute $Q_i = Q_{i(max)}$

If $Q_i^{cal} < Q_{i(min)}$, substitute $Q_i = Q_{i(min)}$

9.Write the most important mode of operation of power system and mention the major problems encountered with it.

Symmetrical steady state is the most important mode of operation of power system. Three major problems are encountered in this mode of operation. They are,

1) Load flow problem
2) Optimal load scheduling problem
3) Systems control problem

10. What is power flow study or load flow study ?

The study of various methods of solution to power system network is referred to as load flow study. The solution provides the voltages at various buses, power flowing in various lines and line-losses.

The load flow study of a power system is essential to decide the best operation of existing system and for planning the future expansion of the system. It is also essential for designing a new power system.

11.Why the load flow studies are important for planning the existing system as well as its future expansion?

The load flow studies are very important for planning, economic scheduling, control and

operation of existing systems as well as planning its future expansion depends upon knowing the effect of interconnections, new loads, new generating stations, or new transmission lines, etc. before they installed.

12.Draw sample power system network.

Power system network consist of following parts namely as

      a. Generator,

      b. Transmission lines,

      c. Transformer,

      d. load

13.Why power flow analysis is made?

Power flow analysis is performed to calculate the magnitude and phase angle of voltages at the buses and also the active power and reactive volt-amperes flow for the given terminal or bus conditions. The variables associated with each bus or node are,

      a. Magnitude of voltage |V|

      b. Phase angle of voltage δ

      c. Active power, P

      d. Reactive voltamperes, Q

14.What are the works involved in a load flow study? (Or) How a load flow study is performed?

The following work has to be performed for a load flow study.

(i)   Representation of the system by single line diagrams.

(ii)   Determining the impedance diagram using the information in single line diagram.

(iii) Formulation of network equations.

(iv) Solution of network equations.

15.What are the information that are obtained from a load flow study?

The information obtained from a load flow study are magnitude and phase angles of bus voltages, real and reactive power flowing in each line and line losses. The load flow solution also gives the initial conditions of the system when the transient behavior of the system is to be studied.

16.Write about ideal load flow problem.

The network configuration and all the bus power injections.

$$P_i = P_G - P_D$$

Where

P_i = bus power injection.

P_G = Bus generation

P_D = Bus demand.

To determine the complex voltages at all the buses.

The state vector X is defined as $X = [V_1, V_2 \dots, V_N, \delta_1, \delta_2 \dots \delta_N]^T$

Once the voltages at all the buses are known, then we can compute slack bus power, power flows

in the transmission lines and power loss in the transmission lines.

17. What is meant by flat voltage start?

In iterative methods of load flow solution, the initial voltages of all buses exept slack bus are assumed as 1+j0 pu. This is referred as flat voltage profile.

$V = |V_{spec}| \angle 0°$ for slack bus

$V = |V_{spec}| \angle 0°$ for generator bus

$V = 1 \angle 0°$ for load bus

18.Write an equation in loop frame of reference for power flow analysis.

$[V_{LOOP}] = [Z_{LOOP}] [I_{LOOP}]$

Where $[Z_{LOOP}]$ = Bus impedance matrix.

$[V_{LOOP}]$ = Voltage matrix

$[I_{LOOP}]$ = Current matrix.

19. Why do we go for iterative methods to solve load flow problems?

The load (or power) flow equations are nonlinear equations and so explicit solution is not possible. The solution of nonlinear equations can be obtained only by iterative numerical techniques. As the number of iteration increases in a load flow problem or power flow problem the solution obtained will be more accurate.

20. What are the operating constraints imposed in the load flow studies & What are the iterative methods used for solution of load flow study ?

The operating constraints imposed in load flow studies are reactive power limits for generator buses and allowable change in magnitude of voltage for load buses.

- *Iterative methods used for load flow study.*
  1. Guass seidal method
  2. Newton Raphson method
  3. Fast decouple method.

21.-Write about practical load flow problem.

The network configuration, complex power demands for all buses, real power generation schedules and voltage magnitudes of all the P-V buses and voltage magnitude of the slack bus.

To determine:

Bus admittance matrix.

Bus voltage phase angles of all buses except the slack bus and bus voltage magnitudes of all the P-Q buses.

The state vector X is defined as $X= [V_1, V_2…,V_N, \delta_1, \delta_2……. \delta_N]$

22. What is a bus?

The meeting point of various components in a power system is called a bus. The bus is a conductor made of copper or aluminum having negligible resistance .At some of the buses power is being injected into the network, whereas at other buses it is being tapped by the system loads.

23. What are the different types of buses in a power system?

The buses of a power system can be classified into three types based on the quantities being specified for the buses, which are as follows:

a. Load bus or PQ bus (P and Q are specified)

b. Generator bus or voltage controlled bus or PV bus (P and V are specified)

c. Slack bus or swing bus or reference bus (|V| and δ are specified)

24. Define Voltage controlled bus.

A bus is called voltage controlled bus if the magnitude of voltage │V│ and real power (P) are specified for it. In a voltage controlled bus the magnitude of the voltage is not allowed to change. The other names for voltage controlled bus are generator bus and PV bus. In this bus the phase angle of the voltages and the reactive power are to be determined. The limits on the reactive power are also specified.

25. What is swing bus?

A bus is called swing bus when the magnitude and phase for bus voltage are specified for it. The swing bus is the reference bus for load flow solution and it is required for accounting line losses. Usually one of the generator bus selected as the swing bus.

Swing bus is also called as Slack bus.

26. What will be the reactive power and bus voltage when the generator bus is treated as load bus?

When the generator bus is treated as load bus the reactive power of the bus is equated to the limit it has violated, and the previous iteration value of bus voltage is used for calculating current iteration value.

If $Q_i > Q_{i(max)}$, then $Q_i = Q_{i(max)}$

If $Q_i < Q_{i(min)}$, then $Q_i = Q_{i(min)}$

Reactive power of the bus has violates the specified limits, then the P-V bus will act as load bus.

27. What is jacobian matrix? How the elements of jacobian matrix are computed?

The matrix formed from the first derivatives of load flow equation is called jacobian matrix and it is denoted by J.

The elements of jacobian matrix will change in every iteration. The elements of the jacobian matrix are obtain matrix are obtained by partially differentiating the load flow equation with respect to a unknown variable and then evaluating the first derivative as using the solution of previous iteration .

29. What is the use of acceleration factor in load flow algorithm.

The acceleration factor is a real quantity and it modifies the magnitude of voltage alone. Since in voltage controlled bus (generator bus), the magnitude of bus voltage is not allowed to change, the acceleration factor is not used for voltage controlled bus. (i.e acceleration factor is used only for load bus)

30. Give the power flow equation in polar form

$P_i = |V_i|^2 |Y_{ii}| \cos \theta_{ii} \sum_{j=1}^{N} |V_i||Y_{ij}||V_j| \cos (\theta_{ij} + \delta_j - \delta_i)$

$Q_i = - |V_i|^2 |Y_{ii}| \sin \theta_{ii} \sum_{j=1}^{N} |V_i||Y_{ij}||V_j| \cos (\theta_{ij} + \delta_j - \delta_i)$

The above equations are called as polar form of the power flow equations.

31.Define primitive network.

Primitive network is a set of unconnected elements which provides information regarding the characteristics of individual elements only. The performance equations of primitive network are given below.

V + E = ZI (In Impedance form)

I + J = YV (In Admittance form)

where V and I are the element voltage and current vectors respectively.

J and E are source vectors.

Z and Y are the primitive Impedance and Admittance matrices respectively.

32. What are the iterative methods mainly used for solution of load flow study?

The Gauss seidal method and Newton Raphson method are the two iterative methods which are mainly used in load flow study. Because Fast decoupled method requires more number of iteration when compared to other two iteration methods and the FDLF is suitable only for large size bus systems.

33. Why it is necessary to use acceleration factor in Gauss Seidal method of load flow studies?

In Gauss Seidal method, the number of iteration required for convergence can be reduced if the voltage computed at each iteration is multiplied by a factor greater than unity called acceleration factor to bring the voltage closer to the value to which it is converging. The range of 1.3 to 1.7 is found to be satisfactory for the typical systems.

$$V_i^{new} = V_i^{old} + \propto [V_i^{new} - V_i^{old}]$$

Where V_i^{old} = Voltage value obtained in previous iteration

V_i^{new} = New value of Voltage value obtained in current iteration

$\propto$ = Acceleration factor

34. Write the load flow equation of Gauss-Seidel method.

$$V_i^{new} = \frac{1}{Y_{ii}} [\frac{P_i - jQ_i}{V_i^{*old}} - \sum_{j=1}^{i=1} Y_{ij} V_j^{new} - \sum_{j=i+1}^{N} Y_{ij} V_j^{old}]$$

Above equation is used to determine the new voltage in load flow analysis in Gauss-Seidel method.

35. Why bus admittance matrix is used in Gauss Seidal instead of bus impedance matrix?

Using bus admittance matrix is amenable to digital computer analysis, because it could be formed and modified for network changes in subsequent cases. Bus admittance matrix is used in Gauss seidal method because of the following reasons.

- It requires less computation time
- Less memory allocation

36. What are the advantages of Gauss seidal method?

The advantages of Gauss seidal method are as follows

    i. Calculations are simple and so the programming task is less.

    ii. The memory requirement is less.

    iii. Useful for small systems

37. What are the disadvantages of Gauss seidal method?

The disadvantages of Gauss seidal method are listed as follows

    i. Requires large no. of iterations to reach converge

    ii. Not suitable for large systems.

    iii. Convergence time increases with size of the system

38. Give the Q limit condition for Gauss seidal load flow method.

If $Q_{i(min)} < Q_{Gi} < Q_{i(max)}$, then $Q_{i(spec)} = Q_i^{cal}$

If $Q_{i(min)} < Q_{Gi}$, then $Q_{i(spec)} = Q_{i(min)} - Q_{Li}$

If $Q_{i(max)} < Q_{Gi}$, then $Q_{i(spec)} = Q_{i(max)} - Q_{Li}$

If Q limit is violated, then treat this bus as P-Q bus till convergence is obtained.

39. What are the advantages of Newton-Raphson method?

The advantages of Newton-Raphson method are,

  i. This load flow method is faster, more reliable and he results are accurate.

  ii. Requires less number of iterations for convergence.

  iii. The number of iterations are independent of the size of the system.

  iv. Suitable for large size systems.

40. What are the disadvantages of Newton-Raphson method?

The disadvantages of Newton-Raphson method are,

  i. Programming is more complex.

  ii. The memory requirement is more.

  iii. Computational time per iteration is higher due to larger number of calculations per iteration.

41. How the disadvantages of N-R method are overcome?

The disadvantage of large memory requirement can be overcome by decoupling the weak coupling between P-δ and Q-V. (i.e., using decoupled load flow algorithm). The disadvantage of large computational time per iteration can be reduced by simplifying the decoupled load flow equations. The simplifications are made based on the practical operating conditions of a power system.

42. Give the Q limit condition for Newton Raphson load flow method.

  for PV bus, Check for Q limit violation

   If $Q_{i(min)} < Q_i{}^{cal} < Q_{i(max)}$, the bus acts as PV bus

   If $Q_i{}^{cal} > Q_{i(max)}$, then $Q_{i(spec)} = Q_{i(max)}$

   If $Q_i{}^{cal} < Q_{i(min)}$, then $Q_{i(spec)} = Q_{i(min)}$, the PV bus will act as PQ bus.

43. Write the load –flow equations for Newton-Raphson method.

$$P_i = \sum_{j=1}^{N}|V_i||Y_{ij}||V_j|cos(\theta_{ij} + \delta_j - \delta_i)$$

$$Q_i = \sum_{j=1}^{N}|V_i||Y_{ij}||V_j|sin(\theta_{ij} + \delta_j - \delta_i)$$

Above equation is used to determine the power flow in load flow analysis in Newton-Raphson method.

44. How approximation is performed in Newton-Raphson method?

In Newton-Raphson method, the set of nonlinear simultaneous (load flow) equations are approximated to a set of linear simultaneous equations using Taylor's series expansion and the terms are limited to first order approximation. The approximation procedure involved is based upon the initial estimate of unknown and simply it is called as successive approximation method.

45. How the convergence of N-R method is speeded up?

The convergence can be speeded up in N-R method by using Fast Decoupled Load Flow (FDLF) algorithm. In FDLF method the weak coupling between P-δ and Q-V are decoupled and then the equations are further simplified equations are further simplified using the knowledge of practical operating conditions of a power system.

46. List out the advantages of Fast Decoupled method.

The advantages of Fast Decoupled method are,

  i. This load flow method is faster, more reliable and the results are accurate.

  ii. Programming is simple

  iii. The memory requirement is less compared to NR method.

iv. Computational time per iteration is less.

47.What are the disadvantages of Fast Decoupled method?

The main disadvantages of Fast Decoupled method are listed as follows,

i. Require more number of iterations.

ii. Suitable only for large bus systems but the number of iteration does not depend upon the size of the system.

48.In contingency analysis, which load flow is preferred? And give reasons for it.

In contingency analysis, fast decoupled method is suitable for performing load flow analysis due to following reasons.

i. Programming is simple.

ii. Computational time per iteration is less.

49. Give the Q limit condition for Fast decoupled load flow method.

for PV bus, Check for Q limit violation

If $Q_{i(min)} < Q_i < Q_{i(max)}$, calculate P_i^{cal}

If $Q_i^{cal} < Q_{i(min)}$, then $Q_{i(spec)} = Q_{i(min)}$

If $Q_i^{cal} > Q_{i(max)}$, then $Q_{i(spec)} = Q_{i(max)}$, the PV bus will act as PQ bus.

50.Compare all the different methods of load flow study.

S.No	G.S	N.R	FDLF
1	Require large number of iterations to reach convergence.	Require less number of iterations to reach convergence.	Require more number of iterations than N.R method.
2	Computation time per iteration is less	Computation time per iteration is more	Computation time per iteration is less
3	It has linear convergence	It has quadratic convergence	No convergency
4	The number of iterations required for convergence increases	The number of iterations are independent of the	The number of iterations are does not dependent of the
5	Less memory requirements.	More memory requirements.	Less memory requirements than N.R. method.

PART – B

1. **For the system shown in Fig., determine the voltages at the end of the first iteration by Gauss – seidel method and also find the slack bus power, line flows, transmission line loss. Assume base MVA as 100.** **(APRIL/MAY 2018)**
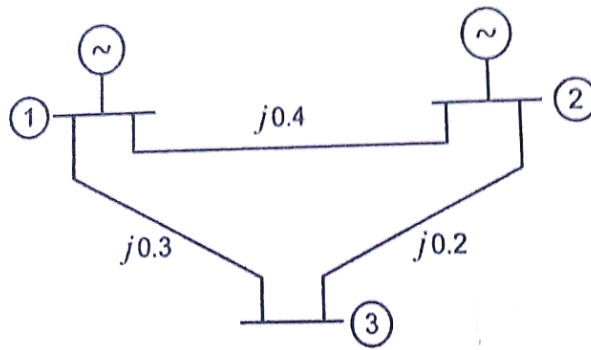
Solution:

Bus No.	Voltage	Generator			Load	Q_{min} MVAR	Q_{max} MVAR
		P	**Q**		**Q**		
1.	1.05∠0° p.u.	-	-		-	-	-
2.	1.02 p.u.	0.3 p.u.	-		-	-10	100
3.	-	-	-		0.2 p.u.	-	-

Step – 1: From Y-bus.

$$Y_{bus} = \begin{bmatrix} \dfrac{1}{Y_{11}} & \dfrac{-1}{Y_{12}} & \dfrac{-1}{Y_{13}} \\ \dfrac{-1}{Y_{21}} & \dfrac{1}{Y_{22}} & \dfrac{-1}{Y_{23}} \\ \dfrac{-1}{Y_{31}} & \dfrac{-1}{Y_{32}} & \dfrac{1}{Y_{33}} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{j0.4} + \dfrac{1}{j0.3} & \dfrac{-1}{j0.4} & \dfrac{-1}{j0.3} \\ \dfrac{-1}{j0.4} & \dfrac{1}{j0.4} + \dfrac{1}{j0.2} & \dfrac{-1}{j0.2} \\ \dfrac{-1}{j0.3} & \dfrac{-1}{j0.2} & \dfrac{1}{j0.3} + \dfrac{1}{j0.2} \end{bmatrix}$$

$$Y_{bus} = \begin{bmatrix} -j5.8333 & j2.5 & j3.3333 \\ j2.5 & -j7.5 & j5 \\ j3.3333 & j5 & -j8.3333 \end{bmatrix}$$

Step – 2: Initialize bus voltages.

$V_1^{old} = 1.05\angle 0°$ p.u.   [Bus 1 is a slack bus i.e., V and δ is specified]

$V_2^{old} = 1.02\angle 0°$ p.u.   [Bus 2 is a PV bus i.e., P and V is specified]

$V_3^{old} = 1.0$ p.u.   [Bus 3 is a load bus i.e., P and Q is specified]

Note

For Slack bus, the specified voltage will not change in any iteration.

For generation bus, calculate V_i^{new} using the formula and write

$$V_i^{new} = V_{specified}\angle\delta_{calculated\ value}$$

Step 3 : Calculate Q value for all generator buses.

Q_i^{cal} = - Im {$V_i^{old}*[\sum_{j=1}^{i-1} Yij\ Vjnew + \sum_{j=i}^{N} Yij\ Vjold]$}

Q_2^{cal} = - Im {$V_2^{old}*[Y_{21}V_1^{new} + Y_{22}V_2^{old} + Y_{23}V_3^{old}]$}

Q_2^{cal} = - Im {$1.02\angle0°*[j2.5X1.05\angle0° + (-j7.5X1.02\angle0°) + j5\ X\ 1\angle0°]$}

Q_2^{cal} = - Im {$1.02\angle0°*j2.625-j7.65+j5]$}

$\boxed{Q_2^{cal} = 0.025\ \text{p.u.}}$

Now $Q_{2(min)} \le Q_2^{cal} \le Q_{2(max)}$

i.e., Q_2^{cal} is within the specified limit.

Step – 4: Calculate V_i^{new} .

$V_i^{new} = 1.05\angle0°$ p.u.

$V_i^{new} = \frac{1}{Y_{ii}}\left[\frac{P_i-jQ_i}{V_i^{old*}} - \sum_{j=1}^{i-1} Y_{ij}\ V_j^{new} - \sum_{J=i+1}^{N} Y_{ij}\ V_j^{old}\right]$

$V_2^{new} = \frac{1}{Y_{22}}\left[\frac{P_2-jQ_2}{V_2^{old*}} - Y_{21}V_1^{new} - Y_{23}\ V_3^{old}\right]$

$P_2 = 0.3$ p.u. (Given); $Q_2 = 0.025$ p.u.

$V_2^{new} = \frac{1}{-j7.5}\left[\frac{0.3-j0.025}{1.02\angle0°} - j2.5\ X\ 1.05\angle0° - j5\ X\ 1\angle0°\right]$

$= 1.0199+j0.0392$

$\boxed{V_2^{new} = 1.0207\angle2.2°}$

$V_2^{new} = V_{2(spec)}\angle\delta_2^{cal} = 1.02\angle2.2° = 1.0192 + j0.0392$

$P_3 = P_{G3} - P_{L3} = 0 - 0.4 = -0.4$ p.u.

$Q_3 = Q_{G3} - Q_{L3} = 0 - 0.2 = -0.2$ p.u.

$V_3^{new} = \frac{1}{Y_{33}}\left[\frac{P_3-jQ_3}{V_3^{old*}} - Y_{31}V_1^{new} - Y_{32}\ V_2^{new}\right]$

$V_3^{new} = \frac{1}{-j8.3333}\left[\frac{-0.4+j0.2}{1.0\angle0°} - j3.3333\ X\ 1.05\angle0° - j5\ X\ 1.02\angle2.2°\right]$

$= \frac{1}{-j8.3333}[-0.4 + j0.2 - j3.4999 - j5.096 + 0.196]$

$= 1.0075 - j0.0244 = 1.0078\ \angle-1.39°$

$\boxed{V_3^{new} = 1.0075 - j0.0244 = 1.0078\ \angle -1.39°}$

Step 5: Slack bus power

$S_1 = P_1 - jQ_1 = V_1^* \sum_{j=1}^{N} Y_{ij}V_j$

$S_1 = V_1^* [Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3]$

$= 1.05[-j5.8333\ X\ 1.05\ \angle0° + j2.5\ X\ (1.0192 + j0.0392) + J3.3333\ (1.0075 - j0.0244)]$

$= -0.0175 - j0.2295$ p.u.

$\boxed{\begin{array}{l}P_1 = -0.0175\ \text{p.u.} = -1.75\ \text{MW} \\ Q_1 = 0.2295\ \text{p.u.} = 22.95\ \text{MVAR}\end{array}}$

Step 6: Line flow

$S_{ij} = P_{ij} + jQ_{ij} = V_i\ [\ V_i^* - V_j^*]Y_{ij}^*_{series} + |Vi|^2Y_{Pi}^*$

Line flow from bus 1 to 2.

$S_{12} = P_{12} + jQ_{12} = V_1[V_1^* - V_2^*]\ Y_{12}^*_{series}$

$= 1.05[(1.05\angle-0°) - 1.0192+j0.0392]\ j2.5$

$= -0.1029 + j0.0808$ p.u.

$S_{21} = P_{21}+jQ_{21} = V_2[V_2^* - V_1^*]\ Y_{21}^*_{Series}$

$= 1.0192 + j0.0392\ [1.0192-j0.0392-1.05]\ j2.5$

$$= 0.1029 - j0.0746 \text{ p.u}$$

$S_{23} = P_{23} + jQ_{23}$

$S_{23} = V_2 [V_2^* - V_3^*] \ Y_{23}^*{}_{series}$

$$= 1.0192 + j0.0392 [1.0192 - j0.0392 - 1.0075 - j0.0244] j5$$

$$= 0.3218 + j0.072 \text{ p.u}$$

$S_{32} = P_{32} + jQ_{32}$

$S_{32} = V_3 [V_3^* - V_2^*] \ Y_{32}^*{}_{series}$

$$= 1.0075 - j0.0244 [1.0075 + j0.0244 - 1.0192 + j0.0392] j5$$

$\boxed{S_{32} \quad = -0.3218 - j0.0512 \text{ p.u}}$

$S_{13} = P_{13} + jQ_{13}$

$S_{13} = V_1 [V_1^* - V_3^*] \ Y_{13}^*{}_{series} = 1.05[1.05\angle\text{-}0° - 1.0075 - j0.0244] j3.3333$

$\boxed{S_{13} = 0.085 + j0.148 \text{ p.u.}}$

$S_{31} = P_{31} + jQ_{31}$

$S_{31} = V_3 [V_3^* - V_1^*] \ Y_{31}^*{}_{series}$

$$= 1.0075 - j0.0244 \times [1.0075 + j0.0244 - 1.05] \times j3.3333$$

$\boxed{S_{31} = -0.085 - j0.1407 \text{ p.u}}$

Transmission Loss

$S_{ij \ Loss} = S_{ij} + S_{ji}$

For line 1-2,

$S_{12} = P_{12 \ Loss} + jQ_{12 \ Loss} = S_{12} + S_{21}$

$S_{12 \ Loss} = - 0.1029 + j0.0808 + 0.1029 - j0.0746 = 0 + j0.0061$

$P_{12 \ Loss} = 0, \ Q_{12 \ Loss} = 0.0061 \text{ p.u.} = 0.61 \text{ MVAR}$

For line 2-3,

$S_{23 \ Loss} = P_{23 \ Loss} + jQ_{23 \ Loss} = S_{23} + S_{32}$

$$= 0.3218 + j0.072 + (- 0.3218 - j0.0512)$$

$$= 0 + j0.021$$

$P_{23 \ Loss} = 0, \ Q_{23 \ Loss} = 0.021 \text{ p.u.} = 2.1 \text{ MVAR}$

For line 1-3,

$S_{13 \ Loss} = P_{13 \ Loss} + jQ_{13 \ Loss} = S_{13} + S_{31}$

$$= 0.085 + j0.148 + [- 0.085 - j0.1407]$$

$$= 0 + j0.00726$$

$P_{13 \ Loss} = 0,$

$\boxed{Q_{13 \ Loss} = 0.00726 \text{ p.u.} = 0.726 \text{ MVAR}}$

2. **Perform two iteration of Newton Raphson load flow method and determine the power flow solution for the given system. Take base MVA as base 100.** (APRIL/MAY 2018)

Solution:

Line Data:

Line	Bus	R(p.u.)	X(p.u.)	Half line charging

	From	To			admittance ($\frac{Y_P}{2}$ (p.u.))
1	1	2	0.0839	0.5183	0.0636

Bus Data:

Bus	P_L	Q_L
1	90	20
2	30	10

Step – 1: $Y_{bus} = \begin{bmatrix} 0.3044 - j1.816 & -0.3044 + j1.88 \\ -0.3044 + j1.88 & 0.3044 - j1.816 \end{bmatrix}$

$Y_{bus} = \begin{bmatrix} 1.842\angle -1.405 & 1.904\angle 1.7314 \\ 1.904\angle 1.7314 & 1.842\angle -1.405 \end{bmatrix}$ {Note: Use in rad mode}

Step – 2: Assume the initial value i.e., $\delta=0$, V=1.0

$[X] = \begin{bmatrix} \delta_2 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1.0 \end{bmatrix}$

Step – 3: Calculate P_2^{cal}, Q_2^{cal}, ΔP_2 and ΔQ_2.

$P_2^{cal} = |V_2|\{|V_1||Y_2| \cos(\theta_{12} + \delta_2 - \delta_1) + |V_2||Y_{22}|\cos(\theta_{22} + \delta_2 - \delta_2) \}$

$= 1.0[1.05 \ X \ 1.904 \cos(1.7314) + 1.842 \cos(-1.405)]$

$= 1.05 \ X \ 1.904(-0.15991) + 1.842(0.16503)$

$= -0.015$ p.u.

$P_{2(spec)} = P_{G2} - P_L$

$= 0 - \frac{30}{100} = -0.3$ p.u.

$P_{2(spec)} = -0.3$ p.u.

$\Delta P_2 = P_{2(spec)} - P_2^{cal}$

$= -0.3 - (-0.015) = -0.285$

$Q_2^{cal} = -V_2\{|V_1||Y_{21}| \sin(\theta_{12} + \delta_1 - \delta_2) + |V_2||Y_{22}| \sin(\theta_{22} + \delta_2 - \delta_2)\}$

$= -1.0[1.05 \ X \ 1.904 \sin(1.7314) + 1.0 \ X \ 1.842 \sin(-1.405)]$

$= -0.157$ p.u.

$\Delta Q_2 = Q_{2(spec)} - Q_2^{cal} = -0.1 - (-0.157)$

$= 0.057$

Step – 4: Form Jacobian matrix

$$\begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial V_2} \\ \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial V_2} \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta V_2 \end{bmatrix} = \begin{bmatrix} \Delta P_2 \\ \Delta Q_2 \end{bmatrix}$$

$\frac{\partial P_2}{\partial \delta_2} = |V_2||V_1||Y_{12}| \sin(\theta_{12} + \delta_1 - \delta_2) + |V_2|^2|Y_{22}| X \ 0$

$= 1.0 \ X \ 1.05 \ X 1.904 \sin(1.7314)$

$= 1.973$

Step – 5: Compute Δx,

$$\begin{bmatrix} \Delta\delta2 \\ \Delta V2 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2^1}{\partial \delta_2} & \frac{\partial P_2}{\partial V_2} \\ \frac{\partial Q2}{\partial \delta_2} & \frac{\partial Q_2}{\partial V_2} \end{bmatrix}^{-1} \cdot \begin{bmatrix} \Delta P_2 \\ \Delta Q_2 \end{bmatrix}$$

$$= \begin{bmatrix} 1.973 & 0.289 \\ -0.3196 & 1.66 \end{bmatrix} \times \begin{bmatrix} -0.285 \\ 0.057 \end{bmatrix}$$

$$= \begin{bmatrix} 0.493 & -0.086 \\ 0.0949 & 0.586 \end{bmatrix} \begin{bmatrix} -0.285 \\ 0.057 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} \Delta\delta2 \\ \Delta V2 \end{bmatrix} = \begin{bmatrix} -0.145 \\ 0.0064 \end{bmatrix}}$$

$$X = X° + \Delta X = \begin{bmatrix} 0 \\ 1.0 \end{bmatrix} + \begin{bmatrix} -0.145 \\ 0.0064 \end{bmatrix} = \begin{bmatrix} -0.145 \\ 0.0064 \end{bmatrix}$$

Iteration – 2: Compute mismatch vectors.

$P_2^{cal} = 1.0064 \,[1.05 \times 1.904 \cos(1.7314 + 0 + (-0.145)) + 1.0064 \times 1.842 \times \cos(-1.405)]$

$= -0.297$

$\Delta P_2 = P_{2(spec)} - P_2^{cal} = -0.3 - (-0.297) = -0.003$

$$\boxed{\Delta P_2 = -0.003}$$

$Q_2^{cal} = -\{1.0064\,[1.05 \times 1.904 \times \sin(1.7314 + 0 - (-0.145)) + 1.0064 \times 1.842 \times \sin(-1.405)]\}$

$= -0.078$

$\Delta Q_2 = Q_{2(spec)} - Q_2^{cal} = -0.1 - (-0.078) = -0.021$

$$\boxed{\Delta Q_2 = -0.021}$$

Compute Jacobian matrix

$\dfrac{\partial P_2}{\partial \delta_2} = 1.0064 \times 1.05 \times 1.904 \sin(1.7314 + 0.145)$

$= 1.919$

$\dfrac{\partial P_2}{\partial V_2} = 1.05 \times 1.904 \cos(1.7314 + 0.145) + 2 \times 1.0064 \times 1.842 \times \cos(-1.405)$

$= 0.011$

$\dfrac{\partial Q_2}{\partial \delta_2} = 1.0064 \times 1.05 \times 1.904 \cos(1.7314 + 0.145)$

$$\boxed{\dfrac{\partial Q_2}{\partial \delta_2} = -0.605}$$

$\dfrac{\partial Q_2}{\partial v_2} = -1.05 \times 1.904 \sin(1.7314 + 0.145) - 2 \times 1.0064 \times 1.842 \times \sin(-1.405)$

$$\boxed{\dfrac{\partial Q_2}{\partial V_2} = 1.75}$$

$$\begin{bmatrix} \Delta\delta2 \\ \Delta V2 \end{bmatrix} = \begin{bmatrix} 1.919 & 0.011 \\ -0.605 & 1.75 \end{bmatrix}^{-1} \cdot \begin{bmatrix} \Delta P_2 \\ \Delta Q_2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.52 & -0.0033 \\ 0.179 & 0.57 \end{bmatrix} \begin{bmatrix} -0.003 \\ -0.021 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} \Delta\delta2 \\ \Delta V2 \end{bmatrix} = \begin{bmatrix} -0.0015 \\ -0.0125 \end{bmatrix}}$$

$$\begin{bmatrix} \delta_2 \\ V_2 \end{bmatrix} = \begin{bmatrix} \delta_2^{\,old} \\ V_2^{\,old} \end{bmatrix} + \begin{bmatrix} \Delta\delta_2 \\ \Delta V_2 \end{bmatrix}$$

$$= \begin{bmatrix} -0.145 \\ 1.0064 \end{bmatrix} + \begin{bmatrix} -0.0015 \\ -0.0125 \end{bmatrix} = \begin{bmatrix} -0.1465 \text{ rad} \\ 0.994 \text{ p.u.} \end{bmatrix} = \begin{bmatrix} -8.39° \\ 0.994 \text{ p.u.} \end{bmatrix}$$

$$\boxed{V_2 = 0.994 \angle\text{-}8.39°}$$

3. **With a neat flowchart, explain the computational procedure for load flow solution using Gauss-seidal load flow solution.** **(NOV/DEC2015, 2017)**

Start

Read line data, tolerance (ε), α
Bus Data
P_i for $i = 2 \dots N$
Q_{min}, Q_{max} for $i = 2 \dots M$
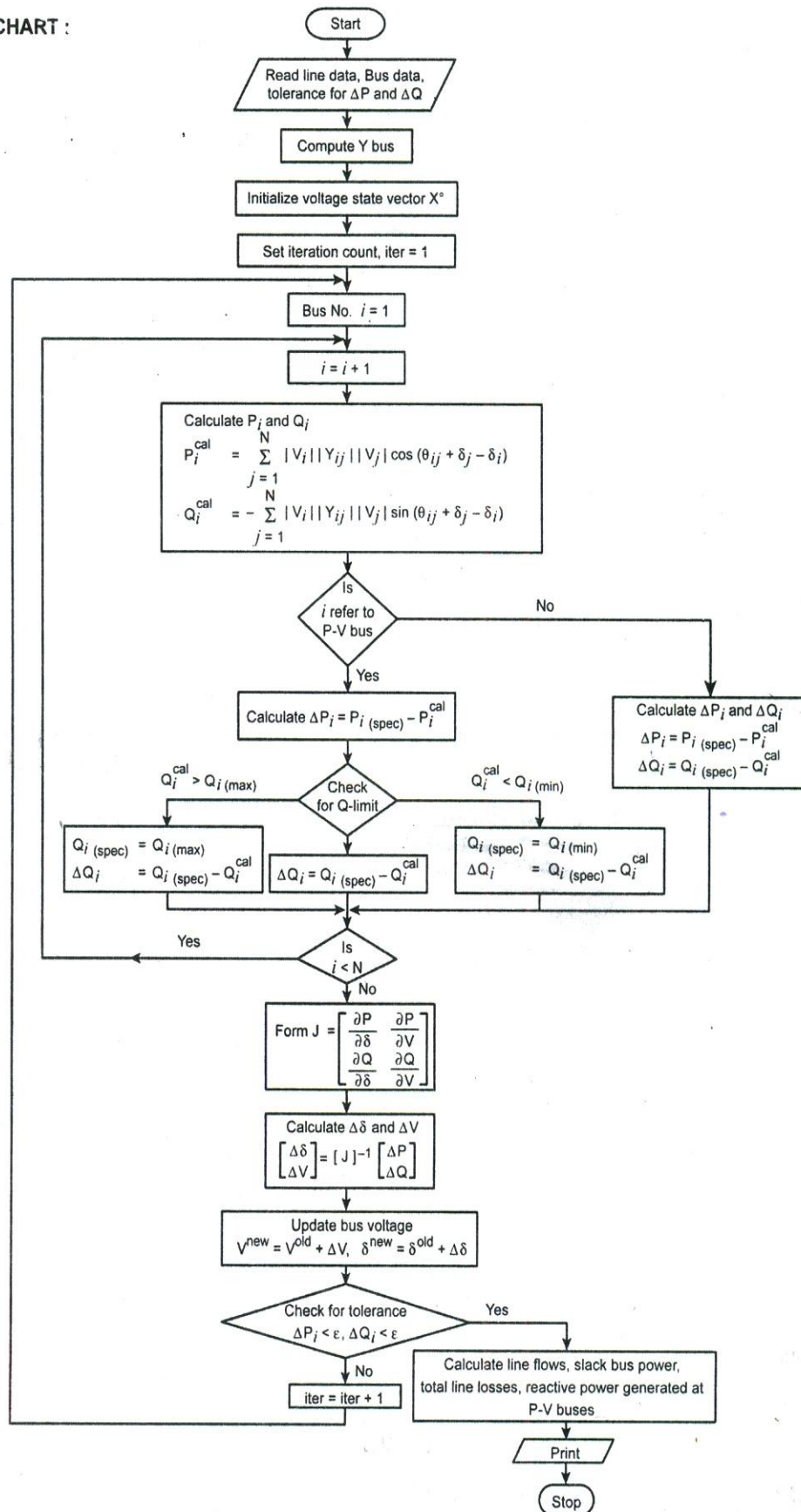Q_i for $i = M + 1 \dots N$
$|V_i|$ for $i = 1, \dots M$

Compute Y bus matrix

Initialize bus voltages

Set iteration count iter = 1

Bus No, $i = 1$

$i = i + 1$

Does i refer to generator bus?

No

Yes

$V^{new} = V_{(spec)} \angle \delta^{cal}$

$Q_i^{cal} = - \text{Im} [V_i^* \sum_{j=1}^{N} Y_{ij} V_j]$
$Q_{Gi} = Q_i^{cal} + Q_{Li}$

Check for Q limit

$Q_{Gi} > Q_{i(max)}$

$Q_{Gi} < Q_{i(min)}$

$Q_{Gi} = Q_{i(max)}$
$Q_{i(spec)} = Q_{i(max)} - Q_{Li}$

$Q_{i(spec)} = Q_i^{cal}$

$Q_{Gi} = Q_{i(min)}$
$Q_{i(spec)} = Q_{i(min)} - Q_{Li}$

$V_i^{new} = \dfrac{1}{Y_{ii}} \left[\dfrac{P_{i(spec)} - Q_{i(spec)}}{V_i^{old *}} - \sum_{j=1}^{i-1} Y_{ij} V_j^{new} - \sum_{j=i+1}^{N} Y_{ij} V_j^{old} \right]$

Is $i < N$

No

Yes

Are all $|V_i^{new} - V_i^{old}| \le \varepsilon$ for $i = 2 \dots N$?

No

Yes

$V_i^{new} = V_i^{old} + \alpha [V_i^{new} - V_i^{old}]$
$V_i^{old} = V_i^{new}$, $i = 2$ to N

iter = iter + 1

Calculate all line flows, total line losses, reactive power generated at P-V buses, slack bus power

Print the results

Stop

4. **Draw a flowchart and explain the algorithm of Newton Raphson iterative method when the system containall types of buses.** **(NOV/DEC 2015)**

FLOW CHART :

Start

Read line data, Bus data, tolerance for ΔP and ΔQ

Compute Y bus

Initialize voltage state vector $X°$

Set iteration count, iter = 1

Bus No. $i = 1$

$i = i + 1$

Calculate P_i and Q_i

$$P_i^{cal} = \sum_{j=1}^{N} |V_i||Y_{ij}||V_j| \cos(\theta_{ij} + \delta_j - \delta_i)$$

$$Q_i^{cal} = -\sum_{j=1}^{N} |V_i||Y_{ij}||V_j| \sin(\theta_{ij} + \delta_j - \delta_i)$$

Is i refer to P-V bus

No

Yes

Calculate $\Delta P_i = P_{i\,(spec)} - P_i^{cal}$

Calculate ΔP_i and ΔQ_i
$\Delta P_i = P_{i\,(spec)} - P_i^{cal}$
$\Delta Q_i = Q_{i\,(spec)} - Q_i^{cal}$

$Q_i^{cal} > Q_{i\,(max)}$

Check for Q-limit

$Q_i^{cal} < Q_{i\,(min)}$

$Q_{i\,(spec)} = Q_{i\,(max)}$
$\Delta Q_i = Q_{i\,(spec)} - Q_i^{cal}$

$\Delta Q_i = Q_{i\,(spec)} - Q_i^{cal}$

$Q_{i\,(spec)} = Q_{i\,(min)}$
$\Delta Q_i = Q_{i\,(spec)} - Q_i^{cal}$

Yes

Is $i < N$

No

$$\text{Form } J = \begin{bmatrix} \dfrac{\partial P}{\partial \delta} & \dfrac{\partial P}{\partial V} \\ \dfrac{\partial Q}{\partial \delta} & \dfrac{\partial Q}{\partial V} \end{bmatrix}$$

Calculate $\Delta \delta$ and ΔV
$$\begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix} = [J]^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$

Update bus voltage
$V^{new} = V^{old} + \Delta V, \quad \delta^{new} = \delta^{old} + \Delta \delta$

Check for tolerance
$\Delta P_i < \varepsilon, \Delta Q_i < \varepsilon$

Yes

No

iter = iter + 1

Calculate line flows, slack bus power, total line losses, reactive power generated at P-V buses

Print

Stop

5. **Write a neat flowchart, explain the computational procedure for load flow solution using Newton Raphson iterative method when the system contains all types of buses.**

The most widely used method for solving simultaneous non linear algebraic equation is the Newton Raphson method.

from the fig the complex power balance at bus i is given by

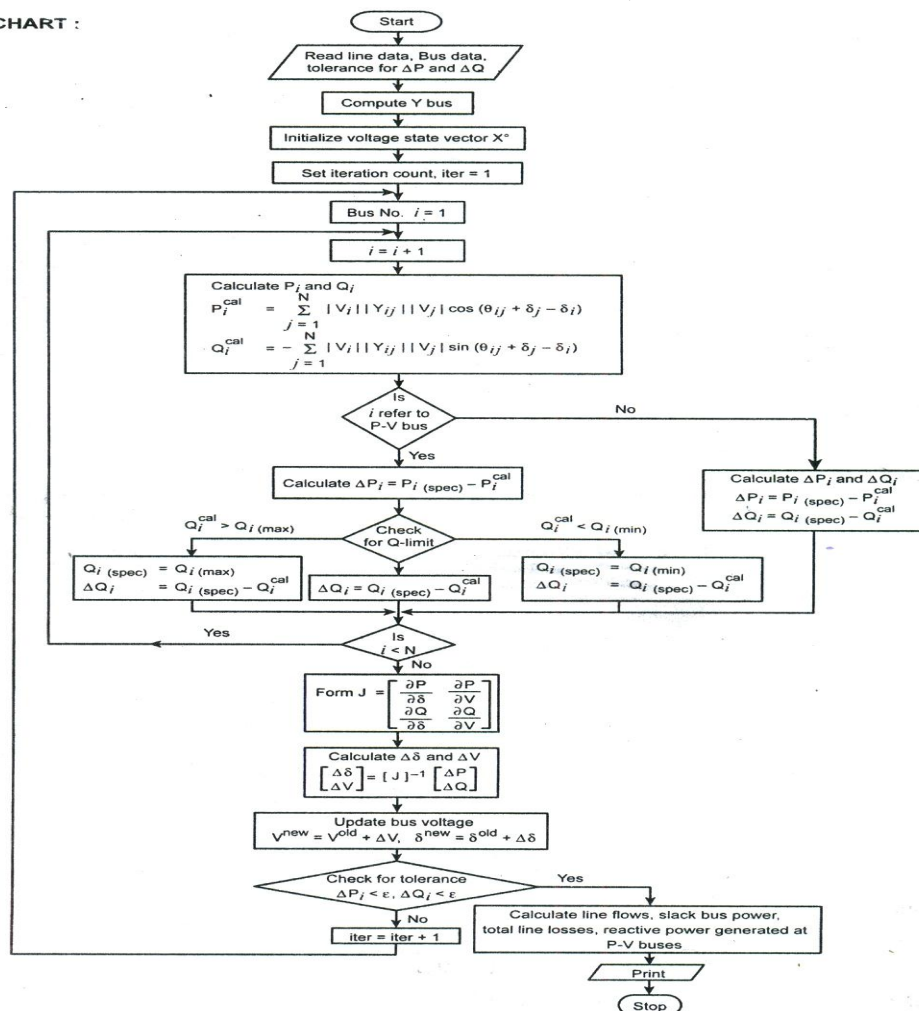$PI_i + jQI_i = P_i + jQ_i$ ………………………………………………………..…(1)
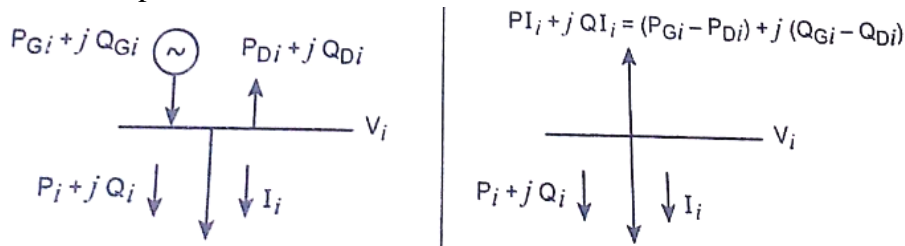
Complex power injection at the ith bus $PI_i + jQI_i$

$= (P_{Gi} - P_{Di}) + (jQ_{Gi} - jQ_{Di})$

Since the bus generation and demand are specified, the complex the complex power injection is a specified quantity and is given by,

(MAY/JUNE 17 & NOV/DE 16)

$PI_{i(spec)} + jQI_{i(spec)} = [P_{Gi(spec)} - P_{Di(spec)}] + j[Q_{Gi(spec)} - Q_{Di(spec)}]$ ………………(a)

The current entering bus i is given by,

$$I_i = \sum_{j=1}^{N} Y_{ij} V_j$$

In polar form

$$I_i = \sum_{j=1}^{N}|Y_{ij}||V_i|\angle(\theta_{ij} + \delta_j)$$ ....(b)

$$[Y_{ij} = |Y_{ij}|\angle\theta_{ij} \; ; V_j = |V_i|\angle\delta_i]$$

Equating the real and imaginary parts

$$P_i = \sum_{j=1}^{N}|V_i||Y_{ij}||V_j|\cos(\theta_{ij} + \delta_j - \delta_i)$$.................................(c)

$$Q_i = \sum_{j=1}^{N}|V_i||Y_{ij}||V_j|\sin(\theta_{ij} + \delta_j - \delta_i)$$ ....(d)

6. Single line diagram of simple power system, with generators at buses 1 and 3 is shown. The magnitude of voltage at bus 1 is 1.05 pu. Voltage magnitude at bus 3 is fixed at 1.04 pu with active power generation of 200MW. A load consisting of 400 MW and 250 MVAR is taken from bus 2. Line impedance are marked in pu on a 100 MVA base and the line charging susecptances are neglected.

Determine the voltage at buses 2 and 3 using G-S method at the end of first iteration. Also calculate slack bus power. **(MAY/JUNE 2017)**
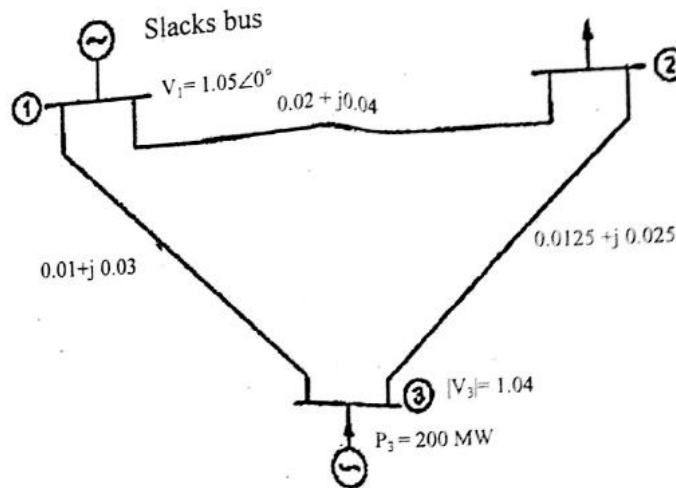
Solution:

    Step1: Formulate Ybus.

    When the switch is open, there is no connection of capacitor at bus 2.

    Take the bus as load bus.

    Step2: Initialize bus voltages

$$V_2^{old} = 1.05\angle 0°$$
$$V_3^{old} = 1.04\angle 0°$$

    Step3: Calculate V_2^{new}.

$V_2^{new} = 1.018\angle\text{-}8.915°$

    Step 4: Calculate V_2^{new} using acceleration factor

    Step 5: Slack bus power

$$S_1 = P_1 - jQ_1$$

    Real power generation $P_{G1} = P_1 + P_{L1}$

3. The figure shows the one line diagram of a simple 3 bus power system with generators at

buses 1 and 3 line impedances are marked in pu on a 100 MVA base. Determine the bus voltage at the end of second iteration using G-S method. (NOV/DEC 2016)
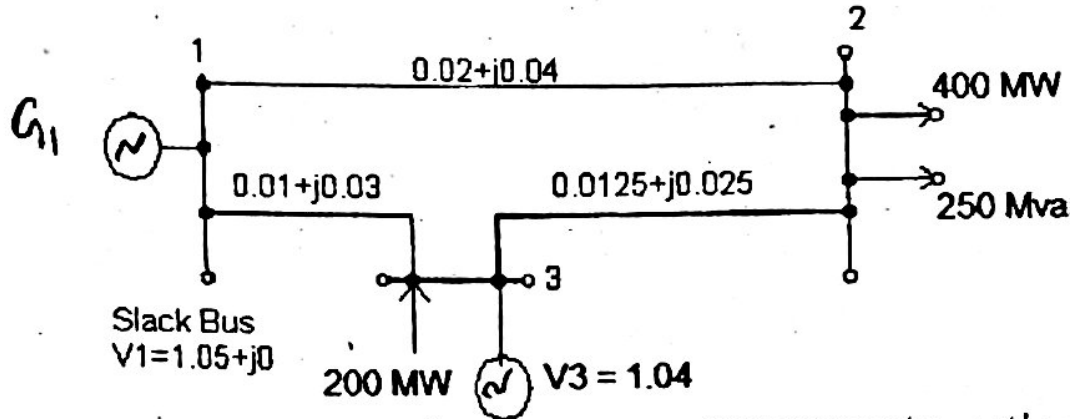
Solution:

Formulate Ybus.

When the switch is open, there is no connection of capacitor at bus 2.

Take the bus as load bus.

Initialize bus voltages

Calculate V_2^{new}.

$$V_2^{new} = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{V_2^{old*}} - Y_{21} V_1^{new} \right]$$

Calculate V_2^{new} using acceleration factor

$$V_2^{new}{}_{acc} = V_2^{old} + \alpha [V_2^{new} - V_2^{old}]$$

Slack bus power

$$S_1 = P_1 - jQ_1$$

Line flow

$$S_{ij} = P_{ij} + jQ_{ij} = V_i [V_i^* - V_j^*] Y_{ij}^*{}_{series} + |Vi|^2 Y_{Pi}^*$$

Line flow from bus 1 to 2.

$$S_{12} = P_{12} + jQ_{12} = V_1 [V_1^* - V_2^*] Y_{12}^*{}_{series}$$

7. **The system data for a load flow solution are given in tables. Determine the voltages at the end of first iteration using G-S method. Take α = 1.6** (MAY/JUNE 2016)

LINE ADMITTANCE

Bus code	Admittance
1-2	2-j8.0
1-3	1-j4.0
2-3	0.666-j2.664
2-4	1-j4.0
3-4	2-j8.0

Schedule of active and reactive powers

Bus code	P in pu	Q in pu	V in pu	Remarks
1	-	-	1.06	SLACK
2	0.5	0.2	1+J0.0	PQ
3	0.4	0.3	1+J0.0	PQ
4	0.3	0.1	1+J0.0	PQ

Solution:

Step1: Formulate Ybus.

When the switch is open, there is no connection of capacitor at bus 2.

Take the bus as load bus.

Step2: Initialize bus voltages

Step3: Calculate V_2^{new}.

$$V_2^{new} = \frac{1}{Y_{22}}\left[\frac{P_2\text{-}jQ_2}{V_2^{old*}} - Y_{21} V_1^{new}\right]$$

Step 4: Slack bus power

$$S_1 = P_1\text{-}jQ_1$$

8. **Draw and explain the step by step procedure of load flow solution for Gauss-seidal method with PV buses are present.** **(MAY/JUNE 2016)**

Step 1 : form Y-Bus

Step 2 : Assume $V_k = V_{k(spec)} \angle 0^0$ at all generator buses.

Step 3 : Assume $V_k = 1\angle 0^0 = 1+j0$ at all load buses.

Step 4 : set iteration count = 1 (iter = 1)

Step 5 : let bus number i = 1

Step 6 : If 'i' refers to generator bus go to step no. 7, otherwise go to step 8.

Step 7(a) : If 'i' refers to slack bus go to step no. 9, otherwise go to step 7(b).

Step 7(b) :compute Q_i using

$$Q_i^{cal} = - Im\ [\ \textstyle\sum_{j=1}^{N}\ V_i^*\ Y_{ij}\ V_j\ ]$$

$Q_{Gi} = Q_i^{cal} + Q_{Li}$

Check for Q limit violation

If $Q_{i(min)} < Q_{Gi} < Q_{i(max)}$, then $Q_{i(spec)} = Q_i^{cal}$

If $Q_{i(min)} < Q_{Gi}$, then $Q_{i(spec)} = Q_{i(min)} - Q_{Li}$

If $Q_{i(max)} < Q_{Gi}$, then $Q_{i(spec)} = Q_{i(max)} - Q_{Li}$

If Qlimit is violated, then treat this bus as P-Q bus till convergence is obtained.

Step 8 : Compute V_i using eqn.

$$V_i^{new} = \frac{1}{Y_{ii}}\ [\frac{P_i - jQ_i}{V_i^{old*}} - \ \textstyle\sum_{i=1}^{j-1}\ Y_{ij}\ V_j^{new} - \textstyle\sum_{i=j+1}^{N}\ Y_{ij}\ V_j^{old}$$

Step 9 : If i is less than number of buses, increment I by 1 and go to step 6.

Step 10 : Compare two successive iteration values for V_i

If $V_i^{new} - V_i^{old} <$ tolerance, go to step 12.

Step 11 : Update the new voltage as

$$V^{new} = V^{old} + \propto (V^{new} - V^{old})$$

$$V^{new} = V^{old}$$

Iter = iter +1; go to step 5.

Step 12: Compute relevant quantities.

Slack bus power, $S_1 = V*I = V_i^* \sum_{j=i}^{N}\ Y_{ij}\ V_j$

line flow losses, $S_{ij} = P_{ij} + jQ_{ij}$

Real power loss, $P_{loss} = P_{ij} + P_{ji}$

Reactive power loss, $Q_{loss} = Q_{ij} + Q_{ji}$

Step 13 : Stop the execution.

9. Derive the development of load flow model in complex variable form and polar variable form.

Solution

The power flow or load flow model in complex form is obtained by writing one complex power matching equation at each bus for the figure shown below.
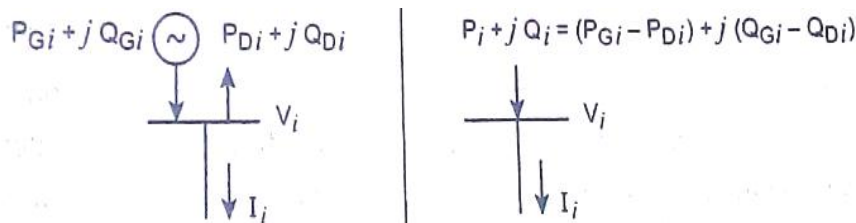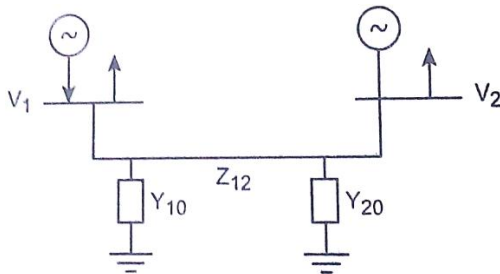

Net power injected into the bus i.

$S_i = S_{Gi} - S_{Di}$

$\quad = P_{Gi} + jQ_{Gi} - (P_{Di} + jQ_{Di})$

$\qquad = P_i + jQ_i$

We know,

$P_i + jQ_i = V_i I_i^*$

Consider two bus system with I_1 and I_2 as net current entering into bus 1 and 2.

$$[I] = [Y][V]$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$Y_{11} = y_{10} + y_{12}$$

$$Y_{22} = y_{20} + y_{21}$$

$$Y_{12} = Y_{21} = - y_{21}$$

In general, $Y_{ij} = |Y_{ij}| \angle \theta_{ij}$

$\quad I_1 = Y_{11} V_1 + Y_{12} V_2$

$\quad I_2 = Y_{21} V_1 + Y_{22} V_2$

In general , the net current entering into i^{th} bus

$I_i = Y_{i1}V_1 + Y_{i2}V_2 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + Y_{iN}V_N = \sum_{j=1}^{N} Y_{ij}V_j$

Substituting the value of I_i in power flow eqn we get.

$\quad S_i = P_i + jQ_i = V_i I_i^*$

$\quad S_i = P_i - jQ_i = V_i I_i^*$

$\quad P_i - jQ_i = V_i^* \sum_{j=1}^{N} Y_{ij}V_j \qquad$ where i = 1,2,3…………..N

There are N complex variable equations for which the N unknown complex variables $V_1, V_2, \ldots V_N$ can be determined.

Substituting Y_{ij} from the above eqn, we get.

$P_i - jQ_i = V_i^* \sum_{j=1}^{N} |Y_{ij}| \angle\theta_{ij} \ V_j$

Where $V_i = |V_i| \angle\delta_i$, $V_i^* = |V_i| \angle - \delta_i$,

$V_j = |V_j| \angle\delta_j$

Therefore net power equations can be written as

$P_i - jQ_i = \sum_{j=1}^{N} |V_i| |Y_{ij}| |V_j| \angle(\theta_{ij} + \delta_j - \delta_i)$

Equating real and reactive parts,

$P_i = \sum_{j=1}^{N} |V_i| |Y_{ij}| |V_j| \cos(\theta_{ij} + \delta_j - \delta_i)$

$Q_i = - \sum_{j=1}^{N} |V_i| |Y_{ij}| |V_j| \sin(\theta_{ij} + \delta_j - \delta_i)$

We can write the above equation as

$$P_i = |V_i|^2 |Y_{ii}| \cos\theta_{ii} \sum_{j=1}^{N} |V_i| |Y_{ij}| |V_j| \cos(\theta_{ij} + \delta_j - \delta_i)$$

$$Q_i = - |V_i|^2 |Y_{ii}| \sin\theta_{ii} \sum_{j=1}^{N} |V_i| |Y_{ij}| |V_j| \cos(\theta_{ij} + \delta_j - \delta_i)$$

The above equations are called as polar form of the power flow equations.

10. **Derive the load flow equation using Gauss seidal method.**

Bus 1 is generator bus take it as reference bus or slack bus. Here the voltages are specified.

In load buses, assume initial value of voltage as $1 \angle 0^0$ and find the new value of voltages.

The calculation starts from bus 2 onwards. In the generator bus first check generator limit and find the voltages.

Injected bus power is given by,

$S_i = P_i - jQ_i = V_i^* I_i$

$= V_i^* \sum_{j=1}^{N} Y_{ij} \ V_j$

$P_i - jQ_i = V_i^* Y_{ii} \ V_i + V_i^* \sum_{j=1\neq i}^{N} Y_{ij} \ V_j$

$V_i = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{j=1\neq i}^{N} Y_{ij} \ V_j \right]$

$i = 1, 2, 3, \ldots \ldots \ldots N$ except slack bus.

Let $V_1^{old}, V_2^{old} \ldots \ldots V_N^{old}$ be initial voltage. On substituting initial values in above eqn we get $V_1^{new}, V_2^{new} \ldots \ldots V_N^{new}$. After calculating each voltages replace the old values by new values.

Therefore $V_i^{new} = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^{*old}} - \sum_{j=1}^{i=1} Y_{ij} \ V_j^{new} - \sum_{j=i+1}^{N} Y_{ij} \ V_j^{old} \right]$ ....(a)

For load bus,

The above equation is applicable to find $|V|$ and δ values.

For slack bus,

The voltage is specified and so it will not change in each iteration.

For PV bus or generator bus,

(i) Q value is not specified for PV bus. So $V_i^{new} = |V_i|_{spec} \angle\delta^{cal}$

(ii) Compute reactive power generation using the V_i^{new} as.

$Q_i^{cal} = - Im\{ V_i^{*old} [\sum_{j=1}^{i=1} Y_{ij} \ V_j^{new} - \sum_{j=i+1}^{N} Y_{ij} \ V_j^{old}]\}$

$Q_{Gi} = Q_i^{cal} + Q_{Di}$

If $Q_{Gi(min)} \leq Q_{Gi} \leq Q_{Gi(max)}$, set $Q_i = Q_{Gi} - Q_{Di}$ then compute V_i^{new}

If $Q_{Gi} < Q_{Gi(min)}$, set $Q_{Gi} = Q_{Gi(min)}$, then compute V_i^{new} using eqn (a)

If $Q_{Gi} > Q_{Gi(max)}$, set $Q_{Gi} = Q_{Gi(max)}$, then compute V_i^{new} using eqn (a)

Acceleration factor ($\propto$)

$$V_i^{new} = V_i^{old} + \propto [V_i^{new} - V_i^{old}]$$

Where V_i^{old} = Voltage value obtained in previous iteration

$\qquad V_i^{new}$ = New value of Voltage value obtained in current iteration

$\qquad \propto$ = Acceleration factor

Computation of transmission loss.

$\qquad S_{ij(loss)} = S_{ij} = S_{ji}$

$\qquad\qquad = P_{ij} + jQ_{ij} + P_{ji} + jQ_{ji}$

Real power loss = $P_{ij} + P_{ji}$

Reactive power loss = $Q_{ij} + Q_{ji}$

11. What is the need for load flow analysis (or) importance of power flow analysis.

Load flow analysis is performed on a symmetrical steady state operating conditions of a power system under normal mode of operation. The solution of load flow gives bus voltages and line/transformer power flows for a given load condition. This information is essential for long term planning and operational planning.

long term planning.

Load flow analysis helps in investigating the effectiveness of alternative plans and choosing the best plan for system expansion to meet the projected operating state.

Operational planning.

It helps in choosing the best unit commitment plan and generation schedules to run the system efficiently for the next day's load condition without violating the bus voltages and line flow operating limits.

Steps for load flow study.

The following work has to be performed for a load flow study.

(i) Representation of the system by single line diagrams.

(ii) Determining the impedance diagram using the information in single line diagram.

(iii) Formulation of network equations.

(iv) Solution of network equations.

12. Derive the load flow equation using Newton Raphson method.

The most widely used method for solving simultaneous non linear algebraic equation is the Newton Raphson method.

from the fig the complex power balance at bus i is given by

$PI_i + jQI_i = P_i + jQ_i$ …………………………………………………..…(1)

Complex power injection at the ith bus $PI_i + jQI_i$

$= (P_{Gi} - P_{Di}) + (jQ_{Gi} - jQ_{Di})$

Since the bus generation and demand are specified, the complex the complex power injection is a specified quantity and is given by,

$$PI_{i(spec)} + jQI_{i(spec)} = [P_{Gi(spec)} - P_{Di(spec)}] + j[Q_{Gi(spec)} - Q_{Di(spec)}] \ldots\ldots\ldots\ldots\ldots(a)$$

The current entering bus i is given by,

$$I_i = \sum_{j=1}^{N} Y_{ij}V_j$$

In polar form

$$I_i = \sum_{j=1}^{N}|Y_{ij}||V_i|\angle(\theta_{ij} + \delta_j)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(b)$$

$$[Y_{ij} = |Y_{ij}|\angle\theta_{ij} ; V_j = |V_i|\angle\delta_i]$$

Complex power at bus i

$$P_i - jQ_i = V_i^* I_i = V_i^* \sum_{j=1}^{N} Y_{ij}V_j$$

Substituting from eqn (b), we get

$$P_i - jQ_i = |V_i|\angle-\delta_i \sum_{j=1}^{N}|Y_{ij}||V_j|\angle(\theta_{ij} + \delta_j)$$

$$= \sum_{j=1}^{N}|V_i||Y_{ij}||V_j|\angle(\theta_{ij} + \delta_j - \delta_i)$$

Equating the real and imaginary parts

$$P_i = \sum_{j=1}^{N}|V_i||Y_{ij}||V_j|cos(\theta_{ij} + \delta_j - \delta_i)\ldots\ldots\ldots\ldots\ldots\ldots\ldots(c)$$

$$Q_i = \sum_{j=1}^{N}|V_i||Y_{ij}||V_j|sin(\theta_{ij} + \delta_j - \delta_i) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(d)$$

The above eqn constitute a set of nonlinear algebraic equations in terms of the independent variables.

Substitute eqn (a),(c),(d) in (1) we get power balance eqns.

$$P_i (\delta,V) - PI_{i(spec)} = 0$$

$$Q_i (\delta,V) - QI_{i(spec)} = 0$$

13. Write the Procedure for load flow solution by Newton Raphson method.

Step 1 : form Y-Bus

Step 2 : Assume flat start voltage solution

$\delta_i^0 = 0$, for i = 1…..N

$|V_i^0| = 1.0$,

$|V_i| = |V_i|_{spec}$

Step 3 : for load buses, calculate $P_i{}^{cal}$ and $Q_i{}^{cal}$

Step 4 : for PV bus, Check for Q limit violation

If $Q_{i(min)} < Q_i{}^{cal} < Q_{i(max)}$, the bus acts as PV bus

If $Q_i{}^{cal} > Q_{i(max)}$, then $Q_{i(spec)} = Q_{i(max)}$

If $Q_i{}^{cal} < Q_{i(min)}$, then $Q_{i(spec)} = Q_{i(min)}$, the PV bus will act as PQ bus.

Step 5 : Compute mismatch vector using.

$$\Delta P_i = P_{i(spec)} - P_i{}^{cal}$$

$$\Delta Q_i = Q_{i(spec)} - Q_i{}^{cal}$$

Step 6 : Compute $\Delta P_{i(max)} = max |\Delta P_i|$ i = 1,2……….N except slack

$$\Delta Q_{i(max)} = max |\Delta Q_i| i = M+1……….N$$

Step 7 : compute jacobian matrix using $J = \begin{bmatrix} \frac{\partial P_i}{\partial \delta} & \frac{\partial P_i}{\partial |V|} \\ \frac{\partial Q_i}{\partial \delta} & \frac{\partial Q_i}{\partial |V|} \end{bmatrix}$

Step 8 : Obtain state correction vector $\begin{bmatrix} \Delta V \\ \Delta \delta \end{bmatrix} = [J]^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$

$V^{new} = V^{old} + \Delta V$

$\delta^{new} = \delta^{old} + \Delta \delta$

Step 9 : Update state vector using .

$V^{new} = V^{old} + \Delta V$

$\delta^{new} = \delta^{old} + \Delta \delta$

Step 10 : This procedure is continued until

$|\Delta P_i| < \varepsilon$ and $|\Delta Q_i| < \varepsilon$, otherwise go to step 3.

14. Explain the significance of load flow analysis or power flow analysis.

The information of load flow is essential for analyzing the effective alternative plans for the system expansion to meet increase load demand.

The load flow studies are very much important for planning, economic scheduling, control and operations of existing systems as well as planning its future expansions depends upon knowing the effect of interconnections, new loads , new generating stations, or new transmission lines, etc., before they are installed.

With help of load flow studies we can determine the best size as well as the most favourable locations for the power system capacitors both for the improvement for the power factor and raising the bus voltages of the electrical network. It helps us to determine the capacity of the proposed generating stations, substations or new lines.

The information obtained from a load flow study are magnitude and phase angles of bus voltages, real and reactive power flowing in each line and line losses. The load flow solution also gives the initial conditions of the system when the transient behavior of the system is to be studied.

The mathematical formulation of the load flow problem results in a system of nonlinear equations. These equations can be written in terms of either the bus admittance matrix or bus impedance matrix. Using bus admittance matrix is amenable to digital computer analysis, because it could be formed and modified for network changes in subsequent cases, and requires less computation time and memory.

The load flow analysis can be carried out for small and medium size power systems. It suits for radial distribution systems with high R/X ratio. The load flow analysis helps to identify the overloaded or under loaded lines and transformers as well as the overvoltage or under voltage buses in the power system network.

It is used to study the optimum location of capacity and their size to improve the unacceptable voltage profile.

15. Draw the flowchart for Fast Decoupled method.

FLOW CHART :

Start

Read line data, Bus data, tolerance for ΔP and ΔQ

Compute Y bus
From Y-bus, find Bus susceptance matrix
B′(imaginary part of Y-bus for all buses except slack)
B″ (I.P of Y-bus for load buses)

Initialize voltage state vector X°

Set iteration count, iter = 1

Bus No. $i = 1$

$i = i + 1$

Calculate P_i and Q_i

$$P_i^{cal} = \sum_{j=1}^{N} |V_i| |Y_{ij}| |V_j| \cos(\theta_{ij} + \delta_j - \delta_i)$$

$$Q_i^{cal} = -\sum_{j=1}^{N} |V_i| |Y_{ij}| |V_j| \sin(\theta_{ij} + \delta_j - \delta_i)$$

Is i refer to P-V bus

No

Yes

Calculate $\Delta P_i = P_{i(spec)} - P_i^{cal}$

$Q_i^{cal} > Q_{i(max)}$

Check for Q-limit

$Q_i^{cal} < Q_{i(min)}$

$Q_{i(spec)} = Q_{i(max)}$
$\Delta Q_i = Q_{i(spec)} - Q_i^{cal}$

$Q_{i(spec)} = Q_{i(min)}$
$\Delta Q_i = Q_{i(spec)} - Q_i^{cal}$

Calculate ΔP_i and ΔQ_i
$\Delta P_i = P_{i(spec)} - P_i^{cal}$
$\Delta Q_i = Q_{i(spec)} - Q_i^{cal}$

Is $i < N$

Yes

No

Calculate $\Delta \delta$ and ΔV

$$\Delta \delta_i = -[B']^{-1}\left[\frac{\Delta P_i}{|V_i|}\right]$$

$$\Delta V_i = -[B'']^{-1}\left[\frac{\Delta Q_i}{|V_i|}\right]$$

Update bus voltage
$V^{new} = V^{old} + \Delta V, \quad \delta^{new} = \delta^{old} + \Delta \delta$

Check for tolerance
$\Delta P_i < \varepsilon, \Delta Q_i < \varepsilon$

Yes

No

iter = iter + 1

Calculate line flows, slack bus power, total line losses, reactive power generated at P-V buses

Print

Stop

16. **Explain the classification of buses.**

The meeting point of various components in a power system is called a bus. At some of the buses power is being injected into the network, whereas at other buses it is being tapped by the system loads.

The buses of a power system can be classified into three types based on the quantities being specified for the buses, which are as follows:

a. Load bus or PQ bus (P and Q are specified)
b. Generator bus or voltage controlled bus or PV bus (P and V are specified)
c. Slack bus or swing bus or reference bus (|V| and δ are specified)

Each bus in a power system is associated with four quantities and they are

Real power (P), Reactive power (Q), magnitude of voltage (V), phase angle of voltage(δ).

Bus type	Quantities specified	Quantities to be obtained
Load bus	P,Q	$\lvert V \rvert$, δ
Generator bus	P, $\lvert V \rvert$	Q, δ
Slack bus	$\lvert V \rvert$, δ	P,Q

Voltage controlled bus. (Generator bus)

A bus is called voltage controlled bus if the magnitude of voltage │V│ and real power (P) are specified for it. In a voltage controlled bus the magnitude of the voltage is not allowed to change. The other names for voltage controlled bus are generator bus and PV bus. In this bus the phase angle of the voltages and the reactive power are to be determined. The limits on the reactive power are also specified.

PQ-bus (load bus)

A bus is called PQ-bus when real and reactive components of power are specified for the bus. In this bus the magnitude and phase angle of voltage are unknown. In PQ bus the voltage is allowed to vary within permissible limits.

Swing bus (Slack bus)

A bus is called swing bus when the magnitude and phase for bus voltage are specified for it. The swing bus is the reference bus for load flow solution and it is required for accounting line losses. Usually one of the generator bus selected as the swing bus.

Need for Slack bus (or) Swing bus.

The slack bus is needed to account for transmission line losses. In a power system the total power generated will be equal to sum of power consumed by loads and losses. In a power system only the generated power and load power are specified for buses. The slack bus is assumed to generate the power required for losses. Since the losses are unknown the real and reactive power are not specified for slack bus. They are estimated through the solution of load flow equations.

When the generator bus is treated as load bus the reactive power of the bus is equated to the limit it has violated, and the previous iteration value of bus voltage is used for calculating current iteration value.

If $Q_i > Q_{i(max)}$, then $Q_i = Q_{i(max)}$

If $Q_i < Q_{i(min)}$, then $Q_i = Q_{i(min)}$

Reactive power of the bus has violates the specified limits, then the P-V bus will act as load

bus.

If the reactive power constraints of a generator bus violates the specified limits then the generator is treated as load bus.

If $Q_i > Q_{i(max)}$, substitute $Q_i = Q_{i(max)}$

If $Q_i^{cal} < Q_{i(min)}$, substitute $Q_i = Q_{i(min)}$

17. Write the step-by-step algorithm to solve the load flow problem using Fast decoupled method.

Step 1 : form Y-Bus and then compute bus susceptance matrices B$'$ and B$''$

Step 2 : Assume flat start for starting voltage solution

$\delta_i^0 = 0$, for i = 1…..N (for all bus except slack bus)

$|V_i^0| = 1.0$, for i = M+1………N (for all PQ buses)

$|V_i| = |V_i|_{spec}$ for all PV bus and slack bus.

Step 3 : for load buses, calculate P_i^{cal} and Q_i^{cal}

$\quad P_i^{cal} = \sum_{j=1}^{N} |V_i||Y_{ij}||V_j| cos(\theta_{ij} + \delta_j - \delta_i)$

$\quad Q_i^{cal} = \sum_{j=1}^{N} |V_i||Y_{ij}||V_j| sin(\theta_{ij} + \delta_j - \delta_i)$

Step 4 : for PV bus, Check for Q limit violation

If $Q_{i(min)} < Q_i < Q_{i(max)}$, calculate P_i^{cal}

If $Q_i^{cal} < Q_{i(min)}$, then $Q_{i(spec)} = Q_{i(min)}$

If $Q_i^{cal} > Q_{i(max)}$, then $Q_{i(spec)} = Q_{i(max)}$, the PV bus will act as PQ bus.

Step 5 : Compute mismatch vector using.

$\quad \Delta P_i = P_{i(spec)} - P_i^{cal}$

$\quad \Delta Q_i = Q_{i(spec)} - Q_i^{cal}$

Step 6 : Compute $\quad \Delta P_{i(max)} = max |\Delta P_i| \quad$ i = 1,2……….N ; $\neq$ except slack

$\quad\quad\quad\quad\quad\quad\quad \Delta Q_{i(max)} = max |\Delta Q_i| \quad$ i = M+1……….N

Step 7 : compute jacobian matrix using J = $\begin{bmatrix} \frac{\partial P_i}{\partial \delta} & \frac{\partial P_i}{\partial |V|} \\ \frac{\partial Q_i}{\partial \delta} & \frac{\partial Q_i}{\partial |V|} \end{bmatrix}$

Step 8 : Calculate $\Delta \delta$ and ΔV using

$\quad [\Delta \delta_i] = -[B']^{-1} . \left[\frac{\Delta P_i}{|V_i|}\right]$

$\quad [\Delta V_i] = -[B'']^{-1} . \left[\frac{\Delta Q_i}{|V_i|}\right]$

Step 9 : Update state vector using .

$\quad V^{new} = V^{old} + \Delta V$

$\delta^{new} = \delta^{old} + \Delta \delta$

Step 10 : This procedure is continued until

$\quad |\Delta P_i| < \varepsilon$ and $|\Delta Q_i| < \varepsilon$,

Otherwise go to step 3.

18. Perform power flow of one iteration for the system as shown in fig. using Gauss - seidel method. Determine slack bus power, line flows and line losses. Take base MVA as 100
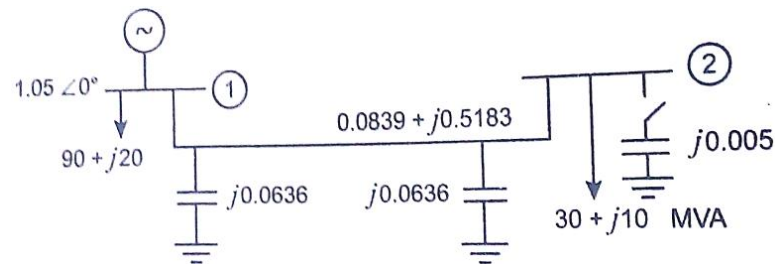
(α=1).

Solution:

Step1: Formulate Ybus.

When the switch is open, there is no connection of capacitor at bus 2.

Take the bus as load bus.

$$Y_{bus}=\begin{bmatrix} 0.3044 - j1.816 & -0.3044 + j1.88 \\ -0.3044 + j1.88 & 0.3044 - j1.816 \end{bmatrix}$$

Step2: Initialize bus voltages

$V_1^{old} = 1.05\angle 0°$

$V_2^{old} = 1.0\angle 0°$

Step3: Calculate V_2^{new}.

$P_2 = -30 \text{ MW} = \frac{-30}{100}\text{p.u.} = -0.3 \text{ p.u.}$

$Q_2 = -10 \text{ MVAR} = \frac{-10}{100}\text{p.u.} = -0.1 \text{ p.u.}$

$V_2^{new} = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^{old*}} - Y_{21} V_1^{new}\right]$

$=\frac{1}{0.3044 - j1.816}[\frac{-0.3 + j0.1}{1.0\angle -0°} - (-0.344 + j1.88)1.05]$

$=1.0054 - j0.1577$

$=1.018\angle -8.915°$

$\boxed{V_2^{new} = 1.018\angle -8.915°}$

Step 4: Calculate V_2^{new} using acceleration factor

$V_2^{new}{}_{acc}=V_2^{old}+\alpha[V_2^{new} - V_2^{old}]$

$=1.0+1.1[1.0054 - j0.1577 - 1]$

$=1.0059 - j0.173$

$\boxed{V_2^{new}{}_{acc} = 1.0207\angle -9.78°}$

Step 5: Slack bus power

$S_1 = P_1 - jQ_1$

$=1.05\angle -0°[(0.3044 - j1.816)1.05 + (-0.3044 + j1.88)(1.0207\angle -9.78°)]$

$=0.3566 + j0.0388 \text{ p.u}$

$=35.56 + j3.88 \text{ MVA}$

$P_1=35.56 \text{ MW}, \quad Q_1=-3.88 \text{ MVAR}$

Real power generation $P_{G1}=P_1+P_{L1}$

$=35.56+90=125.56 \text{ MW}$

$\boxed{P_{G1}=125.56 \text{ MW}}$

Reactive power generation $Q_{G1}=Q_1+Q_{L1}$

$=-3.88+20=16.12 \text{ MVAR}$

Q_{G1}=16.12 MVAR

Step - 6: Line flows

Bus		$S_{ij}=P_{ij}+jQ_{ij}=V_i[V_i^* - V_j^*]Y_{ij\ series}^* +|V_i|^2 Y_{Pi}^*$
From	To	
1	2	$S_{12}=V_1[V_1^* - V_2^*]Y_{12\ series}^* +|V_1|^* Y_{10}^*$
		$=1.05[1.05\angle -0°(1.0059+j0.173)]^*(0.3044+j1.88)+1.05^2*(-j0.0636)$
		$= 0.3556 - j0.0383$ p.u
		$P_{12}= 0.3556$ p.u$=35.56$ MW
		$Q_{12}= - 0.0383$ p.u$= -3.83$ MVAR
2	1	$S_{21}=V_2[V_2^* - V_1^*]Y_{12\ series}^* +|V_2|^2 Y_{20}^*$
		$= (1.0059 - j0.173)[1.0059+j0.173-1.05]x (0.3044+j1.88)+1.0207^2 x(-j0.0636)$
		$=-0.3459-j0.038$ p.u.
		$P_{21}=-0.3459$ p.u. $= -34.59$ MW
		$Q_{21}= - 0.038$ p.u. $= -3.8$ MVAR

Step – 7: Transmission line loss ($S_{ij\ Loss} = S_{ij} + S_{ji}$)

$P_{12}Loss = P_{12} + P_{21} =35.56 -34.59 =0.97$ MW

$Q_{12}Loss = Q_{12} + Q_{21} = - 3.83 + (-3.8) = -7.63$ MVAR

$P_{12}Loss =0.97$ MW

$Q_{12}Loss = -7.63$ MVAR

19. Perform two iteration of Newton Raphson load flow method and determine the power flow solution for the given system. Take base MVA as base 100, Bus 2 is a voltage controlled bus having the rating $P_G = 60$MW, $V_2=1.02$p.u. $-10 < Q_2 < 100$ MVAR. Carry out two iterations and determine bus voltage magnitudes.

Solution:

$$Y_{bus} = \begin{bmatrix} 1.842\angle -80.49° & 1.904\angle 99.2° \\ 1.904\angle 99.2° & 1.842\angle -80.49° \end{bmatrix}$$

$X° = [\delta_2] = 0$ rad

Compute θ_{12} in radians:

$$Y_{bus} = \begin{bmatrix} 1.842\angle -1.405 & 1.904\angle 1.7314 \\ 1.904\angle 1.7314 & 1.842\angle -1.405 \end{bmatrix}$$

Check for Q- Limit:

$Q_2^{cal} = - |V_2|\{|V_1||Y_{21}| \sin(\theta_{12} + \delta_1 - \delta_2) + |V_2||Y_{22}| \sin(\theta_{22})\}$

$= -1.02 \{1.05$ X $1.904 \sin(1.7314) + 1.02$ X $1.842 \sin(-1.405)\}$

$Q_2^{cal} = -0.1239$ p.u.

$Q_{2(min)} < Q_2^{cal} < Q_{2(max)}$

So the bus acts as generator bus.

Compute ΔP_2 :

$P_2^{cal} = |V_2|\{|V_1||Y_{21}| \cos(\theta_{12} + \delta_1 - \delta_2) + |V_2||Y_{22}|\cos(\theta_{22} + \delta_2 - \delta_2) \}$

$= 1.02[1.05$ X $1.904 \cos(1.7314) + 1.02$ X $1.842 \cos(-1.405)]$

$= 1.02(-0.3197 + 0.3100)$

$\boxed{P_2^{cal} = -0.009 \text{ p.u.}}$

ΔP_2 = $P_{2(spec)}$ – P_2^{cal}

$= 0.6 - (-0.009) = 0.609 \text{ p.u.}$

Form Jacobian matrix

$J = \left[\frac{\partial P_2}{\partial \delta_2}\right] = |V_2|\{|V_1||Y_{21}| + \sin(\theta_{12} + \delta_1 - \delta_2)\}$

$= 1.02 \text{ X } 1.05 \text{ X } 1.904 \sin(0-0+1.7314) = 2.013$

Compute $\Delta\delta_2$:

$\Delta\delta_2 = [J]^{-1} [\Delta P_2]$

$= \frac{1}{2.013} \text{ X } 0.609 = 0.303 \text{ rad}$

$\delta_2^{new} = \delta_2^{old} + \Delta\delta_2 = 0+0.303 = 0.303 \text{ rad}$

$\boxed{|V_2^{new}| = 1.02}$

Iteration 2:

$P_2^{cal} = 1.02[1.05 \text{ X } 1.904 \cos(1.7314+0-0.303) + 1.02 \text{ X } 1.842 \cos(-1.405)]$

$= 0.606$

ΔP_2 = $P_{2(spec)}$ – P_2^{cal}

$= 0.6 - 0.606 = -0.006$

$Q_2^{cal} = -1.02 \{1.05 \text{ X } 1.904 \sin(1.7314+0-0.303) + 1.02 \text{ X } 1.842 \sin(-1.405)\}$

$= -0.128$

$\boxed{Q_{2(min)} < Q_2^{cal} < Q_{2(max)}}$

∴ This bus acts as generator bus.

Jacobian matrix

$J = \left[\frac{\partial P_2}{\partial \delta_2}\right] = |V_2|\{|V_1||Y_{21}| + \sin(\theta_{12} + \delta_1 - \delta_2)\}$

$= 1.02 \text{ X } [1.05 \text{ X } 1.904 \sin(1.7314+0-0.303) = 2.0185$

Compute $\Delta\delta_2$:

$\Delta\delta_2 = [J]^{-1} [\Delta P_2]$

$= \frac{1}{2.0185} \text{ X } -0.006 = -0.003$

$\delta_2^{new} = \delta_2^{old} + \Delta\delta_2 = 0.303+(-0.003)$

$\delta_2 = 0.3 \text{ rad} = 17.19°$

$\boxed{|V_2| = 1.02 \text{ p.u.}}$

20. Perform two iterations of decoupled load flow method and determine the power flow solution for the system as shown in Fig. Take base MVA as 100.
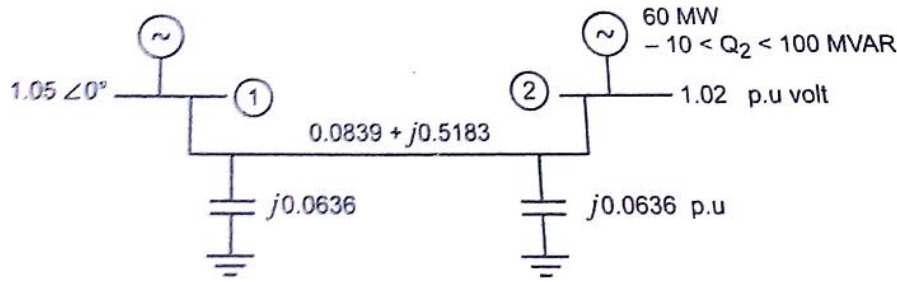
Solution:

Step -1 : Form Y_{bus} matrix

$$Y_{bus} = \begin{bmatrix} 0.3043 - j1.817 & -0.3044 + j1.88 \\ -0.3044 + j1.88 & 0.3043 - j1.817 \end{bmatrix}$$

$$Y_{bus} = \begin{bmatrix} 1.842\angle - 1.405 & 1.904\angle 1.7314 \\ 1.904\angle 1.7314 & 1.842\angle - 1.405 \end{bmatrix} \quad \text{\{Note: Use in rad mode\}}$$

Step – 2:Initialize bus voltages

$$V_1^{old} = 1.05\angle 0 \text{ p.u. (slack bus)}$$
$$V_2^{old} = 1.02\angle 0 \text{ p.u. (P-V bus)}$$

Step – 3: Check for Q-limit violation.

$$Q_2^{cal} = - \{|V_2||V_1||Y_{21}| \sin(\theta_{21} + \delta_1 - \delta_2) + |V_2|^2|Y_{22}| \sin(\theta_{22} + \delta_2 - \delta_2)\}$$
$$= - 1.02[1.05 \text{ X } 1.904 \sin(1.7314) + 1.02^2 \text{ X } 1.842 \sin(-1.405)]$$

$$\boxed{Q_2^{cal} = - 0.1228 \text{ p.u.}}$$

$$Q_{2(min)} = 10 \text{ MVAR} = \frac{10}{100}\text{p.u.} = 0.1 \text{ p.u.}$$

$$Q_2 < Q_{2(min)}; \therefore Q_2 = Q_{2(min)} = 0.1 \text{ p.u.}$$

and the bus 2 will acts as load bus, $V_2 = 1\angle 0$ p.u.

Step – 4: Compute ΔP and ΔQ.

$$P_2^{cal} = |V_2|\{|V_1||Y_{21}| \cos(\theta_{21} + \delta_1 - \delta_2) + |V_2|^2|Y_{22}|\cos(\theta_{22} + \delta_2 - \delta_2) \}$$
$$= 1.0[1.05 \text{ X } 1.904 \cos(1.7314) + 1.0^2 \text{ X } 1.842 \cos(-1.405)]$$
$$= -0.0157 \text{ p.u.}$$

$$P_{2(spec)} = P_{G2} - P_{L2} = \frac{60}{100} = 0.6 \text{ p.u.}$$

$$\Delta P_2 = P_{2(spec)} - P_2^{cal} = 0.6 - (-0.0157) = - 0.6157$$

$$\Delta Q_2 = Q_{2(spec)} - Q_2^{cal} = 0.1 - (-0.1) = 0.2$$

Step – 5: Bus susceptance matrix

$$[B'] = 2\,[1.817]$$

$$[B']^{-1} = \frac{1}{-1.817} = -0.5504$$

$$[B''] = 2\,[1.817], [B'']^{-1} = -0.5504$$

NOTE:

B′ matrix is the imaginary part of Y_{bus} for the buses except slack bus.

B″ matrix is the imaginary part of Y_{bus} for the load buses.

Step – 6: Calculate $\Delta\delta$ and ΔV.

$$[\Delta\delta_2] = -[B']^{-1}\left[\frac{\Delta P_2}{|V_2|}\right] = -(-0.5504)\left[\frac{0.6157}{1.0}\right] = 0.34$$

$$[\Delta V_2] = -[B'']^{-1}\left[\frac{\Delta Q_2}{|V_2|}\right] = -(-0.5504)\left[\frac{0.2}{1.0}\right] = 0.1226 \text{ p.u.}$$

$$\delta_2^1 = \delta_2^0 + \Delta\delta_2 = 0+0.34 = 0.34 \text{ rad}$$

$$V_2^1 = V_2^0 + \Delta V_2 = 1.0 + 0.1226 = 1.1226 \text{ p.u.}$$

$$\boxed{V_2^1 = 1.1226\angle 0.34}$$

Iteration – 2: Check for Q-limit

$$Q_2^{cal} = - [1.1226 \text{ X } 1.05 \text{ X } 1.904 \sin(1.7314 - 0.34)+ 1.1226^2 \text{ X } 1.842 \sin(-1.405)]$$
$$= - (-0.0812) = 0.0812$$

$$0.0812 < 0.1, \ Q_2^{cal} < Q_{2(min)}$$

$$\therefore Q_2 = Q_{2(min)} = 0.1 \text{ p.u. MVAR}$$

Bus 2 again act as load bus. $V_2^{old} = 1.1226\angle 0.34$

$$P_2^{cal} = 1.1226 \text{ X } 1.05 \text{ X } 1.904 \cos(1.7314 - 0.34) + 1.1226^2 \text{ X } 1.842 \cos(-1.405)$$
$$= 0.7836 \text{ p.u.}$$

$$P_{2(spec)} = P_{G2} - P_{L2} = \frac{60}{100} = 0.6 \text{ p.u.}$$

$$\boxed{\begin{aligned}\Delta P_2 &= P_{2(spec)} - P_2^{cal} = 0.6 - (0.7836)= -0.1836\\ \Delta Q_2 &= Q_{2(spec)} - Q_2^{cal} = 0.1 - 0.0812 = 0.0188\end{aligned}}$$

Step – 5: Bus susceptance matrix

$$[B''] = [B'] = [1.817]$$
$$[B']^{-1} = [B'']^{-1} = -0.5504$$

Step – 6: Calculate $\Delta\delta$ and ΔV.

$$[\Delta\delta_2] = -[B']^{-1}\left[\frac{\Delta P_2}{|V_2|}\right] = -(-0.5504)\left[\frac{-0.1836}{1.1226}\right] = -0.09$$

$$[\Delta V_2] = -[B'']^{-1}\left[\frac{\Delta Q_2}{|V_2|}\right] = -(-0.5504)\left[\frac{0.0188}{1.1226}\right] = 0.0092$$

$$\delta_2^2 = \delta_2^1 + \Delta\delta_2 = 0.34+(-0.09) = 0.25 \text{ rad} =14°$$

$$V_2^2 = V_2^1 + \Delta V_2 = 1.1226 + 0.0092 = 1.1318$$

$$\boxed{V_2^1 = 1.1318\angle 14°}$$

PART-C

21. **Obtain the power flow solution(one iteration) for the system shown in Fig. The line admittances are in per unit on a 100MVA base. Use fast decoupled load flow method.**
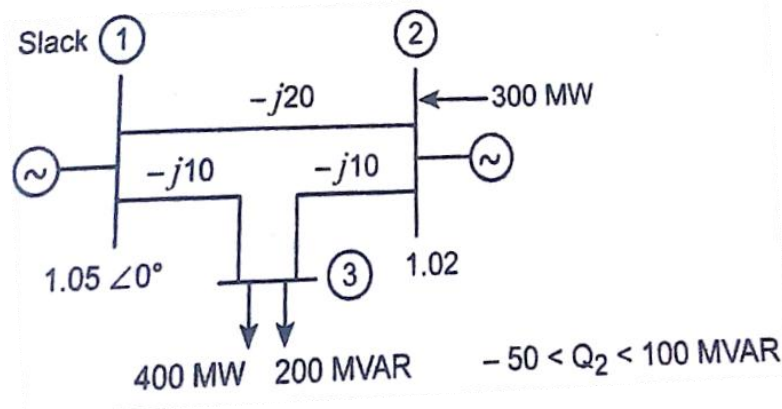
Solution:

Step -1 : Form Y_{bus} matrix

$$Y_{bus} = \begin{bmatrix} Y_{12} + Y_{13} & -Y_{12} & -Y_{13} \\ -Y_{12} & Y_{12} + Y_{23} & -Y_{23} \\ -Y_{13} & -Y_{23} & Y_{13} + Y_{23} \end{bmatrix}$$

$$Y_{bus} = \begin{bmatrix} -j20 + (-j10) & j20 & j10 \\ j20 & -j20 + j10 & j10 \\ j10 & j10 & -j10 + (-j10) \end{bmatrix}$$

$$Y_{bus} = \begin{bmatrix} 30\angle-1.57 & 20\angle1.57 & 10\angle1.57 \\ 20\angle1.57 & 30\angle-1.57 & 10\angle1.57 \\ 10\angle1.57 & 10\angle1.57 & 20\angle-1.57 \end{bmatrix} \qquad \{\text{Note: Use in rad mode}\}$$

Step – 2: Initialize bus voltages

$$V_1^{old} = 1.05\angle0 \text{ p.u.} \quad \text{(slack bus)}$$
$$`V_2^{old} = 1.02\angle0 \text{ p.u.} \quad \text{(P-V bus)}$$
$$V_3^{old} = 1.0\angle0 \text{ p.u.} \quad \text{(P-Q bus)}$$

Step – 3: Check for Q-limit violation for Q bus.

$Q_2^{cal} = - \{|V_2||V_1||Y_{21}| \sin(\theta_{21} + \delta_1 - \delta_2) + |V_2|^2|Y_{22}| \sin(\theta_{22}) + |V_2||V_3||Y_{23}| \sin(\theta_{23} + \delta_3 - \delta_2)\}$

$\qquad = - 1.02[1.05 \text{ X } 20 \sin(1.57-0+0)+ 1.02^2 \text{ X } 30 \sin(-1.57)+1.02 \text{ X } 1.0 \text{ X } 10 \text{ X } \sin(1.57-0+0)]$

$\boxed{Q_2^{cal} = - 0.408 \text{ p.u.}}$

$\dfrac{-50}{100} < -0.408 < \dfrac{100}{100}$

$Q_{2(min)} < Q_2 < Q_{2(max)}, \qquad \therefore \text{Bus 2 acts as P-V bus.}$

Step – 4: Compute ΔP and ΔQ.

$P_2^{cal} = |V_2|\{|V_1||Y_{21}| \cos(\theta_{21} + \delta_1 - \delta_2) + |V_2|^2|Y_{22}| \cos(\theta_{22}) + |V_2||V_3||Y_{23}| \cos(\theta_{23} + \delta_3 - \delta_2) \}$

$\qquad = 1.02 \text{ X } 1.05 \text{ X } 20 \cos(1.57-0+0) + 1.02^2 \text{ X } 30 \cos(-1.57) + 1.02 \text{ X } 1.0 \text{ X } 10 \text{ X } \cos(1.57 - 0 + 0)]$

$\boxed{P_2^{cal} = 0.05}$

$P_3^{cal} = \{|V_3||V_1||Y_{31}| \cos(\theta_{31} + \delta_1 - \delta_3) + |V_3||V_2||Y_{32}| \cos(\theta_{32} - \delta_3 + \delta_2) + |V_3||V_3||Y_{33}| \cos(\theta_{33}) \}$

$\qquad = 1.0 \text{ X } 1.05 \text{ X } 10 \cos(1.57-0+0) + 1.0 \text{ X } 1.02 \text{ X } 10 \cos(1.57- 0 + 0) + 1.0^2 \text{ X } 20 \text{ X } \cos(-1.57)]$

$\boxed{P_3^{cal} = 0.0324}$

$Q_3^{cal} = - \{|V_3||V_1||Y_{31}| \sin(\theta_{31} + \delta_1 - \delta_3) + |V_3||V_2||Y_{32}| \sin(\theta_{32} - \delta_3 + \delta_2) + |V_3||V_3||Y_{33}| \sin(\theta_{33}) \}$

$\qquad = - 1.0 \text{ X } 1.05 \text{ X } 10 \sin(1.57-0+0) + 1.0 \text{ X } 1.02 \text{ X } 10 \sin(1.57 -0 +0) + 1.0^2 \text{ X } 20 \text{ X } \cos(1.57 - 0 + 0)]$

$\boxed{Q_3^{cal} = - 0.7}$

$\Delta P_2 = P_{2(spec)} - P_2^{cal} = 3 - 0.05 = 2.95$

$\Delta P_3 = P_{3(spec)} - P_3^{cal} = - 4 - 0.0324 = - 4.0324$

$\Delta Q_3 = Q_{3(spec)} - Q_3^{cal} = - 2 - (-0.7) = - 1.3$

Step – 5: Bus susceptance matrix

$$[B_3'] = \begin{bmatrix} -30 & 10 \\ 10 & -20 \end{bmatrix}$$

$$[B']^{-1} = \frac{1}{500} \begin{bmatrix} -20 & -10 \\ -10 & -30 \end{bmatrix}$$

$$\boxed{[B']_3^{-1} = \begin{bmatrix} -0.04 & -0.02 \\ -0.02 & -0.06 \end{bmatrix}}$$

$[B''] = 3 \, [-20]$

$[B'']^{-1} = \dfrac{1}{-20} = - 0.05$

$\boxed{[B'']^{-1} = - 0.05}$

Note

B′ matrix is the imaginary part of Y_{bus} matrix for the buses except slack bus.

B″ matrix is the imaginary part of Y_{bus} matrix for the P - V buses only.

Step – 6: Calculate $\Delta\delta$ and ΔV.

$$\begin{bmatrix}\Delta\delta_2\\\Delta\delta_3\end{bmatrix} = -[B']^{-1}\begin{bmatrix}\frac{\Delta P_2}{|V_2|}\\\frac{\Delta P_3}{|V_3|}\end{bmatrix} = -\begin{bmatrix}-0.04 & -0.02\\-0.02 & -0.06\end{bmatrix}\begin{bmatrix}\frac{2.95}{1.02}\\\frac{-4.0324}{1.0}\end{bmatrix} = \begin{bmatrix}-0.04 & -0.02\\-0.02 & -0.06\end{bmatrix}\begin{bmatrix}2.892\\-4.0324\end{bmatrix}$$

$$\begin{bmatrix}\Delta\delta_2\\\Delta\delta_3\end{bmatrix}=\begin{bmatrix}0.035\\-0.184\end{bmatrix}$$

$\delta_2^1 = \delta_2^0 + \Delta\delta_2 = 0+0.035 = 0.035$ rad

$\delta_3^1 = \delta_3^0 + \Delta\delta_3 = 0+(-0.184) = -0.184$ rad

$[\Delta V_3] = -[B'']^{-1}\left[\frac{\Delta Q_3}{|V_3|}\right] = -(-0.05)\left[\frac{-1.3}{1.0}\right] = -0.065$ p.u.

$V_3^1 = V_3^0 + \Delta V_3 = 1.0 + (-0.065) = 0.935$ p.u.

$V_3^1 = 0.935$ p.u.

22. **For the system shown in fig., generators are connected to all the four buses, while loads are at buses 2 and 3. The specifications of the buses are given in table.1 and the values of real and reactive powers are listed in table. Bus 2 be a PV bus with V_2 =1.04 p.u and bus 3 and 4 are PQ bus. Assuming a flat voltage start, find bus voltages and bus angles the end of first Gauss-Seidal iteration. And consider the reactive power limit as $0.2 \leq Q2 \leq 1$.**
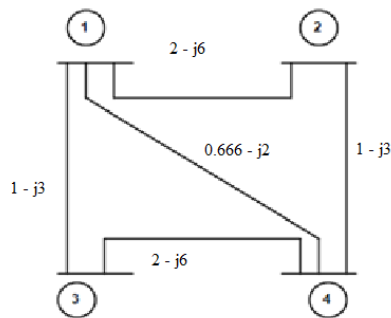
Bus code	P	Q	V	Remarks
1	-	-	$1.04\angle 0^o$	Slack bus
2	0.5	-	1.04 p.u	PV bus
3	-1.0	0.5	-	PQ bus
4	0.3	-0.1	-	PQ bus

Solution

Step – 1: Formulate Y-bus matrix. The given values of admittance are.

$$Y_{bus} = \begin{bmatrix} 3-j9 & -2+j6 & -1+j3 & 0 \\ -2+j6 & 3.666-j11 & -0.666+j2 & -1+j3 \\ -1+j3 & -0.666+j2 & 3.666-j11 & -2+j6 \\ 0 & -1+j3 & -2+j6 & 3-j9 \end{bmatrix}$$

Step – 2: Initialize bus voltages.

$V_1^{old} = 1.04\angle 0°$ p.u. (Slack bus)

$V_2^{old} = 1.04$ p.u. (PV bus)

$V_3^{old} = 1.0$ p.u. $\angle 0°$ p.u. (PQ)

$V_4^{old} = 1.0$ p.u. $\angle 0°$ p.u. (PQ)

Step – 3: Calculate Q2 for the PV bus.

$Q_2^{cal} = -I_m \{V_2^{old} \times [Y_{21}V_1^{new} + Y_{22}V_2^{old} + Y_{23}V_3^{old} + Y_{24}V_4^{old}]\}$

$= -I_m\{1.04 \times [(-2+j6)1.04 + (3.666-j11) \times (-0.0666+j2) \times 1.0 + (-1+j3) \times$

$1.0]\}$

$= -I_m \{0.069 - j0.208\} = 0.208$ p.u

$\boxed{Q_2^{cal} = 0.208 \text{ p.u}}$

$0.2 < Q_2^{cal} < 1$, within limits.

Bus 2 acts as PV bus.

$P_2 = 0.5$ p.u, $Q_2 = 0.208$ p.u

Calculate V_2

$V_2^{new} = \frac{1}{Y_{22}}\left[\frac{P_2-jQ_2}{V_2^{old*}} - Y_{21}V_1^{new} - Y_{23}V_3^{old}\right]$

$= \frac{1}{3.666-j11}[\frac{0.5-j0.208}{1.04} -(-2+j6)\times 1.04 - (-0.666+j2)\,1.0 - (-1+j3)\times 1.0]$

$= 1.051 + j0.0339 = 1.0518\angle 1.846° = 1.0518\angle 0.032$ rad.

$\delta_2 = 0.032$ rad

$\boxed{V_2^{new} = 1.04\angle 0.032 \text{ rad} = 1.0395 + j0.0333}$

$V_3^{new} = \frac{1}{Y_{33}}\left[\frac{P_3-jQ_3}{V_3^{old*}} - Y_{31}V_1^{new} - Y_{33}V_2^{new} - Y_{34}V_4^{old}\right]$

$= \frac{1}{3.666-j11}[\frac{-1.0-j0.5}{1.0} -(-1+j3)\times 1.04 - (-0.666+j2)\times(1.0395-j0.0333) - (-2+j6)\times 1.0]$

$\boxed{V_3^{new} = 1.0317 - j0.0894 \text{ p.u}}$

$V_4^{new} = \frac{1}{Y_{44}}\left[\frac{P_4-jQ_4}{V_4^{old*}} - Y_{41}V_1^{new} - Y_{42}V_2^{new} - Y_{43}V_3^{new}\right]$

$= \frac{1}{3-j9}[\frac{0.3+j015}{1.0} -0\times 1.04 - (-1+j3)\times 1.0395 + j0.0333) - (-2+j6)\times(1.0317-j0.0894)]$

$\boxed{V_4^{new} = \mathbf{1.0343 - j0.015 \text{ p.u}}}$

23. **for the system shown in fig., determine the voltages at the end of the first iteration by Gauss-Seidal method and also find the slack bus power, line flows, transmission loss. Assume base MVA as 100. Resolve the previous problem, the reactive power constraint on generator bus – 2 be changed to $10 \leq Q_2 \leq 100$. Determine slack bus power. [Q_2 in MVAR].**
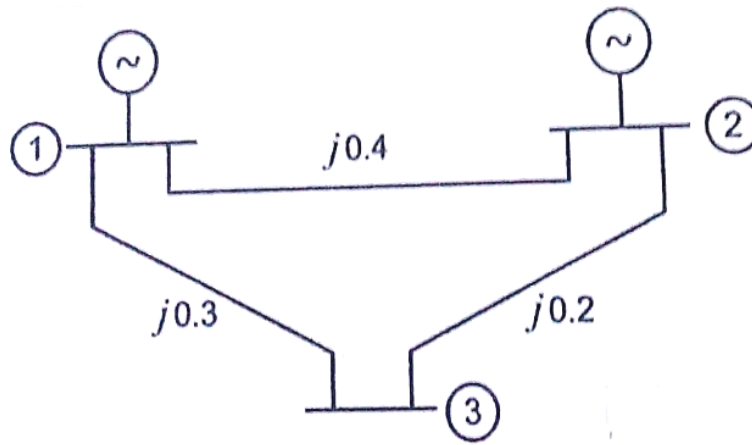
Solution:

Step 1: $Y_{bus} = \begin{bmatrix} -j5.8333 & j2.5 & j3.333 \\ j2.5 & -j7.5 & j5 \\ j3.333 & j5 & -j8.333 \end{bmatrix}$

Step 2: Initialize bus voltage

$$V_1^{old} = 1.05\angle 0^\circ \ p.u$$

$$V_2^{old} = 1.02\angle 0^\circ \ p.u$$

$$V_3^{old} = 1.0\angle 0^\circ \ p.u$$

Step 3: Calculate Q Value for generator bus

$$Q_2^{cal} = -\operatorname{Im}\left[1.02\angle 0^\circ \left[j2.5*1.05\angle 0^\circ + \left(-j.5*1.02\angle 0^\circ\right) + j5*1\angle 0^\circ \right]\right]$$

$$Q_2^{cal} = 0.025 \ \text{p.u}$$

$Q_2^{cal} < Q_{2(\min)}$ [Q_2 exceeds the limit $\therefore$ Bus 2 will act as load bus, i.e. $V_2^{old} = 1.0\angle 0^\circ$]

Substituting $Q_2 = Q_{2(\min)} = 10 MVAR$

$$= \frac{10}{100} = 0.1 p.u$$

$$\boxed{V_2^{old} = 1.0\angle 0^\circ}$$

Step 4: Calculate V_i^{new}

$$V_2^{new} = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^{old*}} - Y_{21}V_1^{new} - Y_{23}V_3^{old} \right]$$

$$= \frac{1}{-j.5}\left[\frac{0.3 - j0.1}{1\angle 0^\circ} - j2.5 \times 1.05\angle 0^\circ - j5 \times 1\angle 0^\circ \right]$$

$$= 1.03 + j0.04 = 1.0308\angle 2.22^\circ$$

$$V_3^{new} = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{old*}} - Y_{31}V_1^{new} - Y_{32}V_2^{new} \right]$$

$$= \frac{1}{-j8.3333}\left[\frac{-0.4 + j0.2}{1.0\angle -0^\circ} - j3.3333 \times 1.05\angle 0^\circ - j5 \times 1.0308\angle 2.22^\circ \right]$$

$$= 1.014 - j0.024 = 1.014\angle -1.36^\circ$$

Slack b
$$V_1^{new} = 1.05\angle 0°$$
$$V_2^{new} = 1.0308\angle 2.22°$$
$$V_3^{new} = 1.014\angle -1.36°$$
$$\left[Y_{12}V_2 + Y_{13}V_3 \right]$$

$$= 1.05\angle 0° \left[-j5.8333 \times 1.05\angle 0° + j2.5 \times \left(1.0308\angle 2.22°\right) + j3.3333\left(1.014\angle -1.36°\right) \right]$$

$$= -0.0206 - j0.1794\,p.u$$

$$P_1 = -0.0206\,p.u = -2.06MW$$
$$Q_1 = 0.1794\,p.u = 17.94MVAR$$

24. Bus 2 is a P-V bus having the rating P_G = 60MW, V_2=1.02 p.u. 10 < Q_2 < 100 MVAR, carry out one iteration. Perform load flow using Newton Raphson method to determine bus voltages. Take base MVA as 100

Line data:

Line	bus		R (p.u)	X (p.u)	Half line charging admittance ($\frac{Y_p}{2}$ (p. u))
	From	To			
1	1	2	0.839	0.5183	0.0636

Bus data:

Bus	P_L	Q_L
1	90	20
2	30	10

Solution:

Step 1: Form Y_{bus}

$$Y_{bus} = \begin{bmatrix} 1.842\angle -1.405 & 1.904\angle 1.7314 \\ 1.904\angle 1.7314 & 1.842\angle -1.405 \end{bmatrix} \qquad \theta_{12} \text{ in rad.}$$

Step 2: Check for Q limit violation.

$$Q_2^{cal} = -\left\{ |V_2||V_1||Y_{21}|\sin\left(\theta_{12} - \delta_2 + \delta_1\right) + |V_2|^2 |Y_{22}|\sin\theta_{22} \right\}$$

$$= -\left\{ 1.02 \times 1.05 \times 1.904 \times \sin\left(1.7314\right) + 1.02^2 \times 1.842 \times \sin\left(-1.405\right) \right\}$$

$` Q_2^{cal} = -0.1239$

$$Q_2^{cal} < Q_{2(min)}$$

$$Q_2 = Q_{2(min)} = \frac{10}{100} = 0.1\,p.uMVAR$$

Now bus will act as load bus.

$$V_2 = 1.0\angle 0\,p.u$$

Step 3: Compute ΔP_2 and ΔQ_2

$$P_2^{cal} = |V_2||V_1||Y_{21}|\cos(\theta_{12} - \delta_2 + \delta_1) + |V_2|^2|Y_{22}|\cos\theta_{22}$$

$$= 1.0 \times 1.05 \times 1.904 \times \cos(1.7314) + 1.0^2 \times 1.842 \times \cos(-1.405)$$

$$P_2^{cal} = -0.0157$$

$$\Delta P_2 = P_{2(spec)} - P_2^{cal} = \frac{60}{100} - (-0.0157) = 0.6157$$

$$\boxed{\Delta Q_2 = Q_{2(spec)} - Q_2^{cal} = 0.1 - (-0.1239) = 0.2239}$$

Step 4: Form Jacobian matrix.

$$\frac{\partial P_2}{\partial \delta_2} = |V_2||V_1||Y_{12}|\sin(\theta_{12} + \delta_2 - \delta_1)$$

$$= 1.0 \times 1.05 \times 1.904 \times \sin(1.7314 - 0 + 0)$$

$$\frac{\partial P_2}{\partial \delta_2} = 1.973$$

$$\frac{\partial P_2}{\partial V_2} = |V_1||Y_{12}|\cos(\theta_{12} - \delta_2 + \delta_1) + 2|V_2||Y_{22}|\cos\theta_{22}$$

$$= 1.05 \times 1.904 \times \cos(1.7314) + 2 \times 1.0 \times 1.842 \times \cos(-1.405)$$

$$\frac{\partial P_2}{\partial V_2} = 0.288$$

$$\frac{\partial Q_2}{\partial \delta_2} = |V_2||V_1||Y_{12}|\cos(\theta_{12} + \delta_2 - \delta_1)$$

$$\frac{\partial Q_2}{\partial \delta_2} = -0.3197$$

$$\frac{\partial Q_2}{\partial V_2} = |V_1||Y_{12}|\sin(\theta_{12} - \delta_2 + \delta_1) + 2|V_2||Y_{22}|\sin\theta_{22}$$

$$= -1.05 \times 1.904\sin(1.314) - 2 \times 1.0 \times 1.842 \times \sin(-1.405)$$

$$\boxed{\frac{\partial Q_2}{\partial V_2} = 1.66}$$

Step 5: Calculate $\Delta\delta$ and ΔV

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta V_2 \end{bmatrix} = \begin{bmatrix} 1.973 & -0.3197 \\ 0.288 & 1.66 \end{bmatrix}^{-1} \begin{bmatrix} \Delta P_2 \\ \Delta Q_2 \end{bmatrix}$$

$$= \frac{1}{3.367} \begin{bmatrix} 1.66 & 0,288 \\ -0.3197 & 1.973 \end{bmatrix} \begin{bmatrix} 0.615 \\ 0.2239 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} \Delta\delta_2 \\ \Delta V_2 \end{bmatrix} = \begin{bmatrix} 0.3227\,rad \\ 0.073\,p.u \end{bmatrix}}$$

$$\delta_2^1 = \delta_2^0 + \Delta\delta_2 = 0 + 0.3227 = 0.3227\,rad = 18.49^\circ$$

$$V_2^1 = V_2^0 + \Delta V_2 = 1.0 + 0.073 = 1.073$$

$$\boxed{V_2^{new} = 1.073\angle 18.49^\circ}$$

UNIT III - FAULT ANALYSIS – BALANCED FAULTS

Importance of short circuit analysis - assumptions in fault analysis - analysis using Thevenin's theorem - Z-bus building algorithm - fault analysis using Z-bus – computations of short circuit capacity, post fault voltage and currents.

PART – A

1. What is the significance of sub transient reactance and transient reactance in short circuit Studies? **(APR/MAY 2017)**

The sub transient reactance is the ratio of induced emf on no-load and the sub transient symmetrical rms current, (i.e, it is the reactance of a synchronous machine under transient condition)

The faults or short circuits are associated with sudden change in currents. Most of the components of the power system have inductive property which opposes any sudden change in currents and so the faults (short circuit) are associated with transients.

2. For a fault at a given location, rank the various faults in the order of severity. (APR/MAY 2017)

In a power system relatively the most severe fault is the three phase fault and less severe fault is the open conductor fault.

The various faults in the order of decreasing severity is given below.

a. 3-phase fault.
b. Double line to ground fault.
c. Line to line fault.
d. Single line to ground fault.
e. Open conductor fault.

3. What is the need of short circuit study? **(NOV/DEC 2016)**

The short circuit studies are essential in order to design or develop the protective schemes for various parts of the system. The protective scheme consists of current & voltage sensing devices, protective relays and circuit breakers. The selection (or proper choice) of these device mainly depends on various currents that may flow in the fault conditions.

4. How the shunt and series faults are classified? **(NOV/DEC 2016)**

Shunt faults

a. Shunt faults are symmetrical in nature.
b. Shunts faults are caused due to short circuits in conductors.
c. The shunt fault is also called as short circuit faults.

Series faults

a. Series faults are unsymmetrical in nature.
b. Series faults are caused due to open conductors.

5. State and explain symmetrical fault. **(MAY/JUNE 2016)**

The currents and voltages at various parts of the system can be estimated by different methods. The fault on the power system which gives rise to symmetrical fault currents (i.e. equal fault current in the lines with 120^0 displacement) is called as symmetrical faults. The symmetrical fault occurs when all the three conductors of phase are brought together simultaneously in short circuit condition.

6. What is bolted fault or solid fault? **(MAY/JUNE 2016)**

1

A fault represents a structural network change be equivalent with that caused by the additional of impedance at the place of fault. If the fault impedance is zero, then the fault is referred as bolted fault or solid fault.

7. Why do faults occur in a power system? **(NOV/DEC 2015)**

In a power system the fault may occur due to the following reasons.

Insulation failure of equipment's., Flashover of lines initiated by a lightning stroke, Switching surges, Sudden releasing of heavy loads at same instant of time, Due to accidental faulty operation of power system operators.

8. What is direct axis reactance? **(NOV/DEC 2015)**

It is the apparent reactance of the armature winding just at the instant of the short circuit occurs at the terminals of the unloaded synchronous generator and it causes heavy currents to flow during the first few cycles.

9. What is meant by a fault?

A fault in a circuit is any failure which interferes with the normal flow of current. The faults are associated with abnormal change in current, voltage and frequency of the power system. The faults may cause damage to the equipment's if it is allowed to persist for a long time. Hence every part of a system has been protected by means of relays and circuit breakers to sense the faults and to isolate the faulty part from the healthy part in the event of fault.

10. Give the general reason for which fault occurs in a power system.

In a power system the fault may occur due to the following reasons.

a. Insulation failure of equipment's.
b. Flashover of lines initiated by a lightning stroke.
c. Switching surges.
d. Sudden releasing of heavy loads at same instant of time.
e. Due to accidental faulty operation of power system operators.

11. Why does symmetrical fault occur in a power system?

The short circuit fault occurs in a power system due to the following reasons.

a. Insulation failure of equipment's,
b. Flashover of lines initiated by a lightning stroke
c. Through accidental faulty operation.
d. Birds shorting out lines.
e. Aircraft colliding with lines.
f. Trees falling over lines.

12. How are the faults classified?

Generally the faults are classified as follows.

1. **Shunt faults**
    d. Shunt faults are symmetrical in nature.
    e. Shunts faults are caused due to short circuits in conductors.
    f. The shunt fault is also called as short circuit faults.
2. **Series faults**
    c. Series faults are unsymmetrical in nature.
    d. Series faults are caused due to open conductors.

13. List the types of Short circuit faults.

Shorts circuit faults can be classified as follows:

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Symmetrical fault or balanced fault

- Three phase fault

Unsymmetrical fault or unbalanced fault

- Line to ground (L-G) fault
- Line to Line (L-L) fault
- Double line to ground (L-L-G) fault.

14. List out the various faults in the order of severity.

In a power system relatively the most severe fault is the three phase fault and less severe fault is the open conductor fault.

The various faults in the order of decreasing severity is given below.

- f. 3-phase fault.
- g. Double line to ground fault.
- h. Line to line fault.
- i. Single line to ground fault.
- j. Open conductor fault.

15. List out the differences in representing the power system for load flow and short circuit studies.

S.No	Load flow studies	Fault analysis
1	Both resistances and reactances are considered.	Resistances are neglected.
2	Bus admittance matrix is used.	Bus impedance matrix is used.
3	The exact voltages and currents are to be determined.	The voltages can be safely assumed as 1 p.u. and the prefault current can be neglected.

16. What is the need for short circuit studies or fault analysis?

- a. The short circuit studies are essential in order to design or develop the protective schemes for various parts of the system.
- b. The protective scheme consists of current & voltage sensing devices, protective relays and circuit breakers.
- c. The selection (or proper choice) of these device mainly depends on various currents that may flow in the fault conditions.

17. What are the objectives of short circuit analysis?

a. To check the MVA ratings of the existing circuit breakers, when new generation are added into the system.

b. To select the rating for fuses, circuit breakers and switch gear in addition to the set up of protective relays.

c. To determine the magnitudes of currents flowing throughout the power system at various time intervals after a fault occurs.

18. State the applications of short circuit analysis.

- a. For proper relay setting and coordination.
- b. To obtain the rating of protective switch gears.

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c. To select the circuit breakers.

d. To perform whenever system expansion is planned.

19. What are the ways to reduce short circuit current?

Following are the two important ways to reduce the short circuit current:

a. By providing neutral reactance.

b. By introducing a large value of shunt reactance between the buses.

20. What are the various period involved in fault calculation?

The fault condition of a power system can be divided into

a. Sub transient period,

b. Transient period,

c. Steady state periods.

The currents in the various parts of the system and in the fault locations are different in these periods. The estimation of these currents for various types of faults at various locations in the system is commonly referred to as fault calculations.

21. How are symmetrical faults analyzed?

The symmetrical faults are analyzed using per unit reactance diagram of the power system. Once the reactance diagram is formed, then the fault is simulated by short circuit or by connecting the fault impedance at the fault point. The currents and voltages at various parts of the system can be estimated by different methods. The fault on the power system which gives rise to symmetrical fault currents (i.e. equal fault current in the lines with 120^0 displacement) is called as symmetrical faults. The symmetrical fault occurs when all the three conductors of phase are brought together simultaneously in short circuit condition.

22. What are the points to be noticed while undergoing symmetrical fault analysis?

The points to be noticed while undergoing symmetrical faults are given as follows.

a. The symmetrical faults rarely occur in practice as majority of the faults are of unsymmetrical nature.

b. The symmetrical fault is the most severe and imposes more heavy duty on the circuit breaker.

23. Why the computation of prefault currents neglected in fault calculation?

The changes in the short circuit currents are limited only by the series impedance elements. Post fault currents are almost purely reactive whereas the pre fault load currents are almost purely real. The total post fault current is therefore obtained as the vectorial sum of two currents having a phase difference of almost 90^0. The magnitude of total current is approximately equal to the magnitude of the largest component. So we can neglect the prefault current.

24. What are the different methods by which faulted network can be solved?

The different methods by which faulted network can be solved are listed as follows.

1) By use of transient and subtransient internal voltages.

2) Using Thevenin's theorem

3) By forming bus impedance matrix.

25. What are symmetrical components?

An unbalanced system of N related vectors can be resolved into N systems of balanced vectors called symmetrical components. The various symmetrical components are as follows.

a. Positive sequence components

b. Negative sequence components

c. Zero sequence components

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This type of fault is defined as the simultaneous short circuit across all the three phases. It occurs infrequently, but it is the most severe type of fault encountered. Because the network is balanced, it is solved by per phase basis using Thevenin's theorem or bus impedance matrix or KVL, KCL laws.

26. What are the needs of sequence network in power system?
   a. Sequence network in power system is very much useful for computing unsymmetrical for computing unsymmetrical faults at different points of the power system network.
   b. Positive sequence network is necessary for load flow studies.
   c. Negative and zero sequence networks are used in stability studies involving unsymmetrical faults**.**

27. What are the assumptions made in short circuit studies of a large power system network?
   a. The phase to neutral emfs of all generators remain constant, balanced and unaffected by the faults.
   b. Each generator is represented by an emf behind either the sub transient or transient reactance depending upon whether the short circuit current is to be found immediately after the short circuit or after about 3 – 4 cycles.
   c. Load currents may often be neglected in comparison with fault currents.
   d. All network impedances are purely reactive. Thus the series resistances of lines and transformers are neglected in comparison with their resistances.

28. Write few words about Positive sequence components.
   Positive sequence components consist of three phasors equal in magnitude, displaced from each other by 120° in phase, and having the same sequence as the original phasors.
   It is denoted as a_1, b_1 and c_1.
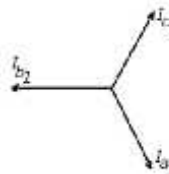
Positive sequence components

29. Write about negative sequence components.
   Negative sequence components consist of three phasors equal in magnitude, displaced from each other by 120° in phase, and having the phase sequence opposite to that of the original phasors. V_{a2}, V_{b2} and V_{c2} are the negative sequence components of V_a, V_b and v_c.
   If a_2, b_2 and c_2 are the three phasors and if a_2 is followed by c_2 followed by b_2 they are said to have negative sequence.
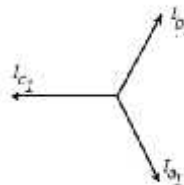
Negative sequence components

30. Write about zero sequence components.
   Zero sequence components consist of three phasors equal in magnitude and with Zero phase displacement from each other. V_{ao}, V_{bo} and V_{co} are the zero sequence components of V_a, V_b and V_c.

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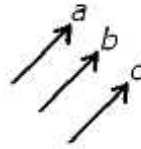
Zero sequence components

31. What are the advantages of symmetrical components?

The advantages of symmetrical components are given as follows:

 a. The unbalance system of n related vectors can be resolved into n system of balanced vectors.

 b. The positive sequence components consists of three vectors equal in magnitude, displace from each other by 120° in phase, and having the same phase sequence as the original vectors.

 c. Unsymmetrical fault analysis can be done by using symmetrical components.

32. In what way does the negative sequence network differ from the positive sequence network?

The negative sequence network is very much similar like that of positive sequence network but they differ in following aspects.

 a. Normally there is no negative sequence e.m.f source.

 b. Negative sequence impedance of rotating machine is generally different from their positive sequence impedances.

 c. The phase displacement of transformer banks for negative sequence is of opposite sign to that of positive sequence.

33. How does the zero sequence networks differ from positive sequence and negative sequence network?

Zero sequence network differ greatly from positive sequence and negative sequence network in the following aspects.

a. Zero sequence reactance of transmission lines is higher than that for positive sequence.

b. Equivalent circuit for transformers is different.

c. The neutral grounding should be considered in zero sequence network.

34. Why delta connected load will not have any zero sequence components?

The current in the neutral is three times the zero sequence line current. A delta connected load provides no path to neutral and hence the line currents flowing to a delta connected load contain zero components. Hence the delta connected load will not have any zero sequence components.

35. Write down the expression of power in terms of symmetrical components.

The expression or equation power in terms of symmetrical components is given by

$$P=3V_0I_0^* +3V_1I_1^* +3V_2I_2^*$$

Where V_0 and I_0 are zero sequence voltage and current.

V_1 and I_1 are positive sequence voltage and current.

V_2 and I_2 are negative sequence voltage and current.

36. What is symmetrical components of three phase system?

Symmetrical components of three phase system is given as:

For voltage

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & u & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

For current.

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$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

37. Write the relative frequency of occurrence of various types of faults.

Types of fault	Relative frequency of occurrence of faults
Three phase fault	5%
Double line to ground fault	10%
Line to Line fault	15%
Line to ground fault	70%

38. Name the reactance used in the analysis of symmetrical faults on the synchronous machines as its equivalent reactance.

The reactance used in the analysis of symmetrical faults on the synchronous machines as its equivalent reactance are given below. They are.

a. Subtransient reactance.
b. Transient reactance.
c. Synchronous reactance.

39. Define synchronous reactance?

The synchronous reactance is the ratio of induced emf and the steady state rms current (i.e. it is the reactance of a synchronous machine under steady state condition). It is the sum of leakage reactance and the reactance representing armature reaction.

It is given by,

$X_s = X_l + X_a$

Where,

X_s = Synchronous reactance

X_l = Leakage reactance

X_a = Armature reaction reactance.

40. What is the reason for transients during short circuits?

The faults or short circuits are associated with sudden change in currents. Most of the components of the power system have inductive property which opposes any sudden change in currents and so the faults (short circuit) are associated with transients. The transient reactance is used to estimate the transient state fault current. Most of the circuit breakers open their contacts only during this period. Therefore for a circuit breaker used for fault clearing (or protection), the interrupting short circuit current rating should be less than the transient fault current.

41. Define sub transient reactance and give its significance.

The sub transient reactance is the ratio of induced emf on no-load and the sub transient symmetrical rms current, (i.e, it is the reactance of a synchronous machine under transient condition)

$$\text{Sub transient reactance} = \frac{\text{Induced emf on no-load}}{\text{Sub transient symmetrical rms current}}$$

The sub-transient reactance can be used to estimate the initial value of fault current

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immediately on the occurrence of the fault. The maximum momentary short circuit current rating of the circuit breaker used for protection or fault clearing should be less than this initial fault current.

42. List the advantages of symmetrical components.
   a. This method is simple.
   b. This method leads to accurate prediction of system behavior.
   c. Unsymmetrical faults on the transmission system are studied by the method of symmetrical components.
   d. They suitable to determine the currents and voltages in all parts of the power system after the occurrence of fault**.**

43. How transients occur during short circuits?
   A sudden change or sudden disturbance in the voltage and current rating of a system then the system is said to be in transient. The faults or short circuits are associated with sudden change in currents. Most of the components of the power system have inductive property which opposes any sudden change in currents, so the faults are associated with transients. The main reasons for transients are insulation failure, switching surges, improper earthing, flashover,etc.

44. What is the purpose of analyzing unsymmetrical fault?
   Analysis of unsymmetrical fault is very important for the following reasons. They are given as follows
   a. Relay setting,
   b. Single phase switching
   c. System stability studies

45. Write down the equation to find the fault current in bus-k and change in voltages in other buses due to a 3-phase fault in bus-k using impedance matrix.

   The fault current in bus-k, $I_f = \frac{V_{pf}}{Z_{kk}}$

   Where, V_{pf} = prefault voltage in bus-k, (normally 1 p.u)

   The change in bus-q voltage due to a 3 phase fault in bus-k, $\Delta V_q = -I_f Z_{qk}$; for q=1,2,3…n

46. Define direct axis subtransient reactance and transient reactance.
   It is the apparent reactance of the armature winding just at the instant of the short circuit occurs at the terminals of the unloaded synchronous generator and it causes heavy currents to flow during the first few cycles.
   The effective reactance after the damper winding currents have died out is called transient reactance of the machine. It determines the fault current after several cycles.

47. Define direct axis synchronous reactance and state which fault is the severe fault when compared to all.
   Direct axis synchronous reactance is the apparent reactance of the armature winding and it comes into action only after the transient period is over and steady state condition is reached.
   Three phase short circuit occurs rarely but it is the most severe type of fault involving largest fault currents.

48. What are sequence impedances and sequence networks?
   The sequence impedances are the impedances offered by the devices or Components for the like sequence component of the current.

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The single phase equivalent circuit of a power system consists of impedances and current at any one sequence only is called sequence network.

49. Define transient reactance and DC off set current

Transient reactance

The synchronous reactance is the ratio of induced emf on no load and the transient symmetrical rms current.

DC off set current.

The unidirectional transient component of short circuit current is called DC off set current.

50. Define short circuit capacity of power system (or) fault level.

Short circuit capacity or short circuit MVA of fault level at a bus is defined as the product of the magnitudes of the perfect bus voltage and the post fault current.

Uses of short circuit capacity

a. It is used for determining the dimension of a bus bar
b. It is used for determining the interrupting capacity of a circuit breaker.
c. It helps to find out the magnitude of fault current.

PART – B

1. **A 3 phase 5 MVA 6.6 KV alternator with a reactance of 8% is connected toa afeeder series impedance (0.12 + j0.48)ohm/phase/km through a step up transformer. The transformer is rated at 3 MVA, 6.6 KV/33 KV and has a reactance of 5%. Determine the fault current supplied by the generator operating under no load with a voltage of 6.9 KV, when a 3 phase symmetrical fault occurs at a point 15KM along the feeder.** (APR/MAY 2017)

∴ Actual value of fault current, I_f= p.u. value of $I_f \times I_b$ = 9.3179∠-84.9° × 67.4773

> I_f= 45.2∠-97.6° amps.

Result:

Fault current = 7.9∠-17.6° p.u. or 465.2∠-87.6° A.

2. **Draw the detailed flowchart, which explains how a symmetrical fault can be analyzed using Z - bus.** (APR/MAY 2017)

Step 1 : Start.

Step 2 : Read line impedance, generator impedance and fault impedance.

Step 3 : Form the Z_{bus} matrix using step by step assembly using 4 types of modifications.

Step 4 : Set all bus voltages as 1∠0° p.u.

Step 5 : Connect Z_f in series with faulted bus.

Step 6 : Calculate fault current for ith bus.

$$I_i(F) = \frac{V_{i(o)}}{Z_{ii} + Z_f}$$

Step 7 : Calculate the generator current $I_G = Z_{eq} I_{Gi}$

Step 8 : Calculate change in voltage ΔV_i and $V_i(F)$

$$V_i(F) = V_{i(o)} + \Delta V_i$$

Step 9 : Calculate $V_j(F) = V_{j(o)} = -\sum_{\substack{j=1 \\ \neq i}}^{n} Z_{ij} I_i(F)$

Step 10 : Calculate $I_{ij}(F) = \frac{V_i(F) - V_j(F)}{Z_{ij}}$

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Step 11 : Print the result.

Step 12 : Stop.

3. **A 100 MVA, 11 KV generator with X"=0.20p.u is connected through a transformer and line to a bus bar that supplies three identical motor as shown in fig and each motor has X"=0.20 p.u and X"=0.25p.u on a baase of 20 MVA, 33 KV, the bus voltage at the motors is 33KV when three phase balanced fault occurs at the point F. calculate (i) sub transient current in the fault (ii) sub transient current in the circuit breaker (iii) Momentary current in the circuit breaker (iv) the current to be interrupted by C.B.B in 5 cycles.** **(APR/MAY 2017)**
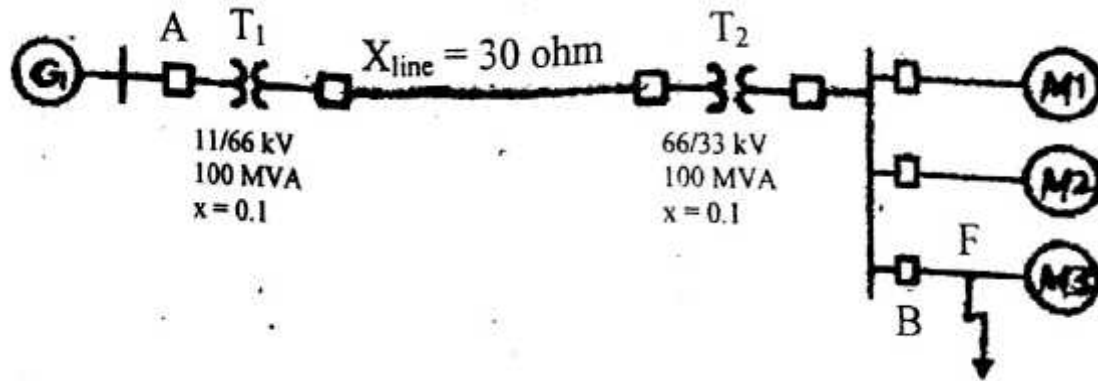
Actual value of current in the fault during subtransient state $I_f" = 10.1\angle-80° \times 156.16$

$I_f" = 50.56\angle-90° = 16.752\angle-20°$ kA

$I_f" = 50.56\angle-90°$ kA

4. **For the radial network shown in fig. 3 phase fault occurs at point F. determine the fault current and the line voltage at 11.8 KV bus under fault condition.** **(NOV/DEC 2016)**
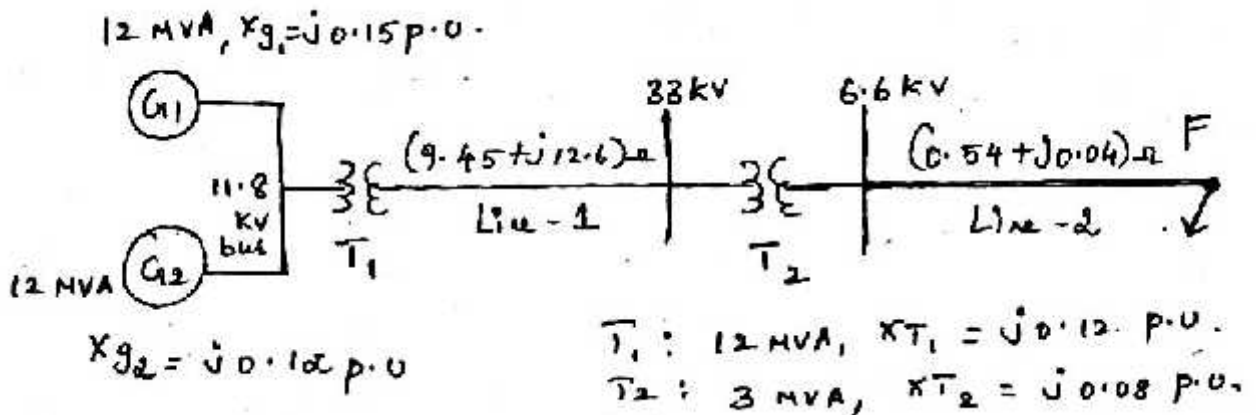
$I_f" = 12.0\angle-90°$

$I_f" = 12.0\angle-90°$ kA

5. **A 3-phase, 5 MVA, 6.6 kV alternator with a reactance of 8% is connected to a feeder of series impedance of 0.12+j0.48 ohms/phase per km. The transformer is rated at 3 MVA, 6.6kV/3.3 kV and has a reactance of 5%. Determine the fault current supplied by the generator operating under no-load with a voltage of 6.9kV, when a 3-phase symmetrical fault occurs at a point 15km along the feeder. 1.For the two bus system as shown in fig. Determine the fault current at the fault point and in other element and post fault voltage, for a bolted fault at bus 4.The subtransient reactance of the generators and positive sequence reactance of other elements are given.** **(NOV/DEC 2016)**

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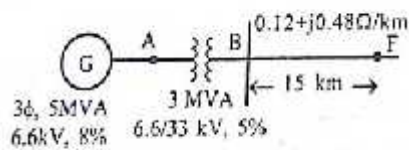
Single line diagram
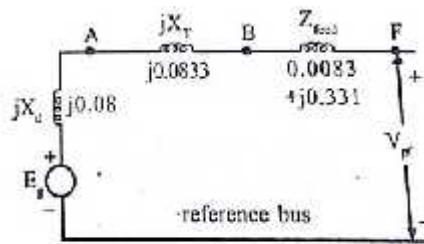
Prefault reactance diagram

Let us choose generator rating as base values.

$\therefore MVA_b = 5$ MVA and $kV_b = 6.6$ kV

To find generator reactance:

$\therefore$ p.u. reactance of the generator, $X_d = 8\% = 0.08$ p.u.

$X_d = 8\% = 0.08$ p.u.

To find transformer reactance:

$$X_{pu,new} = X_{pu,old} \times \left(\frac{kV_{b\,old}}{kV_{b\,new}}\right)^2 \times \frac{MVA_{b\,new}}{MVA_{b\,old}}$$

Here $X_{pu,old} = 5\% = 0.05$ p.u.

$\therefore$ p.u. reactance of transformer, $X_T = 0.05 \times \frac{66}{6.6} \times \frac{5}{3} = 0.0833$ p.u.

$X_T = 0.0833$ p.u.

To find feeder reactance:

$$Z_{feed} = (0.12 + j0.48) \times 15 = 1.8 + j7.2\,\Omega/phase$$

$\therefore$ p.u. value of the impedance of the feeder, $Z_{feed} = \frac{Actual\ impedance}{Base\ impedance} = \frac{1.8 + j7.2}{217.8}$

$Z_{feed} = 0.0083 + j0.0331$ p.u.

To find fault current:

$$Z_{th} = 0.1966\angle 87.6° \text{ p.u.}$$

The fault in the feeder can be represented by a short circuit as shown. Now the current If through the short circuit is the fault current.

$\therefore$ p.u. value of fault current, $I_f = \frac{V_{th}}{Z_{th}} = \frac{1.0455\angle 0°}{0.1966\angle 87.6°} = 5.3179\angle -87.6°$ p.u.

$I_f = 5.3233\angle -90°$ p.u.

It is important to note that the error in neglecting the resistive component is negligible.

Base current, $I_b = \frac{kVA_b}{\sqrt{3}\,kV_L} = \frac{MVA_b \times 1000}{\sqrt{3}\,kV_L} = \frac{5 \times 1000}{\sqrt{3} \times 33} = 87.4773$ A.

Result:

Fault current = $5.3179\angle -87.6°$ p.u. or $465.2\angle -87.6°$ A.

6. **Generator G1 and G2 are identical and rated 11KV, 20MVA and have a transient reactance of 0.25p.u at own MVA base. The transformers T1 and T2 are also identical and are rated 11/66 KV, 5 MVA and have a reactance of 0.06p.u to their own MVA base. A 50KM long transmission line is connected betweek the two generators. Calculate three phase fault current, when fault current occurs at middle of the line as shown in fig.** **(MAY/JUNE 2016)**

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Assuming bolted fault or solid fault, $Z_f = 0$. Prefault voltage $E_{Th} = 1\angle 0^0$

Fault current $I_f = \frac{E_{Th}}{Z_f + Z_{Th}} = \frac{E_{Th}}{Z_f + Z_{Th}} = \frac{1\angle 0^0}{0 + j0.144} = 9.68\angle -90^0$ p.u

Fault current $I_f = 9.68\angle -90^0$ p.u

7. **A synchronous generator and motor are rated for 30000kVA, 13.2kV and both have subtransient reactance of 20%. The line connecting them has a reactance of 10% on the base of machine ratings. The motor is drawing 20000kW at 0.8pf leading. The terminal voltage of the motor is 12.8kV. When a symmetrical three phase fault occurs at motor terminals, find the subtransient current in generator, motor and at the fault point.   (MAY/JUNE 2016)**
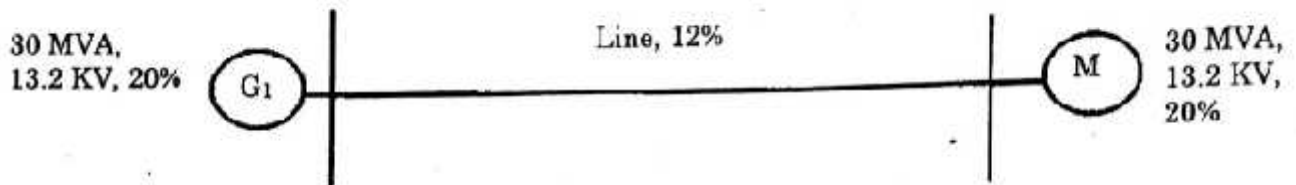
The base values are, $MVA_b = 30$ MVA, $kV_b = 13.2$ kV

Base current, $I_b = \frac{kVA_b}{\sqrt{3}kV_b} = \frac{30 \times 1000}{\sqrt{3} \times 13.2} = 1312.16$ A

$I_b = 1312.16$ A

Prefault condition:

The voltages and currents in the various elements of the system just before the fault are shown in fig. in this circuit V_{tm} and I_L are known values and using these values the subtransient internal voltages E_g'' and E_m'' can be calculated by Kirchoff's voltage Law(KVL) as shown below.

Fig. Prefault current and voltages.

By applying KVL in the circuit of fig, we get,

$E_g'' = j0.2 \ I_L + j0.1 \ I_L + V_{tm}$

$= j0.3 \ I_L + V_{tm}$

$\therefore E_m'' = V_{tm} - j0.2 \ I_L = 0.9697\angle 0° - (0.2\angle 90° \times 0.8594\angle 36.9°)$

$= 0.9697\angle 0° - (0.1719\angle 12.9°) = 0.9697 - (-0.1032 + j0.1375)$

$= 1.0729 - j0.1375 = 1.0817\angle -7.3°$ p.u.

$E_m'' = 1.0817\angle -7.3°$ p.u.

$I_g'' = 2.802\angle -75.8°$ p.u.

$I_m'' = 5.4085\angle -97.3°$ p.u.

Actual value of current in the fault during subtransient state $I_f'' = 8.081\angle -90° \times 1312.16$

$I_f'' = 10603.56\angle -90° = 10.60356\angle -90°$ kA

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$$I_f'' = 10.60356\angle-90° \text{ kA}$$

8. **A generating station feeding a 132 KV system is shown in fig. determine the total fault current, fault level and fault current supplied by each alternator for a 3 phase fault at the receiving end bus. The line is 200KM long.** **(NOV/DEC 2015)**
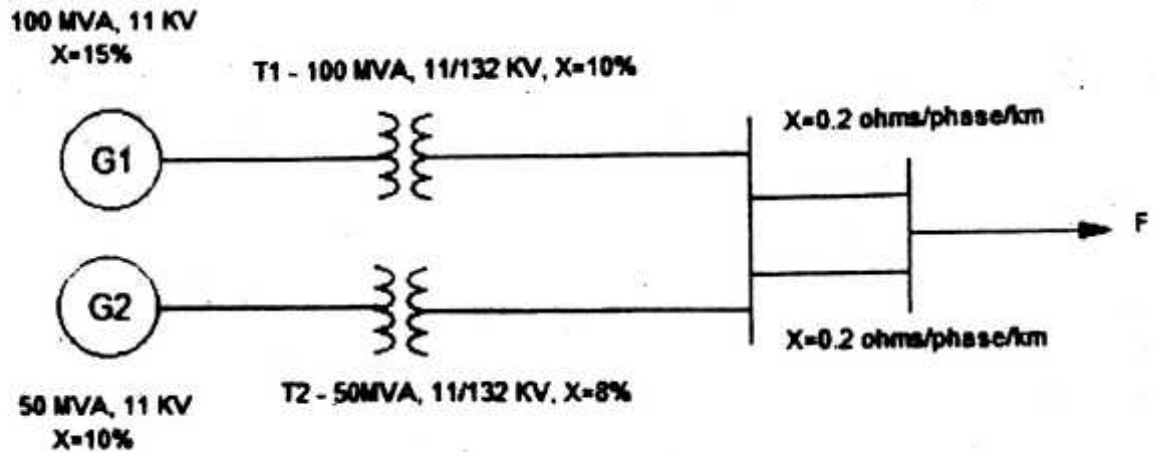
Assuming bolted fault or solid fault, $Z_f = 0$. Prefault voltage $E_{Th} = 1\angle0°$

Fault current $I_f = \frac{E_{Th}}{Z_f + Z_{Th}} = \frac{E_{Th}}{Z_f + Z_{Th}} = \frac{1\angle0°}{0 + j0.144} = 6.94\angle - 90°$ p.u

Fault current $I_f = 6.94\angle - 90°$ p.u

9. **A symmetrical fault occurs on bus 4 of system shown in fig. Determine the fault current, post fault voltages and line currents.** **(NOV/DEC 2015)**
   G_1, G_2 : 100 MVA, 20 KV, $X^+=15\%$.
   Transformer: $X_{leak}=9\%$
   L_1, L_2: $X^+=10\%$.
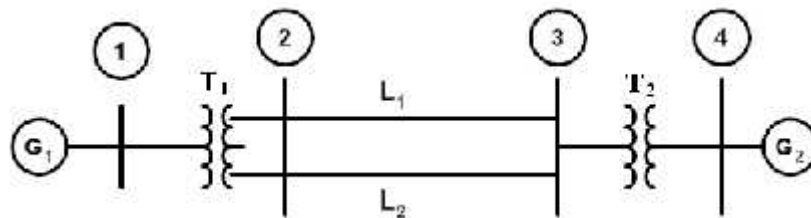
$$= \begin{bmatrix} j0.1075 & j0.172 & j0.068 & j0.0424 \\ j0.172 & j0.13 & j0.108 & j0.068 \\ j0.068 & j0.108 & j0.13 & j0.082 \\ j0.0424 & j0.13 & j0.082 & j0.075 \end{bmatrix}$$

Step 1: fault current

$$I_f = \frac{V^0}{Z_{qq} + Z_f} = \frac{1\angle0°}{j0.1075} = 9.3\angle - 90° \text{ p.u}$$

$I_f = 9.3\angle - 90°$ p.u

Actual current in KA = p.u value x base current.

$$= 9.3\angle - 90° \times \frac{MVA}{\sqrt{3} \times KV}$$

$$= 9.3\angle - 90° \times \frac{100}{\sqrt{3} \times 20} = 26.85 \text{ KA}$$

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Actual current in KA = 26.85 KA

Step 2 : Post fault line currents.

$$I^f_{ij} = \frac{v^f_i - v^f_j}{z_{ij\ series}} \quad ; \quad I^f_{12} = \frac{v^f_1 - v^f_2}{z_{12}} = \frac{0.6056 - 0.3686}{j\ 0.09}$$

$\left|I^f_{12}\right|$ = 2.634 p.u

$$I^f_{23} = \frac{v^f_2 - v^f_3}{z_{23}} = \frac{0.3686 - 0.2374}{j\ 0.05}$$

$\left|I^f_{23}\right|$ = 2.63 p.u.

$$I^f_{34} = \frac{v^f_3 - v^f_4}{z_{34}} = \frac{0.2374 - 0}{j\ 0.09}$$

$\left|I^f_{34}\right|$ = 2.637 p.u.

10. For the two bus system shown in fig. determine the fault current, bus voltages, line currents during the fault when a 3 phase fault impedance Z_f = j0.15. p.u occurs on Bus 4.
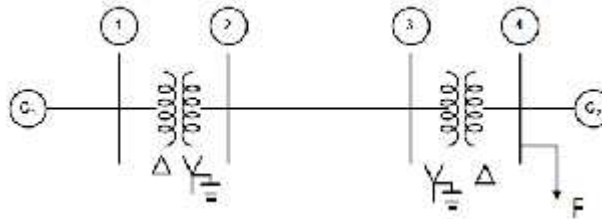
Therefore $Z_{Th} = \frac{j0.15 \times j0.85}{j0.15 + j0.85}$ = j 0.1275

Z_{Th} = j 0.1275

Thevenin's equivalent circuit.

Prefault voltage $E_{Th} = 1\angle 0^0$

Fault current $I_f = \frac{E_{Th}}{Z_{Th} + Z_f} = \frac{1\angle 0^0}{j01.1275 + j0.15}$

$$= -j3.604 = 3.604 \angle - 90^0 \text{ p.u}$$

Fault current I_f = 3.604$\angle - 90^0$ p.u

I_{G1} = 0.5406 $\angle - 90^0$ p.u
I_{G2} = 3.0634 $\angle - 90^0$ p.u

14
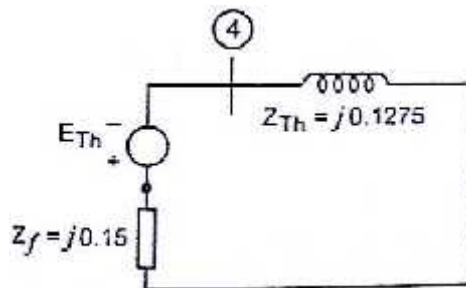
Line flows (post fault):

$$I_{12} = \frac{V_1 - V_2}{Z_{12}} = \frac{0.9189 - 0.8108}{j0.2} = -j0.5406 \text{ p.u}$$

$$I_{23} = \frac{V_2 - V_3}{Z_{23}} = \frac{0.8108 - 0.6486}{j0.3} = -j0.5406 \text{ p.u}$$

$$I_{34} = \frac{V_3 - V_4}{Z_{34}} = \frac{0.6486 - 0.5406}{j0.2} = -j0.54 \text{ p.u}$$

I_{12} = - j0.5406 p.u

I_{23} = - j0.5406 p.u

I_{34} = - j0.54 p.u

11. For the two bus system shown in fig. determine the fault current at the fault point and in other elements for a fault at bus 2 with Z_f =0. The subtransient reactance of the generators and positive sequence reactance of other elements are given.

Generator X=10%, Transmission line X=20%, Transformer X=12 %.

p.u
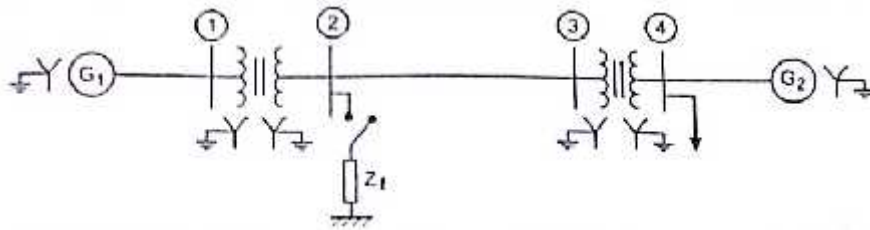
Fault current I_f = 6.94$\angle - 90^0$ p.u

Step 1: Current contribution from generators.
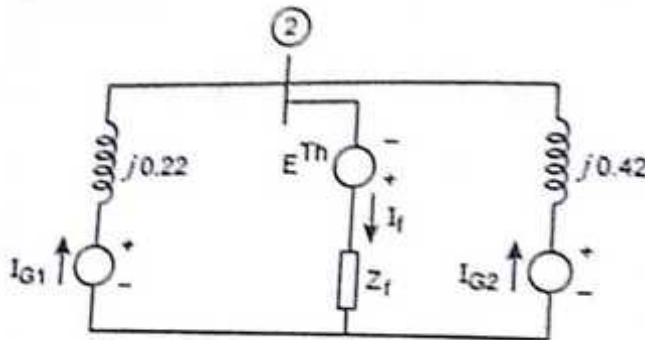
In general, $I_G = I_{Total} \times \frac{Z_{parallel}}{Z_{Total}}$

$I_{G1} = I_f \times \frac{j0.42}{j0.22 + j0.42} = -j4.55 \text{ p.u} = 4.55 \angle - 90^0 \text{ p.u}$

I_{G1} = 4.55 $\angle - 90^0$ p.u

$I_{G2} = I_f \times \frac{j0.42}{j0.42 + j0.22} = -j2.39 \text{ p.u} = 2.39 \angle - 90^0 \text{ p.u}$

$I_{G2} = 2.39 \angle - 90^0$ p.u

E_{Th} = 7.843 p.u

$$SCC = \frac{1}{X_{Th}} \text{ p.u.MVA} = \frac{1}{0.144} = 6.94 \text{ p.u. MVA}$$

SCC = 6.94 p.u. MVA

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12. For the radial network shown in the figure, three phase fault occurs at F. Determine the fault current and the line voltage at 11KV bus under fault conditions.
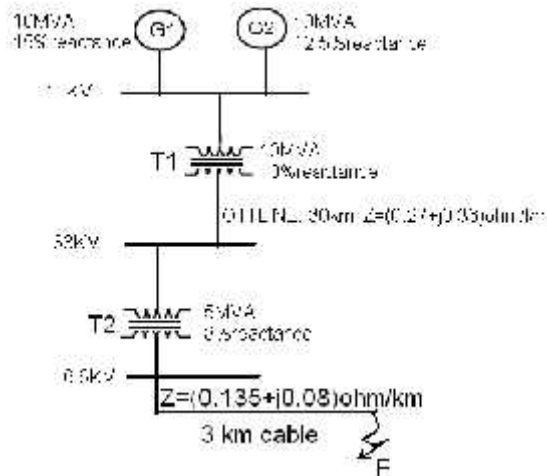
Base MVA = 10 MVA

Base KV = 11KV

Voltage at 11 KV bus : $Z_{AF} = j0.1 + 0.0744 + j0.0992 + j0.16 + 0.093 + j0.55$

$= 0.1674 + j0.4142$

$= 0.447 \angle 67.99^0$ p.u

Voltage at 11 KV bus $= Z_{AF \, p.u} \times I_{f \, p.u}$

$= 0.447 \angle 67.99^0 \times 1.959 \angle -70.85^0$

$= 0.874 - j0.0436 = 0.875$ p.u

$= 0.875 \times$ Base voltage

$= 0.875 \times 11 = 9.615$ KV

Voltage at 11 KV bus = 9.615 KV

13. A 60 MVA, Y connected 11 kv synchronous generator is connected to a 60 MVA, 11/132 KV /Y transformer. The subtransient reactance X_d'' of the generator is 0.12 p.u on a 60 MVA base, while the transformer reactance is 0.1 p.u. on the same base. The generator is unloaded when a symmetrical fault is suddenly place at point P as shown in fig. Find the subtransient fault current in p.u amperes and actual amperes on both sides of the transformer. Phase to neutral voltage of the generator at no load is 1.0 p.u.
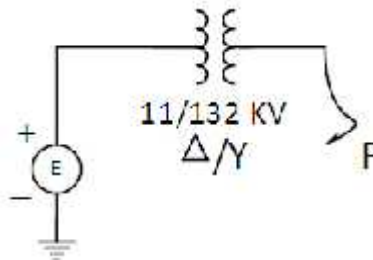
Secondary side of transformer:

Base KV$_{new}$ = KV$_{old}$ x $\frac{H.V \; side \; rating}{L.V \; side \; rating}$

$= 11 \times \frac{132}{11} = 132$ KV

Base KV$_{new}$ = 132 KV

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$$\text{Base current} = \frac{Base\ MVA}{\sqrt{3}\ x\ KV_b} = \frac{60}{\sqrt{3}\ x\ 132} = 0.262\ KA$$

Base current = 0.262 KA

$$\text{Actual current} = I_{F\ p.u}\ x\ \text{Base current}$$

$$= 4.54\ x\ 0.262 = 1.189\ KA$$

Actual current = 1.189 KA

14. A 3 phase transmission line operating at 33kv and having resistance and reactance of 5 ohms and 15 ohms respectively is connected to the generating station bus-bar through a 5000 KVA step up transformer which has a reactance of 0.05 p.u. Connected to the bus-bars are two alternators, are 10,000 KVA having 0.08 p.u. reactance and another 5000 KVA having 0.06 p.u. reactance. Calculate the KVA at a short circuit fault between phases occurring at the high voltage terminals of the transformers.
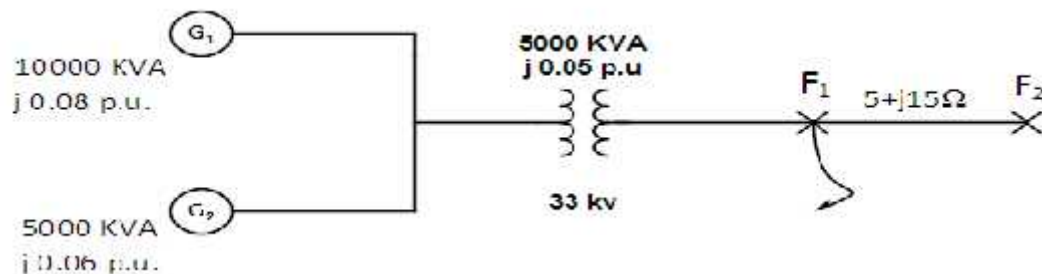
a. Total impedance upto the fault F_1, $Z_{1\ p.u} = -j0.148$ p.u

Short circuit KVA fed into the fault at F_1 = $|KVA_{1\ S.C}| = \frac{|KVA_b|}{|Z_{1\ p.u}|}$

$$= \frac{10,000}{0.148} = 67567.56\ KVA$$

$$= 67.568\ MVA$$

Short circuit KVA fed into the fault at F_1 = 67.568 MVA

b. Total impedance upto the fault F_2, $Z_{2\ p.u} = 0.0459 + j0.2857$ p.u

$$\boxed{|Z_{2\ p.u}| = 0.289\ p.u}$$

Short circuit KVA fed into the fault at F_2 = $|KVA_{2\ S.C}| = \frac{|KVA_b|}{|Z_{2\ p.u}|}$

$$= \frac{10,000}{0.289} = 34602\ KVA$$

$$= 34.602\ MVA$$

Short circuit KVA fed into the fault at F_2 = 34.602 MVA

15. A symmetrical fault occurs on bus 4 of system shown in fig. compute the fault current, post fault voltage, line flow.

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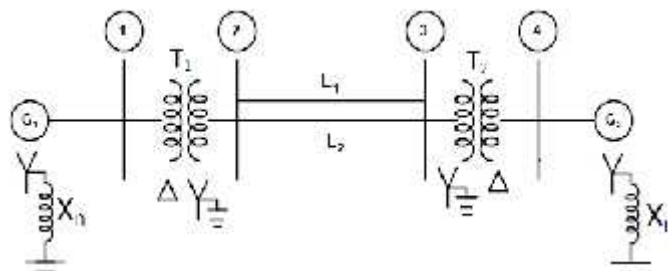
Generator: G_1, G_2: 100 MVA, 20 KV, X^+ =15%.
Transformer: T_1, T_2: X_{leak} = 9%.
Transmission line L_1, L_2: X^+ =10%.
Thevenin's equivalent circuit.
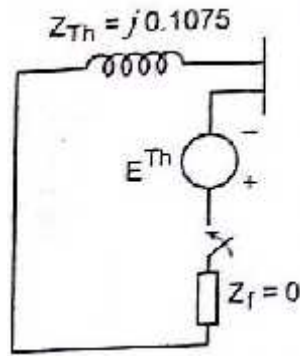
I_{12} = - j2.632 p.u

$$I_{23} = \frac{V_2 - V_3}{Z_{23}} = \frac{0.2683 - 0.2367}{j0.05} = \text{-j2.632 p.u}$$

I_{23} = - j2.632 p.u

$$I_{34} = \frac{V_3 - V_4}{Z_{34}} = \frac{0.2367 - 0}{j0.09} = \text{-j2.6 p.u}$$

I_{34} = - j2.6 p.u

16. **A 25 MVA, 11 KV, 3 generator has a subtransient reactance of 20 %. The generator supplies two motors over a transmission line with transformers at both ends as shown in fig. The motors have rated input of 15 and 7.5 MVA, both 10 KV and 20% subtransient reactance. The 3 transformers are both rated 50 MVA, 10.8/121 KV with delta-star connection with leakage reactance of 10% each. The series reactance of the line is 80 . Draw the impedance diagram of the system with reactances marked in p.u. when symmetrical fault occurs at bus 2 and calculate fault current.**
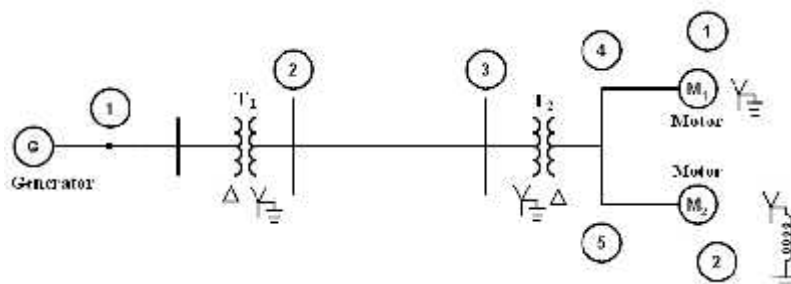
I_f = 5.28 $\angle -90^0$ p.u

Base current $I_{base} = \frac{MVA_b}{\sqrt{3} \, x \, KV_b} = \frac{25 \, x \, 10^6}{\sqrt{3} \, x \, 123.24 \, x \, 10^3}$

      [KV_b for secondary of transformer 1 or transmission line]

      = 117.119 Amp.

Base current I_{base} = 117.119 Amp.

Actual fault current = I_f x I_{base} = 5.28 x 117.119 = 618.388 Amp.

Actual fault current = 618.388 Amp.

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17. The generator at buses 1 and 3 of the network have impedances j1.5 p.u. If a 3 phase short circuit fault occurs at bus 3, when there is no load (all bus voltages are equal to 1.0 0° p.u), find initial symmetrical current in fault in the line 1-3, and post fault voltages when a fault at bus 2 using bus building algorithm.
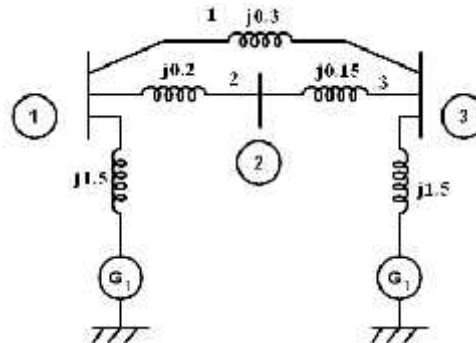
Line flows.

$$I_{13} = \frac{V_1 - V_3}{Z_{13\ series}} = \frac{0.1103 - 0.0969}{j0.3} = \text{-j } 0.0447 \text{ p.u}$$

$I_{13} = \text{- j } 0.0447 \text{ p.u}$

Fault is at bus 3, so, $Z_{Th} = Z_{33} = j0.7879$ p.u

$$E_{Th} = 1\angle 0^0 \ ; \ Z_f = 0$$

$$I_f = \frac{E_{Th}}{Z_{Th} + Z_f} = \frac{1\angle 0^0}{j0.7879 + 0} = \text{-j}1.2692 = 1.2692 \angle -90^0 \text{ p.u}$$

$I_f = 1.2692 \angle -90^0 \text{ p.u}$

$$V_1 = 1\angle 0^0 - I_1 Z_{1f} = 1\angle 0^0 - (-j1.2692) x\ j0.7111 = 0.09747\ p.u$$
$$V_2 = 1\angle 0^0 - I_2 Z_{2f} = 1\angle 0^0 - (-j1.2692) x\ j0.7551 = j0.4163\ p.u$$

$V_1 = 0.09747\ p.u$

$V_2 = j0.4163\ p.u$

$$V_3 = 1\angle 0^0 - I_3 Z_{3f} = 1\angle 0^0 - (-j1.2692) x\ j0.7879 = 0\ p.u.$$

$V_3 = 0\ p.u.$

Line flows.

$$I_{13} = \frac{V_1 - V_3}{Z_{13}} = \frac{0.09747 - 0}{j0.3} = \text{-j } 0.3249 \text{ p.u}$$

$I_{13} = \text{- j } 0.3249 \text{ p.u}$

18. Three 11 KV generators, A,B and C each of 20% leakage reactance and MVA ratings 50,75 and 100 respectively are interconnected electrically as shown in fig. by a tie bar through current limiting reactor, each of 25% reactance based upon the bus-bar of generator A at a line voltage of 11 KV. The feeder has a resistance of 0.08 /ph and an inductive reactance of 0.16 /ph. Estimate the maximum MVA that can be fed into a symmetrical short circuit at the end of the feeder.
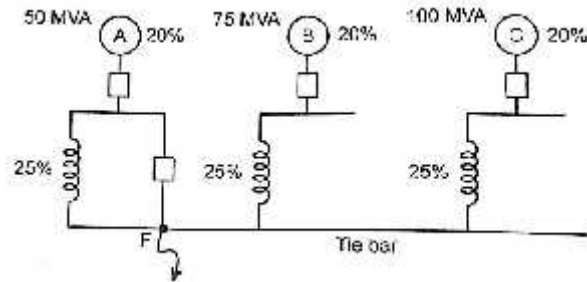
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Current limiting reactor at A :

$$Z_{\text{p.u. new}} = Z_{\text{pu,given}} \times \left[\frac{kV_{b,given}}{kV_{b,new}}\right]^2 \times \left[\frac{MVA_{b,new}}{MVA_{b,given}}\right]$$
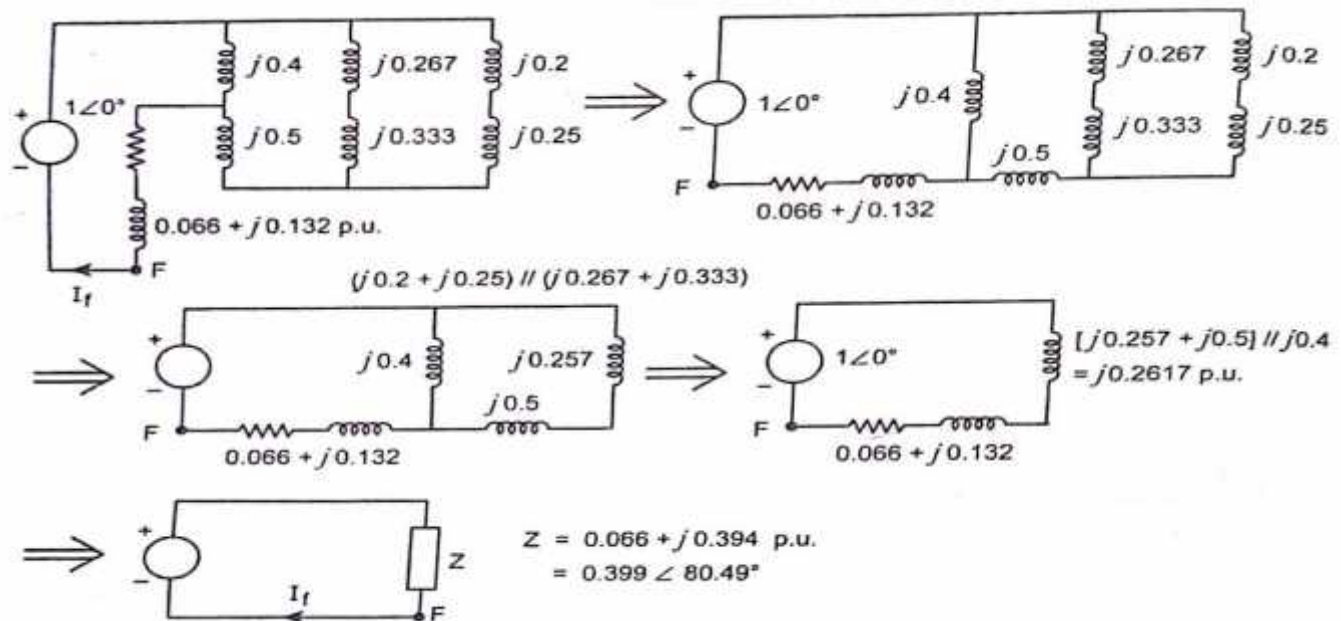
$$= j\,0.25 \times (11/11)^2 \times (100/50)$$

$Z_{\text{new}} = j\,0.5$ p.u

Current limiting reactor at B :

$$Z_{\text{p.u. new}} = Z_{\text{pu,given}} \times \left[\frac{kV_{b,given}}{kV_{b,new}}\right]^2 \times \left[\frac{MVA_{b,new}}{MVA_{b,given}}\right]$$

$$= j\,0.25 \times (11/11)^2 \times (100/75)$$

$Z_{\text{new}} = j\,0.33$ p.u

Current limiting reactor at A :

$$Z_{\text{p.u. new}} = Z_{\text{pu,given}} \times \left[\frac{kV_{b,given}}{kV_{b,new}}\right]^2 \times \left[\frac{MVA_{b,new}}{MVA_{b,given}}\right]$$

$$= j\,0.25 \times (11/11)^2 \times (100/100)$$

$Z_{\text{new}} = j\,0.25$ p.u

Short circuit MVA $= \dfrac{1}{z_{Th}}$ p.u MVA

$$= \frac{1}{0.399} = 2.506 \text{ p.u MVA}$$

$$= 2.506 \times 100 = 250.6 \text{ MVA}$$

Short circuit MVA = 250.6 MVA

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19. A 100 MVA, 11KV, 50 Hz, star connected three phase synchronous generator connected to a 11/220 KV, 100 MVA, Δ - Y connected transformer. The reactances are in per unit on the same base.

Reactance of generator: Xd = 0.9 p.u. X_d' = 0.2 p.u; X_d'' = 0.1 p.u

Reactance of transformer is 0.2 p.u

A three phase load of 100 MVA, 0.8 p.f. lagging is connected to the transformer secondary side as shown in figure. The line to line voltage at the load terminals is 220 KV. A 3 phase short circuit occurs at the load terminals. Find the generator transient current including the load current.
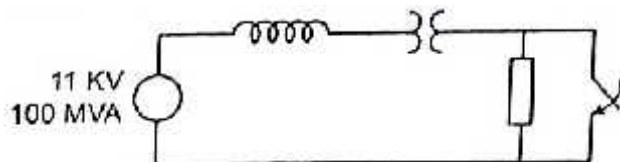
Generator transient current = $I_f + I_L$

$$= -j2.5 + 0.8 - j0.6$$
$$= 0.8 - j3.1 \text{ p.u}$$
$$= 3.2 \angle -75.53^U \text{ p.u}$$

Generator transient current = 3.2 $\angle -75.53^U$ p.u

20. A synchronous generator and synchronous motor each rated 30 MVA, 11 KV having 20% subtransient reactance are connected through transformers and line as shown in fig. the transformers are rated 30 MVA, 11/66 KV and 66/11 KV with leakage reactance of 10% each. The line has a reactance of 10% on a base of 30 MVA, 66 KV. The motor is drawing 20MW at 0.8 p.f leading and a terminal voltage of 10.6 KV when a symmetrical three phase fault occurs at the motor terminals. Find subtransient current in generator and motor.
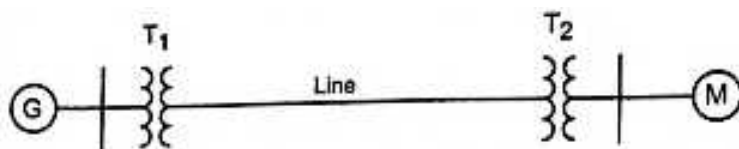
Generator: Voltage behind sub transient reactance

$$E_g'' = V^o + j I^o X_{dg}'' = 0.9636\angle 0^\circ + j0.865\angle 36.87^o \times 0.5$$
$$= 0.704 + j0.346 \text{ p.u}$$

$E_g'' = 0.704 + j0.346$ p.u

Under faulted condition $I_g'' = \dfrac{E_g''}{Z \, upto \, fault \, point \, from \, Generator}$

$$= \dfrac{0.704 + j0.346}{j0.5} = 0.642 - j1.408 \ p.u$$

$I_g'' = 0.642 - j1.408 \ p.u$

Motor: Voltage behind sub transient reactance:

$$E_m'' = V^o - jI^o \times X_{dm}'' = 0.9636\angle 0 - j0.865 \angle 36.87^0 \times 0.2$$

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$$=1.0674 - j0.1384 \ p.u$$

Under faulted conditions, $\quad I_m^{"} = \dfrac{E_m^{"}}{Z \, upto \ fault \ point \ from \ motor}$

$$=\dfrac{1.0674 - j0.1384}{j0.2} = -0.692 - j5.337 \ p.u$$

$$\boxed{I_m^{"} = \ -0.692 - j5.337 \ p.u}$$

Fault current $I_f = I_g^{"} + I_m^{"} = -j6.745 \ p.u$

Base current (Gen/motor) $= \dfrac{MVA_b}{\sqrt{3} \times KV_b} = \dfrac{30}{\sqrt{3} \times 11} = 1.5746 \ KA$

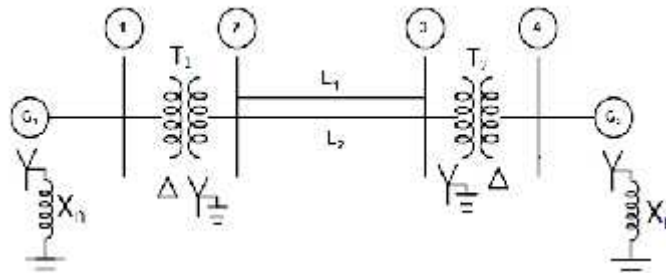$$I_g^{"} = (0.692 - j1.408) \times 1.5746 = 1.086 - j2.217 \ KA$$

$$I_m^{"} = -0.692 - j5.337 \times 0.5746 = -1.089 - j8.404 \ KA$$

$$I_f = -j6.745 \times 1.5337 = -j10.62 \ KA$$

$$\boxed{I_f = -j10.62 \ KA}$$

PART-C

21. A symmetrical fault occurs on bus 4 of system shown in fig. When Zf = j0.14 p.u., determine fault current and current supplied by the generators.

Fault current I_f = 4.04 ∠-90° p.u

Actual fault current = p.u value x base current.

$$= 4.04 \ x \ \frac{MVA}{\sqrt{3} \ KV} = \frac{4.04 \ x \ 100}{\sqrt{3} \ x \ 20} = 11.66 \ KA$$

Actual fault current = 11.66 KA

Current contribution from the Generators.

$$I_{G1} = I_f \ x \ \frac{j0.15}{j0.15 + j0.38} = 1.1434 \ \angle - 90^0 \ p.u$$

$$I_{G2} = I_f \ x \ \frac{j0.38}{j0.15 + j0.38} = 2.8966 \ \angle - 90^0 \ p.u$$

$I_{G1} = 1.1434 \ \angle - 90^0$ p.u

$I_{G2} = 2.8966 \ \angle - 90^0$ p.u

22. Draw the flowchart for symmetrical fault analysis using Z_{bus}

22
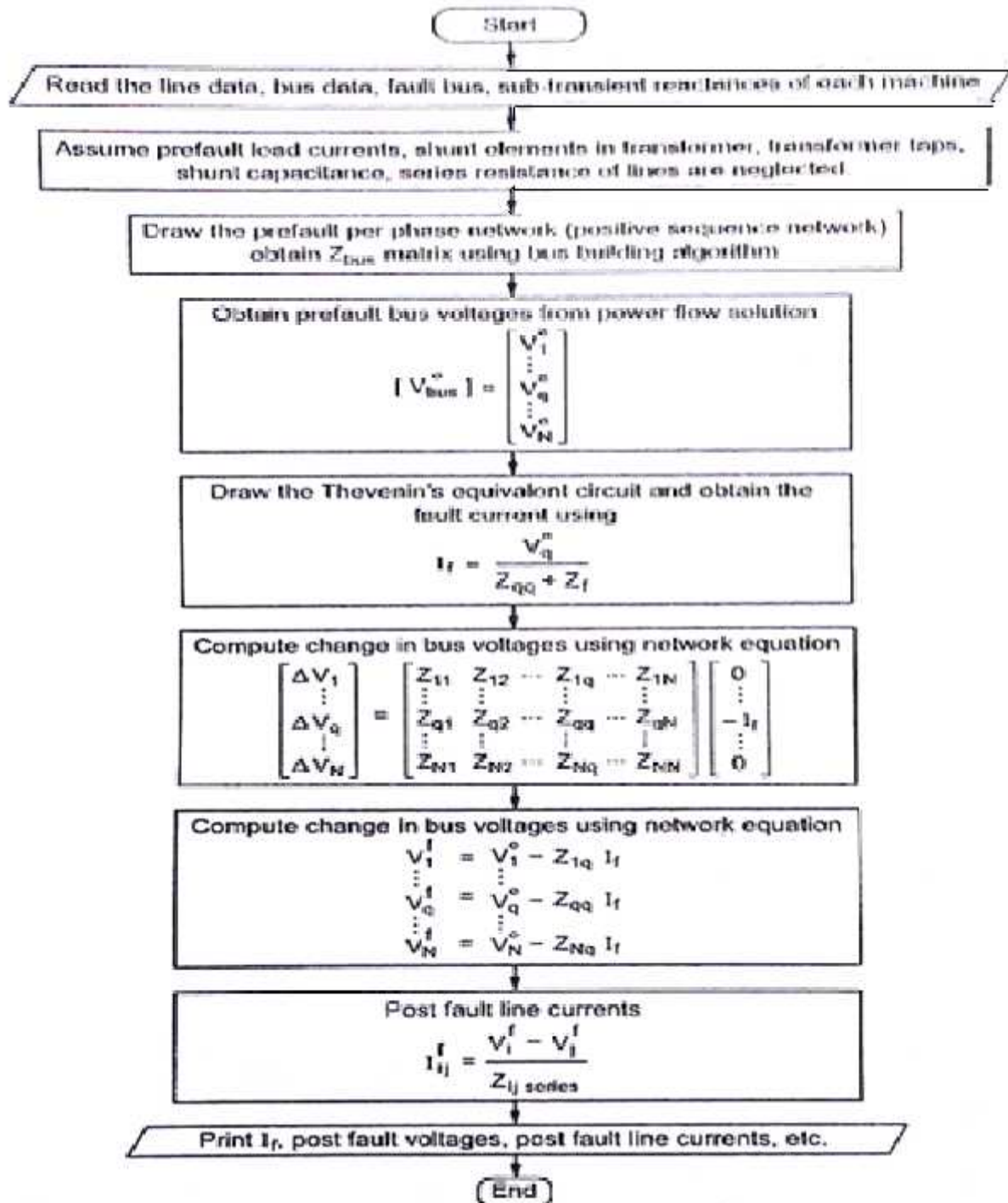
23. A generator is connected through a transformer to a synchronous motor. The subtransient reactances of generator and motor are 0.15 and 0.35 respectively. The leakage reactance of the transformer is 0.1 p.u. All the reactances are calculated on a common base. A three phase fault occurs at the terminals of the motor when the terminal voltage of the generator is 0.9 p.u. The output current of generator is 1 p.u. and 0.8 pf leading. Find the subtransient current in p.u. in the fault, generator and motor, Use the terminal voltage of the generator as reference vector.

Using Thevenin's theorem,

To find fault current:

23
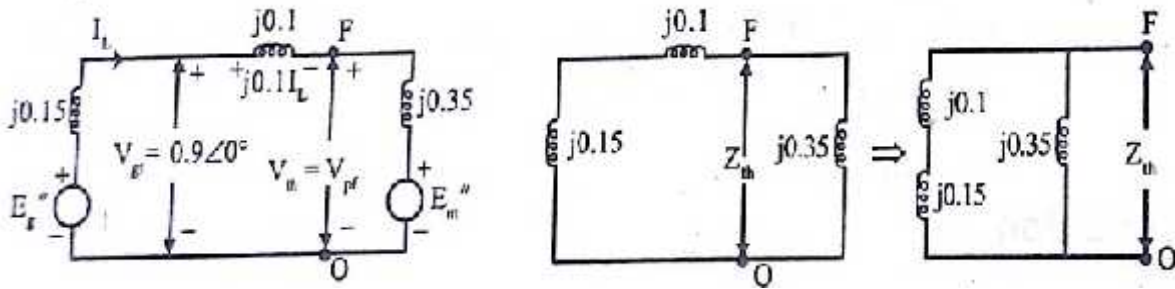
In fig. using KVL we get,

$$V_{tg} = j0.1I_L + V_{th}$$

Thevenin's voltage, $V_{th} = V_{tg} – j0.1I_L$

$$= 0.9\angle0° - (0.1\angle90° \times 1\angle36.9°)$$
$$= 0.9\angle0° - (0.1\angle126°) = 0.9 – (-0.06 + j0.08)$$
$$= 0.96 – j0.08 = 0.9633 \angle-4.8° \text{ p.u.}$$

Thevenin's equivalent impedance, $Z_{th} = \dfrac{(j0.15+j0.1)j0.35}{(j0.15+j0.1)+j0.35} = j0.1458$ p.u.

$$\boxed{Z_{th} = j0.1458 \text{ p.u.}}$$

The Thevenin's equivalent of the circuit with respect to fault point is shown. Now short circuiting the terminals of the Thevenin's equivalent circuit as shown is equivalent to the fault condition. The current flowing through the short is the fault current.

Prefault thevenin's equivalent at fault point   *Thevenin's equivalent under fault condition*
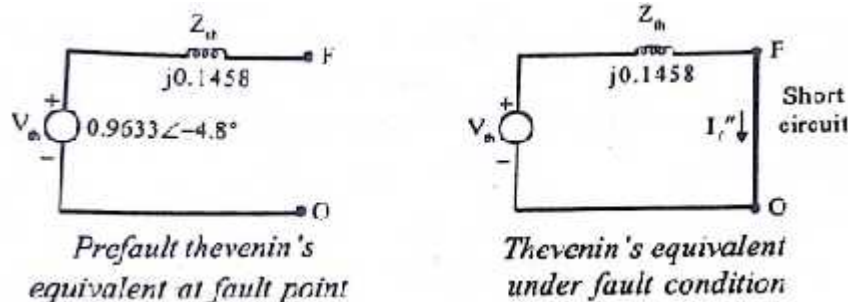
Current in the fault, $I_f" = \dfrac{V_{th}}{Z_{th}} = \dfrac{0.9633\angle-4.8°}{0.1458\angle90°} = 6.606\angle-94.8°$ p.u.

$$\boxed{I_f" = 6.606\angle-94.8° \text{ p.u.}}$$

24. Two synchronous motors are connected to the bus of a large system through a transmission line as shown, The ratings of the various components are,

> **Motor each: 1 MVA, 440 V, 0.1 p.u. transient reactance**
> **Line: 0.0 ohm reactance**
> **Large system: Short circuit MVA at its bus at 440 V is 8.**

When the motors are operated at 400V, calculate the short circuit current fed into a three phase fault at motor bus.

$$\boxed{I_f = 21.6969\angle-90° \text{ p.u.}}$$

24
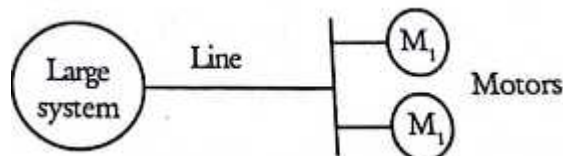
Base current, $I_b = \frac{kVA_b}{\sqrt{3}kV_i} = \frac{1 \times 1000}{\sqrt{3} \times 0.44} = 1312.16$ A $= 1.3122$ kA.

$\therefore$ Actual value of fault current, $I_f = $ p.u. value of $I_f \times I_b = 21.6969\angle-90° \times 1.3122$
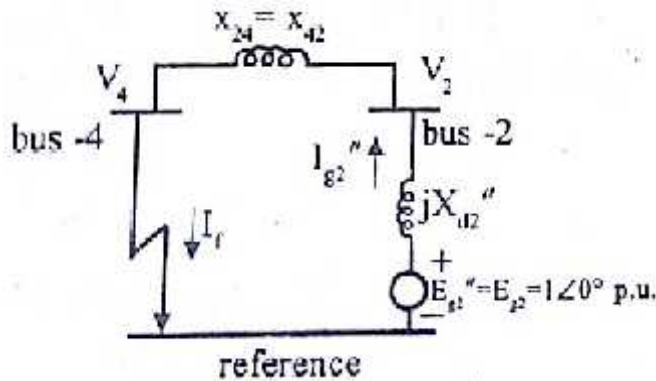
$$I_f = 28.4707\angle-90° \text{ kA.}$$

Result:

Fault current $= 21.6969\angle-90°$ p.u. or $28.4707\angle-90°$ kA.

25. The bus impedance matrix of four bus system with values in p.u. is given by,

$$Z_{bus} = j \begin{bmatrix} 0.15 & 0.08 & 0.04 & 0.07 \\ 0.08 & 0.15 & 0.06 & 0.09 \\ 0.04 & 0.06 & 0.13 & 0.05 \\ 0.07 & 0.09 & 0.05 & 0.12 \end{bmatrix}$$

In this system generators are connected to buses 1 and 2 and their subtransient reactances were included when finding Zbus. If prefault current is neglected, find subtransient current in p.u. in the fault on a bus 4. Assume prefault voltage as 1 p.u. If the subtransient reactance of generator in bus 2 is 0.2 p.u. find the subtransient fault current supplied by generator.

With reference to fig.

The subtransient fault current delivered by the generator at bus -2, $I_{g2}'' = \frac{E_{g2}'' - V_2}{jX_{d2}''}$

$$I_{g2}'' = \frac{1\angle0° - 0.25\angle0°}{j0.2} = \frac{1 - 0.25}{0.2\angle90°} = 3.75\angle-90° \text{ p.u.}$$

$$\boxed{I_{g2}'' = 3.75\angle-90° \text{ p.u.}}$$

Note: $I_f = I_{g1}'' + I_{g2}''$

Result:

The subtransient fault current in the bus -4 $= I_f'' = 8.333\angle-90°$ p.u.

The voltage at bus – 2 when there is a 3 phase fault in bus – 4 $= V_2 = 0.75\angle0°$ p.u.

The subtransient fault current delivered by the generator – 2 when there is a 3 – phase fault in bus – 4 $\Big\}$ $I_{g2}'' = 3.75\angle-90°$ p.u.

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IV - FAULT ANALYSIS – UNBALANCED FAULTS

Introduction to symmetrical components – sequence impedances – sequence circuits of synchronous machine, transformer and transmission lines - sequence networks analysis of single line to ground, line to line and double line to ground faults using Thevenin's theorem and Z-bus matrix.

PART – A

Introduction to symmetrical components

1. **What is meant by unsymmetrical fault?**

When the system is unsymmetrically faulted or loaded, neither the phase currents nor the phase voltages will possess three phase symmetry i.e the system remains unbalanced with unequal displacement. If the insulation of the system fails at a point or if a conducting object comes in contact with a bare conductor, an unsymmetrical fault is said to occur. If unsymmetrical fault occurs, the balanced currents will flow in the system. Symmetrical components are used for analyzing the unsymmetrical faults.

2. **List out the different types of unsymmetrical faults.**

The types of unsymmetrical faults are listed as follows.
- Line to Line fault (L-L)
- Line to Ground fault (L-G)
- Double line to Ground fault (L-L-G)
- Open conductor fault.

3. What is the purpose of analyzing unsymmetrical fault?

Analysis of unsymmetrical fault is very important for the following reasons. They are given as follows
- Relay setting,
- Single phase switching
- System stability studies

4. For a fault at a given location, rank the various faults in the order of severity.

In a power system, the most severe fault is three phase fault and less severe fault is open conductor fault. The various faults in the order of decreasing severity are,

- Three phase fault
- Double line to ground fault
- Line to line fault
- Single line to ground fault
- Open conductor fault

5. Give the steps followed in short circuit analysis of unbalanced low order systems?

The different steps to be followed in analyzing short circuit of an unbalanced low order system are as follows:

- Draw the positive, negative and zero sequence networks with their appropriate description.
- Choice of type of fault (L-G, L-L, or L-L-G) and location of fault and mathematical description for the particular type of fault.
- Using Thevenin's theorem or bus impedance matrix, determine the solution of the network equation. Fault current, post currents, post fault voltages are found at the point of fault, all the bus voltages, and the line flows.

6. What are the different symbols used in unsymmetrical fault calculation?

The following symbols are used in unsymmetrical fault calculation.

- Superscript f represents post fault or fault values.
- Super script +, - and 0 represents positive, negative and zero sequence voltages, current and impedance.
- A number subscript following this positive (+), negative (-) and zero (0) represents the bus code.

- Phase values of voltages and currents are indicated collectively by subscript p and individually by the subscript a, b and c.

Sequence Impedances – Sequence Networks

7. What is sequence network? (M/J'11). What are sequence impedances?

An unbalanced system of n related phasors can be resolved into n systems of balanced phasors called symmetrical components. Symmetrical components are positive, negative and zero sequence components. Hence these sequence component creates the network called as **Sequence network.**

Sequence impedances are the impedances offered by the power system components or elements to +ve, -ve and zero sequence current

8. What are the features of zero sequence current? (M/J'13)

It consists of three phasors equal in magnitude and with zero phase displacement from each other.

Zero sequence phasors a, b, c can be written as

$$I_a^0 = I_b^0 = I_c^0$$

Where I_a^0, I_b^0, I_c^0 are the sequence components of $I^a, I^b \text{ and } I^c$

9. Define negative sequence impedance. (M/J'13, N/D'11)

The impedance offered to the flow of negative sequence currents is known as the negative sequence impedance and it is denoted by Z^-. The negative sequence impedance is occurred in all the fault condition and it is important to find the fault current. The positive sequence impedance and negative sequence impedance are same for transformers and power lines. But it in case of rotating machines the positive and negative sequence impedances are different.

10. Write the symmetrical component currents of phase 'a' in terms of three phase currents. (M/J'14)
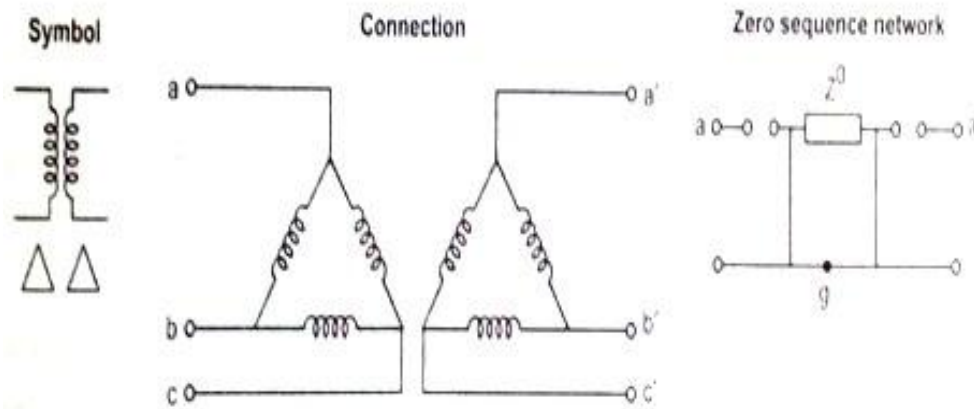
The symmentrical componets of currents are,

$$\begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$
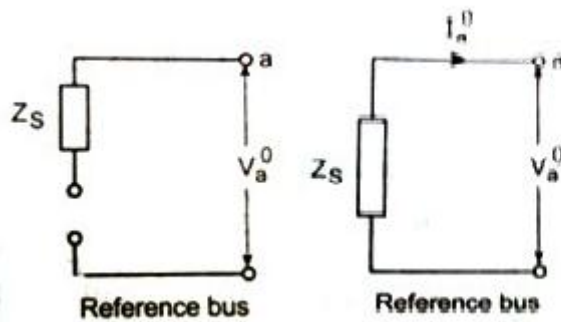
$$I_a^0 = \frac{1}{3}\left[I_a + I_b + I_c\right]$$

$$I_a^+ = \frac{1}{3}\left[I_a + aI_b + a^2 I_c\right]$$

$$I_a^- = \frac{1}{3}\left[I_a + a^2 I_b + aI_c\right]$$

11. Draw the sequence network for Δ – Δ connected transformer. (N/D'12)

12. Draw zero sequence impedance of generator. (M/D'12)

13. Write down the equation for symmetrical component of current vector of a three phase system.

$$I_a = I_a^0 + I_a^+ + I_a^-$$
$$I_b = I_b^0 + I_b^+ + I_b^-$$
$$I_c = I_c^0 + I_c^+ + I_c^-$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix}$$

where $a = 1 \angle 120°$ *and* $a^2 = 1 \angle 240°$

14. Draw the sequence network for Y – Δ connected transformer. (M/J'10)
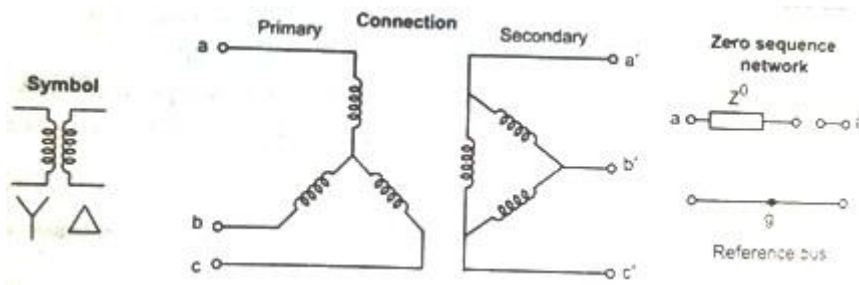
15. Write the symmetrical components of a 3 Phase system. (M/J'11)

In a 3 phase system, three unbalanced vectors can be resolved into three balance system of vectors.

- Positive sequence components
- Negative sequence components

- Zero sequence components.

16. What are positive sequence components?

The positive sequence components of a three phase unbalanced vectors consists of three components of equal magnitude, displaced each other by 120° in phase, and having the phase sequence same as the original vectors.

Three phases are written as

$I_a^+ = I_a^+ \angle 0°$

$I_b^+ = I_a^+ \angle 240° = a^2 I_a^+$

$I_c^+ = I_a^+ \angle 120° = a I_a^+$

$where \ I_a^+, I_b^+, I_c^+ \ are \ the \ Positive \ sequence \ component \ of \ I_a, I_b \ and \ I_c$

17. What are negative sequence components and zero sequence components?

The negative sequence components of a three phase unbalanced vectors consists of three components of equal magnitude, displaced each other by 120° in phase, and having the phase sequence opposite to that of the original vectors.

The zero sequence components of a three phase unbalanced vectors consists of three vectors of equal magnitude and with zero phase displacement from each other.

18. Write down the equations to convert symmetrical quantities into phase quantities.

$Let \ I_a, I_b, I_c \ be \ the \ unbalanced \ phase \ currents.$

$Let \ I_a^0, I_a^+, I_a^- \ be \ the \ symmetrical \ components \ of \ phase.$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix}$$

$where \ a = 1 \angle 120° \ and \ a^2 = 1 \angle 240°$

19. What is positive, negative and zero sequence impedance?

- Impedance offered to the flow of positive sequence current is known as positive sequence impedance and it is denoted by Z^+
- Impedance offered to the flow of negative sequence current is known as negative sequence impedance and it is denoted by Z^-
- Impedance offered to the flow of zero sequence current is known as Zero sequence impedance and it is denoted by Z^0

20. What are the causes of unsymmetrical faults?

The various causes for unsymmetrical faults are listed as follows.

- Lightning
- Wind damage
- Tree falling across lines
- Vehicles colliding with towers or poles, birds.
- Shorting lines
- Breaking due to excessive ice loading or snow loading, salt spray.

21. What is meant by fault calculations?

The fault condition of a power system can be divided into transient, sub-transient and steady state periods. The currents in the various parts of the system and in the fault locations are different in these periods. The estimation of these currents for various types of faults at various locations in the system can be commonly referred to as fault calculations.

22. Name the fault involving ground.

The fault which involve ground are given as follows.

- Line to ground fault
- Double line to ground fault
- 3phase to ground fault

Representation of single line to ground fault

23. What are the observation made from the analysis of various fault ? (N/D'13)

- Finding out the value of Fault current under different fault conditions
- Zero sequence is not presence in LL fault conditions
- Positive and negative sequence component of currents are opposite and equal to each other in LL fault condition
- Positive and negative sequence component of voltage are equal in LLG fault condition

24. Write the boundary condition for the single line to ground fault. (N/D'13)

The boundary condition for the single line to ground fault are

- If the generator is solidly grounded, $Z_n = 0$ and for bolted fault or solid fault, $Z_f = 0$.
- If the neutral of the generator is ungrounded, the zero sequence network is open circuited.

$$\therefore I_a^+ = I_a^- = I_a^0 \text{ and } I_f = 0$$
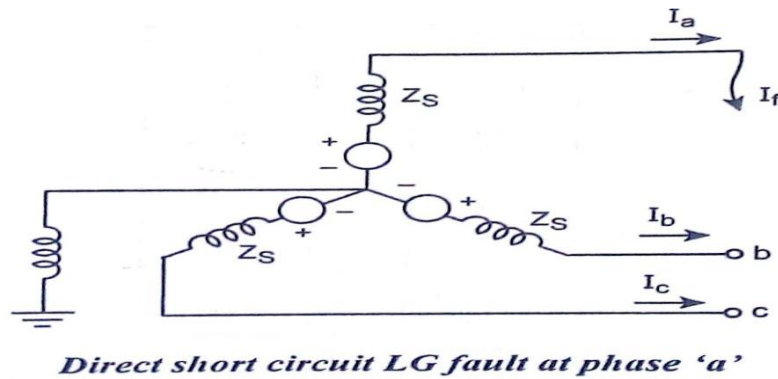
25. Draw the diagram for L-G fault at phases.
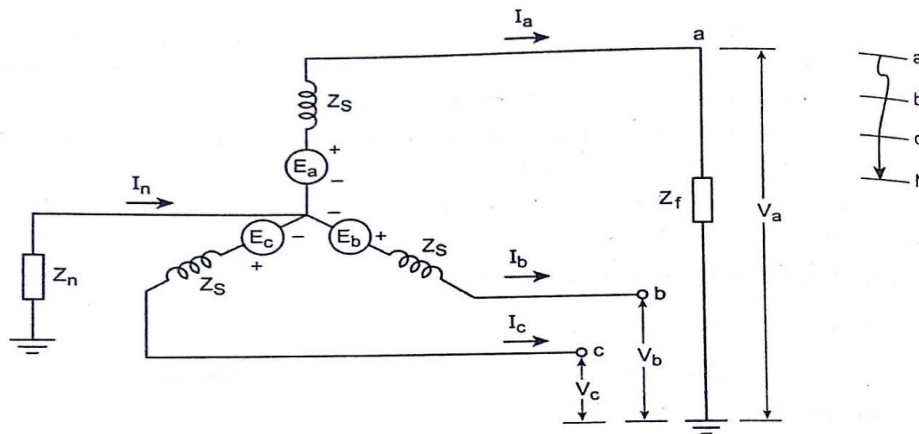
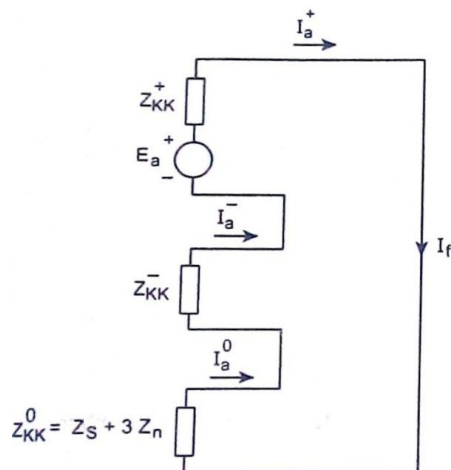*Single line to ground fault at phase 'a'*

26. Draw the equivalent sequence network diagram for LG fault.
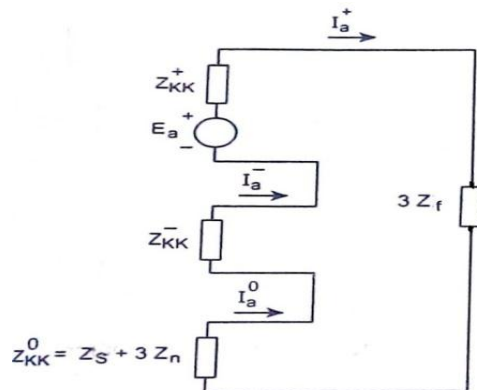
27. Draw the figure showing direct short circuit or bolted line to ground fault.

Direct short circuit LG fault at phase 'a'

28. Draw the equivalent sequence network diagram for direct short circuit or bolted L-G fault.

29. Name the fault in which all the sequence components are to be presents and give the reason for occurrence.

Single line to ground fault is the fault in which all the sequence components are present.

The reason for occurrence of fault are given as follows

*   Lightning
*   Conductors making contact with grounded structures like towers or poles, etc.

30. Write down the expression for fault current in LG fault.

The expression for fault current in LG fault is given as follows

$$I_f = \frac{3E_a}{Z_{kk}^0 + Z_{kk}^+ + Z_{kk}^- + 3Z_f}$$

$Where\ Z_{kk}^0 = Zero\ sequence\ impedance$
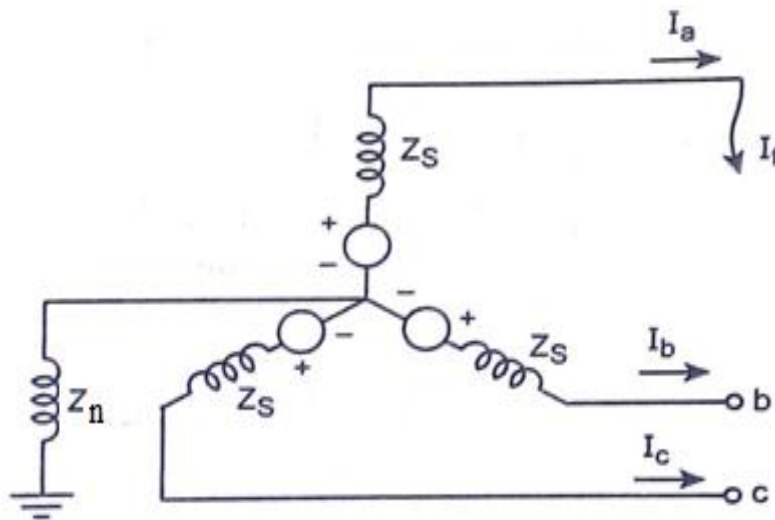
$Z_{kk}^+ = Positive\ sequence\ impedance$

$Z_{kk}^- = Negative\ sequence\ impedance$

$E_a = prefault\ voltage$

$Z_f = Fault\ impedance$

31. In which type of fault at phase 'a', there is no current flows through 'b' and 'c' phases.

Line to ground fault is the only fault in which there is no flow of current between the phases

From the above figure b and c phases are open. Therefore no current flows through b and c phases.

32. Write the general equation to determine post fault voltages.

Post fault positive sequence bus voltages :

$$V_f^+ = V_0^+ + Z_{ik}^+ I_f = V_0^+ - Z_{ik}^+ I_f^+$$

Post fault negative sequence bus voltages :

$$V_f^- = -Z_{ik}^- I_f^-$$

Post fault positive sequence bus voltages :

$$V_f^0 = -Z_{ik}^0 I_f^0$$

33. Write the general equation to determine sequence line currents.

$$Positive\ sequence\ current\ I_{ij}^+ = \frac{V_{fi}^+ - V_{fj}^+}{Z_{ij}^+}$$

$$Negative\ sequence\ current\ I_{ij}^- = \frac{V_{fi}^- - V_{fj}^-}{Z_{ij}^-}$$

$$Zero\ sequence\ current\ I_{ij}^0 = \frac{V_{fi}^0 - V_{fj}^0}{Z_{ij}^0}$$

__Representation of line to line fault__

34. Name the faults which do not have zero sequence current flowing (N/D'11) and zero sequence components.

Double line fault (LL fault) because voltage through zero sequence network is zero and there are no zero sequence sources and $I_a^0 = 0$, current is not being injected into that network due to the fault. Hence LL fault does not involve zero sequence network.

The faults which does not have zero sequence components are given as follows. They are:

- Three phase fault

- Line – Line fault

35. Draw the equivalent sequence network for L-L bolted fault in power system. (M/J'10)
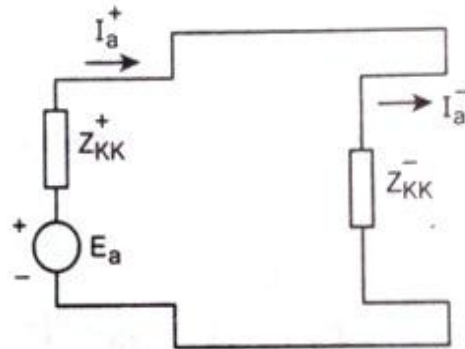
36. Which type of fault has + and – sequence current are same and opposite in direction?
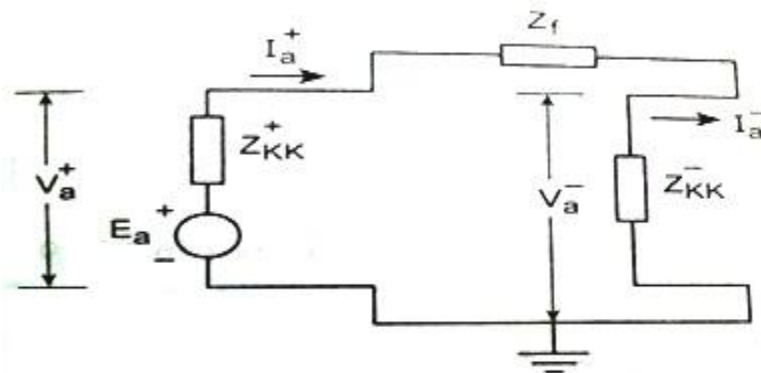
Line to Line fault.

$$I_a^+ = \frac{1}{3}\left[aI_b - a^2 I_b\right]$$

$$I_a^- = \frac{1}{3}\left[a^2 I_b - aI_b\right]$$

$$\therefore I_a^+ = -I_a^-$$

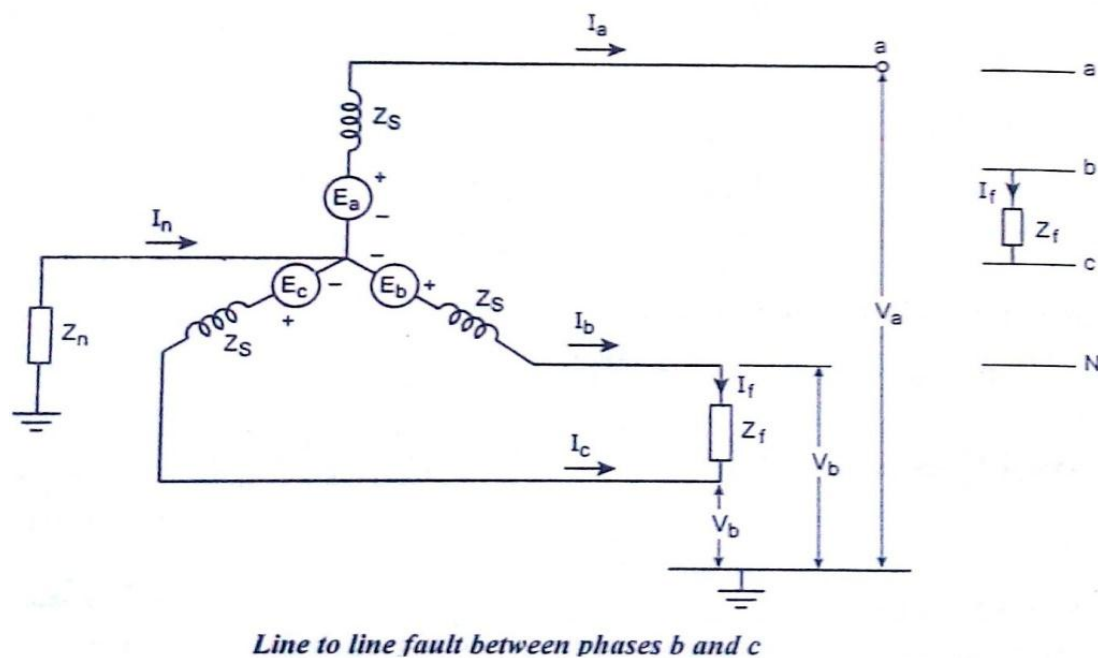## 37. Draw the sequence network connection to LL fault. (M/J'13)

38. Write down the expression for fault current in L-L fault.

$$I_f = I_b = \frac{-j\sqrt{3}E_a}{Z_{kk}^+ + Z_{kk}^- + Z_f}$$

Where

$Z_{kk}^+ = Positive\ sequence\ impedance$

$Z_{kk}^- = Negative\ sequence\ impedance$

$E_a = prefault\ voltage$

$Z_f = Fault\ impedance$

39. Draw the figure showing L-L fault between two phases.

Line to line fault between phases b and c

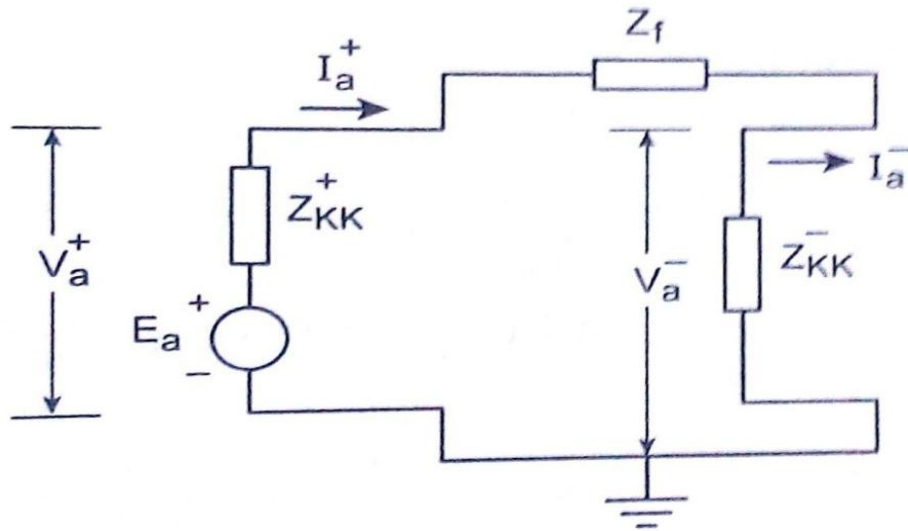**40. Name the fault in which + and – sequence voltage are equal.**

Double line to ground fault

$$V_a^+ = \frac{1}{3}[V_a - V_b]$$

$$V_a^- = \frac{1}{3}[V_a - V_b]$$

$$\therefore V_a^+ = V_a^-$$

41. Draw the equivalent sequence network for L-L fault.

Representation of line to line to ground fault

42. Write the equation to determine fault current for L-L-G fault with fault impedance between phase 'b' and 'c'.

$$Fault\ current\ I_f = 3I_a^0 = -3\left[\frac{E_a - Z_{KK}^+ I_a^+}{Z_{KK}^0 + 3Z_f}\right]$$

$Where\ I_a^+ = Positive\ sequence\ current$

$\qquad E_a = prefault\ voltage$

$\qquad Z_{KK}^0 = Zero\ sequence\ impedance$

$\qquad Z_{KK}^+ = Positive\ sequence\ impedance$
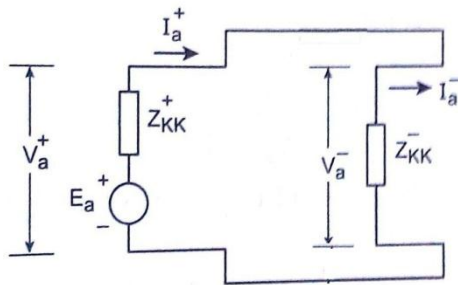
$\qquad Z_f = Fault\ impedance$

43. What type of fault occurs when fault impedance is infinite for LLG fault?
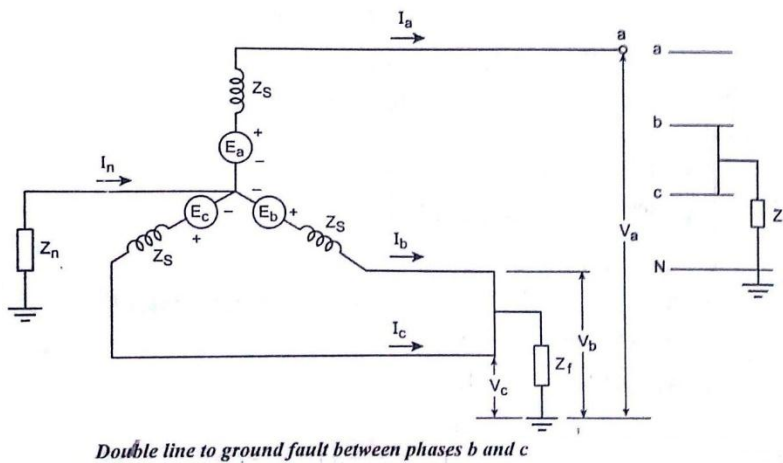
Line to line fault occurs, because

$Z_f = \infty$

$I_a^0 = 0$

Hence the sequence network becomes

44. Draw the diagram for L-L-G fault between phases.

Double line to ground fault between phases b and c

45. Give the expression for fault current in L-L-G fault.

$$I_f = \frac{-3}{Z_{KK}^0 + 3Z_f} \left[\frac{E_a \times Z_{KK}^- (Z_{KK}^0 + 3Z_f)}{Z_{KK}^- \times Z_{KK}^0 + 3Z_f Z_{KK}^+ + Z_{KK}^+ Z_{KK}^- + Z_{KK}^- Z_{KK}^0 + 3Z_f Z_{KK}^-} \right]$$

Where
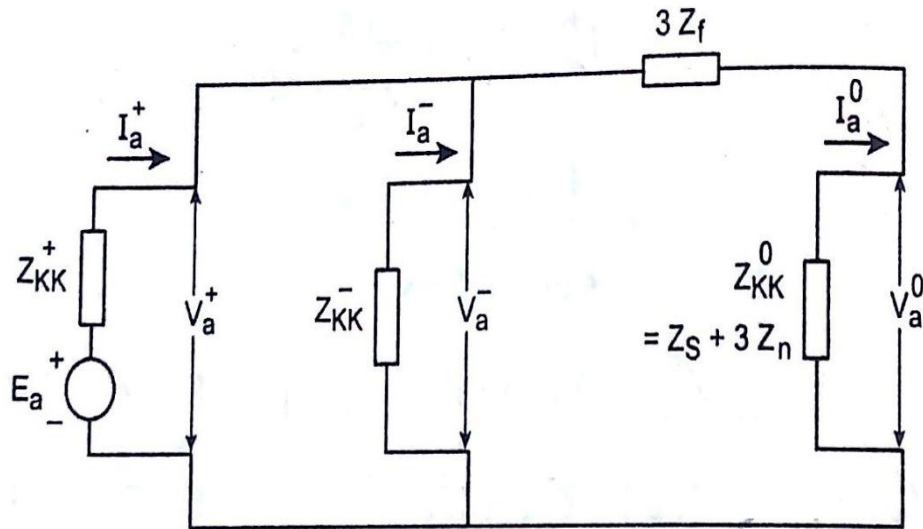
$Z_{kk}^+ = Positive\ sequence\ impedance$

$Z_{kk}^- = Negative\ sequence\ impedance$

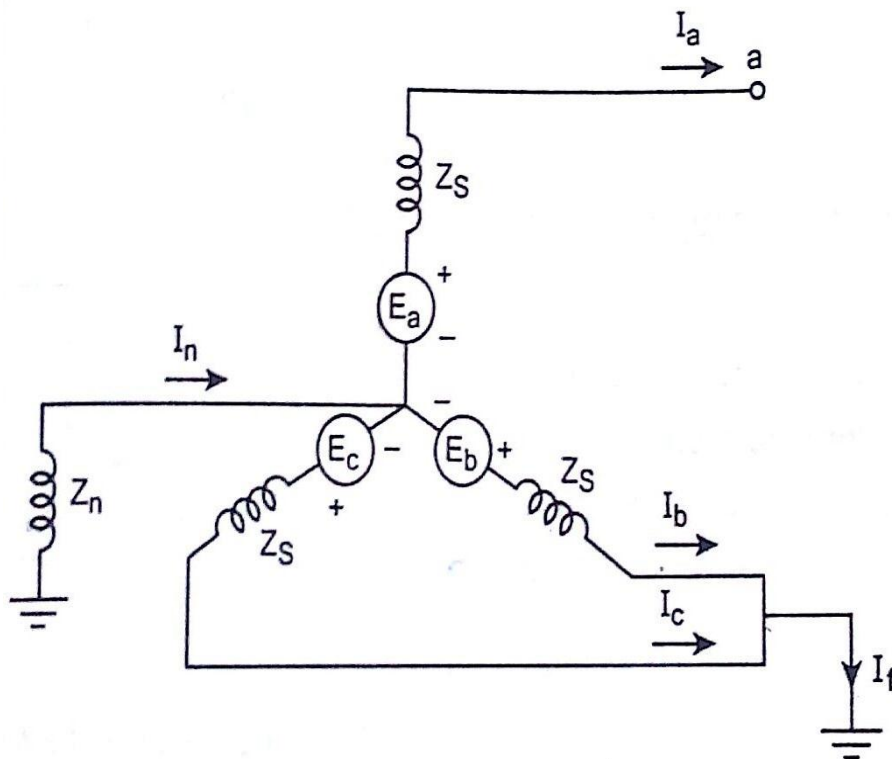$Z_{kk}^0 = Zero\ sequence\ impedance$

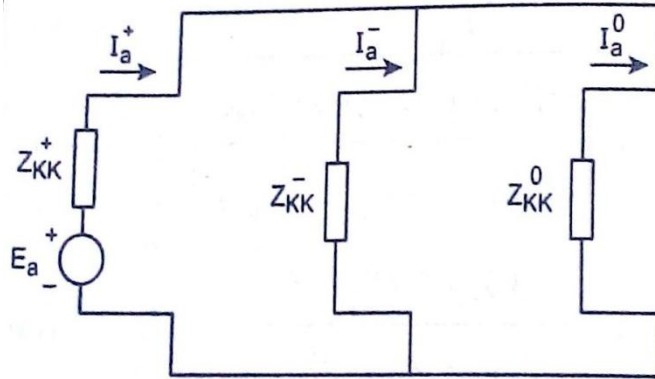$E_a = prefault\ voltage$

$Z_f = Fault\ impedance$

46. Draw the equivalent sequence network diagram for L-L-G fault.

47. Draw the figure showing direct short circuit or bolted L-L-G fault.

48. Draw the equivalent sequence network diagram for bolted L-L-G fault.

49. Find the fault if prefault voltage at the fault point is 0.97 p.u.

Solution:

j0.2 and j0.15 are in series

Its becomes, j0.2 + j0.15 = j0.35

j0.35 is in parallel with j0.15.

$$Z_{Th} = \frac{j0.35 \times j0.15}{j0.5 + j0.15} = j0.105 \ p.u.$$

$$Fault \ current \ I_f = \frac{V_{Th}}{Z_{Th}} = \frac{0.97}{j0.105} = -j9.238 \ p.u.$$

50. What is the need for short circuit study?

Whenever a fault occurs in an electrical power system, relatively high currents flow, producing large amounts of destructive energy in the forms of heat and magnetic forces. A short circuit study ensures that protective device ratings within a power system are adequate for maximum currents that flow during a fault.

A short circuit study is performed to:

1. Make certain protective devices have adequate interrupting current capability;
2. Ensure power system components can withstand mechanical and thermal stresses that occur during a fault; and
3. Calculate current data for protective device coordination studies.

51. Express short circuit KVA in terms of base of KVA and per unit reactance.

$$Short\ circuit\ capacity = \frac{1}{X_{Th}}\ p.u.MVA$$

$$= \frac{1}{X_{Th}} \times MVA_b \quad MVA$$

$$= \frac{1}{X_{Th}} \times KVA_b \quad KVA$$

where $X_{Th} = Thevenin\ equivalent\ reac\tan ce.$

$KVA_b = Base\ KVA.$

52. Define transient reactance.

It is the ratio of induced emf on no-load and the transient symmetrical rms current.

It is given by,

$$Transient\ reac\tan ce, X_d^{'} = \frac{|E_g|}{|I'|} = X_l + \frac{1}{\dfrac{1}{X_a} + \dfrac{1}{X_f}}$$

where $X_l = Leakage\ reac\tan ce$

$X_f = Field\ winding\ reac\tan ce$

$X_a = Armature\ reaction\ reac\tan ce$

53. What is meant by sub transient reactance?

The sub transient reactance is the ratio of included emf on no load and the sub transient symmetrical rms current. It is given by

$$X_d^{''} = \frac{|E_g|}{|I''|} = X_l + \frac{1}{\dfrac{1}{X_a} + \dfrac{1}{X_f} + \dfrac{1}{X_{dw}}}$$

54. What is the significance of sub transient reactance and transient reactance in short circuit studies?

The sub transient reactance can be used to estimate the initial value of fault current immediately on the occurrence of fault. The maximum momentary short circuit of the current rating of the circuit breaker used for protection of fault clearing should be less than its initial fault current.

The transient reactance is used to estimate the transient state fault current. Most of the circuit breakers open their contacts only during this period. Therefore for a circuit breaker used for fault clearing, its interrupting short circuit current rating should be less than the transient fault current.

55. Draw the positive sequence network of an unloaded synchronous generator with its neutral grounded through reactor.
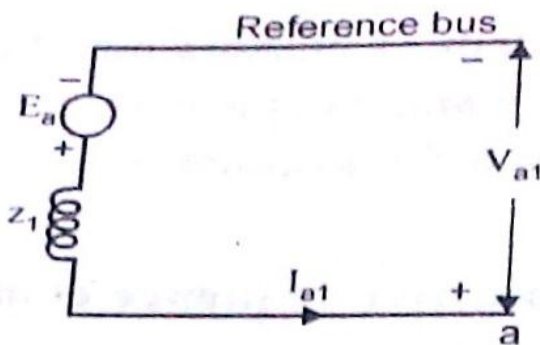
56. Draw the negative sequence network of an unloaded synchronous generator with its neutral grounded through reactor.
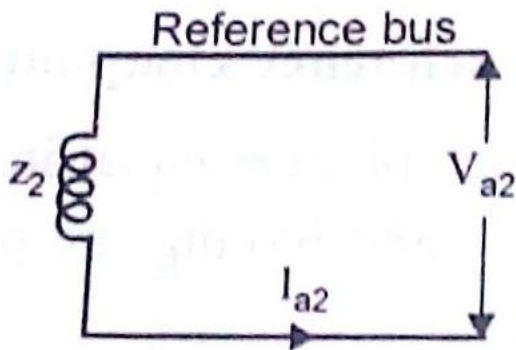
57. Draw the zero sequence network of an unloaded synchronous generator with its neutral grounded through reactor.

Reference bus
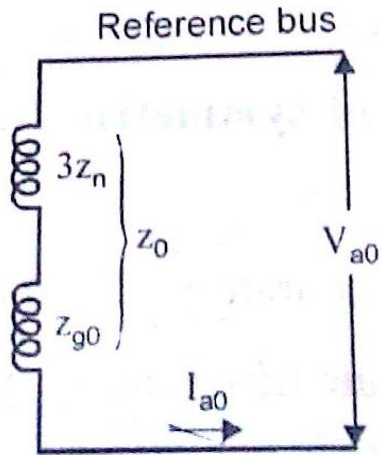
58. Why delta connected load will not have any zero sequence components?

Delta connected load will not have zero sequence components because the current in the neutral is three times the zero sequence line current. A delta connected load provides no path to neutral and hence line currents flowing to a delta connected load cannot contain zero sequence components.

59. Write about the lightning effect on electrical installations.

Lightning damages electrical and electronic systems in particular: transformers, electricity meters and electrical appliances on both residential and industrial premises.
The cost of repairing the damage caused by lightning is very high. But it is very hard to assess the consequences of the following:

- Disturbances caused to computers and telecommunication networks;
- Faults generated in the running of programmable logic controller programs and control systems.

Moreover, the cost of operating losses may be far higher than the value of the equipment destroyed.

60. Write about the electric strokes impact on a building.

Lightning strokes can affect the electrical (and/or electronic) systems of a building in two ways:

- By direct impact of the lightening stroke on the building
- By indirect impact of the lightning stroke on the building

a. A lightning stroke can fall on an overhead electric power line supplying a building. The over current and overvoltage can spread several kilometers from the point of impact.

b. A lightning stroke can fall near an electric power line. It is the electromagnetic radiation of the lightning current that produces a high current and an overvoltage on the electric power supply network.

In the latter two cases, the hazardous currents and voltages are transmitted by the power supply network.

c. A lightning strike can fall near a building. The earth potential around the point of impact rises dangerously.

61. How does the open conductor fault occurs?

When one or two of a three phase circuit is open due to accidents, storms, etc.., then unbalance is created and the asymmetrical currents flow. Such types of faults that come in series with the lines are referred as the open conductor faults. The open conductor faults can be analyzed by using the sequence networks drawn for the system under consideration as seen from the point of fault, F. These networks are then suitably connected to simulate the given type of fault.

62. How is the analysis of unsymmetrical faults done on power systems?

The analysis of unsymmetrical fault in power systems is done in a similar way as that followed thus far for the case of a fault at the terminals of a generator. Here, instead of the sequence impedances of the generator, each and every element is to be replaced by their corresponding sequence impedances and the fault is analyzed by suitably connecting them together to arrive at the Thevenin's equivalent impedance if that given sequence.

PART - B

1. Explain about the concept of symmetrical component.   (N/D'14) (16)

One of the most powerful tools for dealing with unbalanced polyphase circuits is the method of symmetrical components. An unbalanced system of n related phasors can be resolved

into n systems of balanced phasors called symmetrical components. Symmetrical components are positive, negative and zero sequence components.

Balanced System   (2)

The load impedance is the same in all 3Φ and the voltage and currents are characterized by complete three phase symmetry. It is given by

$$I_a + I_b + I_c = I_n = 0$$

Unbalance fault

In an unsymmetrical fault or loaded system, neither the phase currents nor the phase voltages possess three-phase symmetry.

The algebraic sum of the phase current is equal to the neutral current flowing in the system. It is given by,

$$I_a + I_b + I_c = I_n$$

Where

$I_a, I_b, I_c \text{ are phase current}$
$I_n \text{ is the neutral current.}$

Phase Sequence   (3)

In three phase system, the phase sequence is defined as the order in which they pass through a positive maximum.

Consider the unbalanced current I_a, I_b, I_c shown in figure. These current are resolved into three symmetrical components. They are positive, negative and zero sequence.

Positive Sequence Components

It consists of three components of equal magnitude, displaced each other by 120° in phase, and having the phase sequence abc as shown in figure.

Let I_a^+ be the reference phasor.

Positive sequence phasors a,b,c can be written in terms of I_a^+ as,

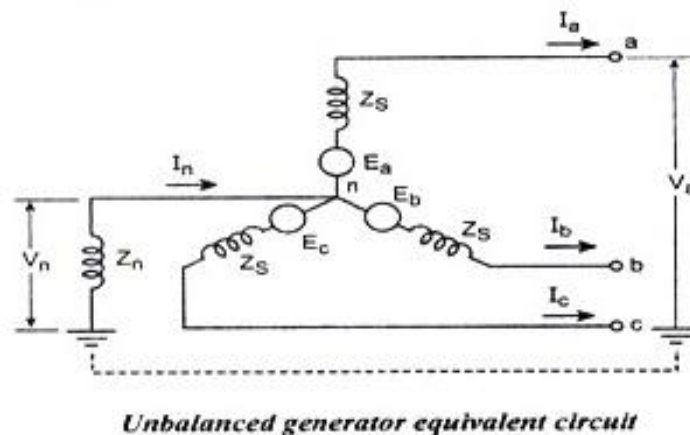$$I_a^+ = I_a^+ \angle 0°$$
$$I_b^+ = I_a^+ \angle 240° = a^2 I_a^+$$
$$I_c^+ = I_a^+ \angle 120° = a I_a^+$$
$$where\ I_a^+, I_b^+, I_c^+\ are\ the\ Positive\ sequence\ component\ of\ I_a, I_b\ and\ I_c$$

Negative Sequence components (3)

It consists of three components of equal magnitude, displaced by 120° in phase, and having the phase sequence abc as shown in figure.

Unbalanced generator equivalent circuit

Let I_a^- be the reference phasor.

Negative sequence phasors a,b,c can be written in terms of I_a^- as,

$$I_a^- = I_a^- \angle 0°$$
$$I_b^- = I_a^- \angle 120° = a I_a^+$$
$$I_c^- = I_a^- \angle 240° = a^2 I_a^+$$
$$where\ I_a^-, I_b^-, I_c^-\ are\ the\ negative\ sequence\ component\ of\ I_a, I_b and\ I_c$$

Zero Sequence Components (2)

It consists of three phasors equal in magnitude and with zero displacement from each other as shown in figure.

Zero sequence phasors a,b,c can be written as

$$I_a^0 = I_b^0 = I_c^0$$

where I_a^0, I_b^0, I_c^0 are the zero sequence components of I_a, I_b and I_c

Symmetrical component transformation   (6)

The three phase unbalanced currents I_a, I_b and I_c can be represented in terms of sequence currents as

$$I_a = I_a^0 + I_a^+ + I_a^-$$
$$I_b = I_b^0 + I_b^+ + I_b^-$$
$$I_c = I_c^0 + I_c^+ + I_c^-$$

According to the definition of symmetrical components, we can rewrite above equation in terms of phase a components.

$$I_a = I_a^0 + I_a^+ + I_a^-$$
$$I_b = I_b^0 + a^2 I_b^+ + a I_b^-$$
$$I_c = I_c^0 + a I_c^+ + a^2 I_c^-$$

Write the above equation in matrix form,

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix}$$

where $a = 1\angle 120°$ and $a^2 = 1\angle 240°$

In simple form,
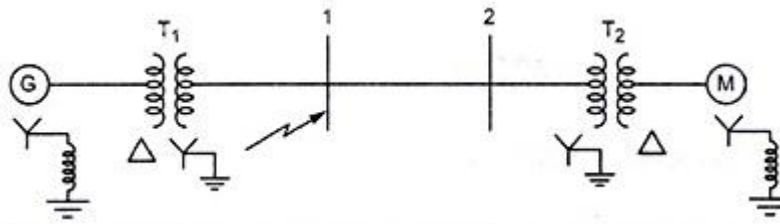
$$\left[I_p \right] = \left[T \right] \left[I_s \right]$$

$$where\ I_p = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$I_s = \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix} T = Symmetrical\ component\ transformation\ matrix$$

**2. A single line to ground fault occurs on the bus 1 of the power system of fig. shown below.**

Find:

    i.   **Current in the fault**

    ii.   **SC current in phase a of generator**

    iii.   **Voltage of the healthy phases of the bus 1 using Z_{bus} method.**

Given values: Rating of each machine 1200 KVA, 600 V with $X_1 = X_2 = 10$ % and $X_0 = 5$%. Each three phase transformer is rated 1200 KVA, 600 V / 3300 V (Δ/Y) with leakage reactance of 5%. The reactance of transmission line are $X_1 = X_2 = 20$ % and $X_0 = 40$ % on the base of 1220 KVA, 3300 V. The reactance of neutral grounding reactance are 5% on the KVA and voltage base of the machines. (N/D'14) (16)
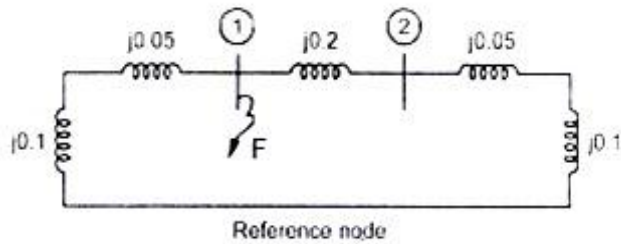
Solution :

Positive sequence (1)
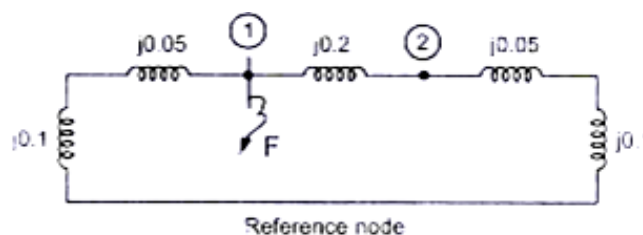
Negative sequence (1)

Formulate Z_{bus} :

$$Z_{bus}^{new} = \begin{bmatrix} j0.15 & j0.15 \\ j0.15 & j0.35 \end{bmatrix}$$

Adding an element between existing node

$$Z_{bus}^{new} = \begin{bmatrix} j0.15 & j0.15 & j0.15 \\ j0.15 & j0.35 & j0.35 \\ j0.15 & j0.35 & j0.5 \end{bmatrix}$$

(4)

Apply Kron reduction, $Z_{bus}^{+} = Z_{bus}^{-} = \begin{bmatrix} j0.105 & j0.045 \\ j0.045 & j0.105 \end{bmatrix}$

$Z_{11}^{+} = Z_{11}^{-} = j0.105$

Zero sequence   **(2)**

$Z_{bus} = \begin{bmatrix} j0.05 \end{bmatrix}$

Adding an element $j0.4$ *between nodes* (1) *and* (2)

$$Z_{bus} = \begin{bmatrix} j0.05 & j0.05 \\ j0.05 & j0.45 \end{bmatrix}$$

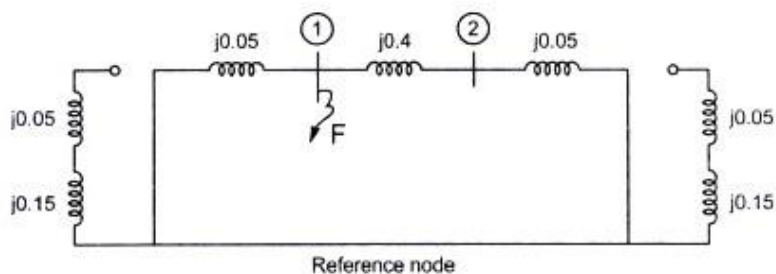*Adding an element* $j0.05$ *between nodes* (2) *and ref node*,

$$Z_{bus} = \begin{bmatrix} j0.05 & j0.05 & j0.05 \\ j0.05 & j0.45 & j0.45 \\ j0.05 & j0.45 & j0.5 \end{bmatrix}$$

(2)

Apply Kron reduction,

$$Z_{bus}^0 = \begin{bmatrix} j0.045 & j0.005 \\ j0.005 & j0.045 \end{bmatrix}$$

$$Z_{11}^0 = j0.045$$

Current in the fault $I_f = 3I_a^+$

$$I_a^+ = \frac{1\angle 0°}{Z_{11}^+ + Z_{11}^- + Z_{11}^0 + 3Z_f}$$

$$= \frac{1\angle 0°}{j0.105 + j0.105 + j0.045 + 0} = -j3.92 \ p.u.$$

Current in the fault $I_f = 3 \times (-j3.92) = -j11.7 \ p.u.$

Current in the fault $I_f = 3I_a^+$

$I_a^+ = -j3.92 \ p.u.$

Current in the fault $I_f - j11.7 \ p.u.$

---

ii) short circuit current on transmission lines

Positive sequence post fault bus voltages,

$$V_f^+ = V_f^+ - Z_{11}^+ I_f^+$$

$$= 1.0 - j0.105 \times (-j3.92) = 0.5884$$

$$V_{f2}^+ = V_0^+ - Z_{12}^+ I_f^+$$

$$= 1.0 - j0.045 \times (-j3.92) = 0.8236$$

(3)

Negative sequence post fault bus voltages

$$V_f^- = -Z_{11}^- I_f^-$$

$$= -j0.105 \times (-j3.92) = -0.4116$$

$$V_{f2}^- = -Z_{12}^0 I_f^0$$

$$= -j0.045 \times (-j3.92) = -0.1764$$

Zero sequence post fault bus voltages

$V_f^0 = -Z_{11}^0 I_f^0$

$\quad = -j0.045 \times (-j3.92) = -0.1764$

$V_{f2}^0 = -Z_{12}^0 I_f^0$

$\quad = -j0.005 \times (-j3.92) = -0.0196$

Positive sequence current $I_{12}^+ = \dfrac{V_{f1}^+ - V_{f2}^+}{Z_{12(line)}^+}$

$$= \frac{0.5884 - 0.8236}{j0.2} = j1.176 \ p.u$$

$$I_{12}^- = \frac{V_{f1}^- - V_{f2}^-}{Z_{12(line)}^-} = \frac{-0.4116 - (-0.1764)}{j0.2} = j1.176 \ p.u$$

$$I_{12}^0 = \frac{V_{f1}^0 - V_{f2}^0}{Z_{12(line)}^0} = \frac{-0.1764 - (-0.0196)}{j0.4} = j0.392 \ p.u$$

Positive sequence current $I_{12}^+ = j1.176 \ p.u$

$$I_{12}^- = j1.176 \ p.u$$
$$I_{12}^0 = j0.392 \ p.u$$

iii) Voltage of healthy phase of bus 1:

$V_a = 0$

$V_b = a^2 V_1^+ + a V_1^- + V_1^0$

$\quad = 1\angle 240° \times 0.5884 + 1\angle 120° \times (-0.4116) + (-0.1764)$

$\quad = -0.2646 - j0.866$

$\quad = 0.9056\angle -107°$

$V_c = a V_1^+ + a^2 V_1^- + V_1^0$

$\quad = 1\angle 120° \times 0.5884 + 1\angle 240° \times (-0.4116) + (-0.1764)$

$\quad = 0.9056\angle 107°$

(3)

*Voltage of healthy phase of bus 1:*

$V_a = 0$

$V_b = 0.9056\angle -107°$

$V_c = 0.9056\angle 107°$

3. **A 25 MVA, 13.2 KV alternator with solidly grounded neutral has a sub transient reactance of 0.25 p.u. The negative and zero sequence reactance are 0.35 and 0.01 p.u respectively. If a double line to ground fault occurs at the terminal of the alternator, determine the fault current and line to line voltage at the fault.**

(M/J'14) (16)

Solution : Sequence network is **(2)**
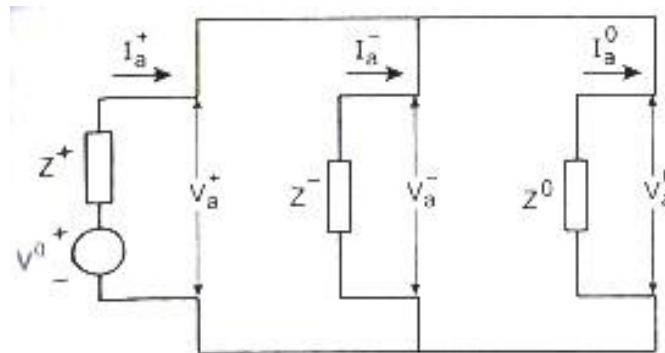
Prefault voltage = E_a = V_0 = $1\angle240°$

$$Positive\ sequence\ current\ I_a^+ = \frac{V^0}{Z^+ + \left(\frac{Z^- \times Z^0}{Z^- + Z^0}\right)}$$

(4)

$$= \frac{1\angle0°}{j0.25 + \left(\frac{j0.35 \times j0.1}{j0.35 + j0.1}\right)} = -j3.0508\ p.u$$

$$I_a^- = -I_a^+ \times \frac{Z^0}{Z^- + Z^0}$$

$$= -(-j3.0508) \times \frac{j0.1}{j0.35 + j0.1} = j0.678\ p.u$$

$$I_a^0 = -I_a^+ \times \frac{Z^-}{Z^- + Z^0}$$

$$= -(-j3.0508) \times \frac{j0.35}{j0.35 + j0.1} = j2.373 \ p.u$$

Fault current $= 3I_a^0 = 3 \times j2.373 = j7.119 \ p.u$ **(3)**

Base current $= \dfrac{MVA}{\sqrt{3} \times KV_b} = \dfrac{25 \times 10^3}{\sqrt{3} \times 132} = 1093.466 \, Amp$

$I_f \, in \, Amp = j7.119 \times 1093.466 = j7.784 \, Amp$

Fault current $= j7.119 \ p.u$

Base current $= 1093.466 \, Amp$

$I_f \, in \, Amp = j7.784 \, Amp$

Symmetrical component of voltages :

$$V_a^0 = -Z^0 I_a^0$$
$$= -j0.1 \times j2.373 = 0.2373 \ p.u$$

$$V_a^+ = E_a - Z^+ I_a^+$$
$$= 1\angle 0° - j0.25 \times -j3.0508 = 0.2373 \ p.u$$

$$V_a^- = -Z^- I_a^- = -j0.35 \times j0.678$$
$$= 0.2373 \ p.u$$

$$\therefore V_a^+ = V_a^-$$

(4)

Symmetrical component of voltages :

$V_a^0 = 0.2373 \ p.u$

$V_a^+ = 0.2373 \ p.u$

$V_a^- = 0.2373 \ p.u$

Phase voltages :

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix}$$

(3)

$V_a = 0.2373 + 0.2373 + 0.2373 = 0.7119$

$V_b = 0.2373 + (-0.5 - j0.866)0.2373 + (-0.5 + j0.866)0.2373 + 0$

$V_c = 0$

$\therefore V_b = V_c = 0$

Line – line voltage

$V_{ab} = V_a - V_b = 0.7119 - 0 = 0.7119 \, p.u$

$V_{bc} = V_b - V_c = 0.7119 - 0 = 0.7119 \, p.u$

$V_{bc} = V_b - V_c = 0 - 0 = 0 \, p.u$

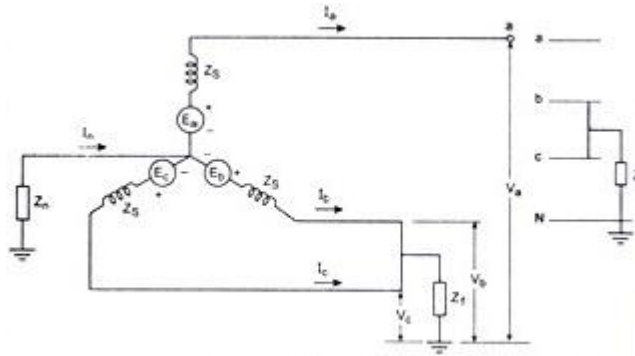$\boxed{\begin{aligned} &Line – line \, voltage \\ &V_{ab} = 0.7119 \, p.u \\ &V_{bc} = 0.7119 \, p.u \\ &V_{bc} = 0 \, p.u \end{aligned}}$

4. Obtain the expression for fault current for a line to line fault taken place through an impedance Z_b in a power system. **(M/J'14, N/D'13) (16)**

Solution:- **(2)**

$I_b = -I_c$

$I_a = 0 \, (unloaded \, generator)$

$V_b - V_c = Z_f I_b \Rightarrow V_c = V_b - Z_f I_b$

Substitute for $I_b = -I_c, I_a = 0,$ the symmetrical components of current are :

$$\begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$\begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix}$$

(3)

Substitute the value of I_b, we get

$(a^2 - a)\left[V_a^+ - V_a^-\right] = (a^2 - a)I_a^+ Z_f$

$V_a^+ - V_a^-$

$$I_a^0 = \frac{1}{3}[0 + I_b - I_b] = 0$$

$$I_a^+ = \frac{1}{3}\left[aI_b - a^2 I_b\right]$$

$$I_a^- = \frac{1}{3}\left[a^2 I_b - aI_b\right]$$

$$\therefore I_a^+ = -I_a^- \text{ and } I_a^0 = 0$$

From sequence network of the generator, the symmetrical voltage are give by

$$\begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} \begin{bmatrix} Z_{kk}^0 & 0 & 0 \\ 0 & Z_{kk}^+ & 0 \\ 0 & 0 & Z_{kk}^- \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix}$$

$$V_a^0 = -Z_{kk}^0 I_a^0 = -Z_{kk}^0 \times 0 = 0$$

$$V_a^+ = E_a - Z_{kk}^+ I_a^+$$

$$V_a^- = -Z_{kk}^- I_a^- = Z_{kk}^- I_a^+$$

The Phase current are given by

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ I_a^+ \\ -I_a^- \end{bmatrix} \quad \textbf{(4)}$$

$$I_a = 0, I_b = a^2 I_a^+ - a I_a^- = I_a^+ (a^2 - a)$$

$$I_c = a I_a^+ - a^2 I_a^+ = I_a^+ (a - a^2) = -I_b$$
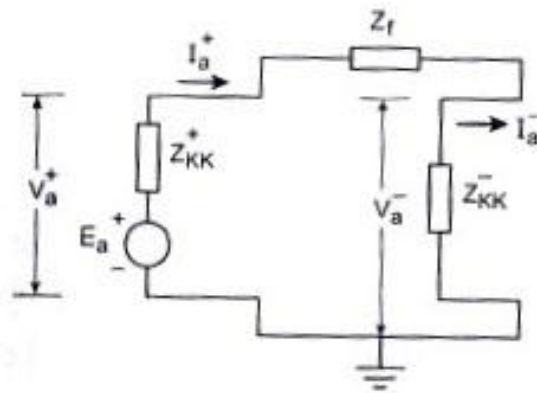
Sequence network for LL fault with Z_f   **(3)**

The Phase voltage are

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ V_a^+ \\ V_a^- \end{bmatrix}$$

$$V_a^0 = 0$$

$$V_a = V_a^+ + V_a^-$$

$$V_b = a^2 V_a^+ + a V_a^-$$

$$V_c = a V_a^+ + a^2 V_a^-$$

From the condition $V_b - V_c = Z_f I_b$

Substituting V_b *and* V_c, *we get*

$$(a^2 - a)\left[V_a^+ - V_a^- \right] = Z_f I_b$$

Substitute the value of I_b, *we get*

$$(a^2 - a)\left[V_a^+ - V_a^- \right] = (a^2 - a)I_a^+ Z_f$$

$$V_a^+ - V_a^-$$

Substitue V_a^+, V_a^-, *we get,*

$$E_a = \left[Z_K^+ + Z_K^- + Z_f \right] I_a^+$$

$$I_a^+ = \frac{E_a}{Z_K^+ + Z_K^- + Z_f}$$

$$I_a^- = -I_a^+$$

$$I_a^0 = 0$$ (4)

Current phase do min e

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix} = \begin{bmatrix} 0 + I_a^+ - I_a^+ \\ 0 + (a^2 - a)I_a^+ \\ 0 + (a + a^2)I_a^+ \end{bmatrix} \begin{bmatrix} 0 \\ (a^2 - a)I_a^+ \\ -(a + a^2)I_a^- \end{bmatrix}$$

The fault current is $I_b = -I_c = (a^2 - a)I_a^+$

$$= (-0.5 - j0.866 + 0.5 - j0.866)I_a^+ = -j1.732 I_a^+$$

$$= -j\sqrt{3} I_a^+$$

Substituting I_a^+ *we get,*

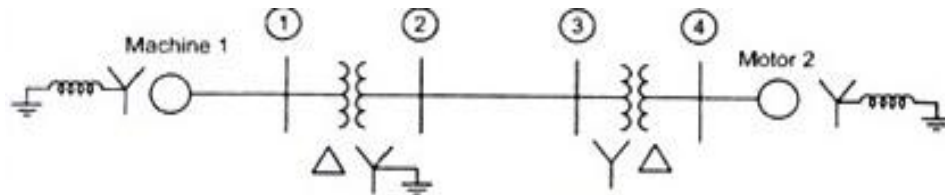$$I_f = I_b = \frac{-j\sqrt{3} E_a}{Z_{KK}^+ + Z_{KK}^- + Z_f}$$

5. Two synchronous machines are connected through three phase transformers to the transmission line shown in fig. The rating and reactance of the machines and transformers are:

Machine 1 and 2: 100 MVA, 20 KV; $X''_d = X_1 = X_2 = 20\%$; $X_0 = 4\%$, $X_n = 5\%$

Transformer T1 and T2 : 100MVA, $20\Delta/345$ Y KV ; X=8%

On a chosen base of 100 MVA, 345 KV in transmission line circuit, line reactance are $X_1 = X_2 = 15\%$ and $X_0 = 50\%$.

Draw each of three sequences networks and find the zero sequence bus impedance matrices by means of Z_{bus} building algorithm. (16)

Solution: Positive sequence network: (8)
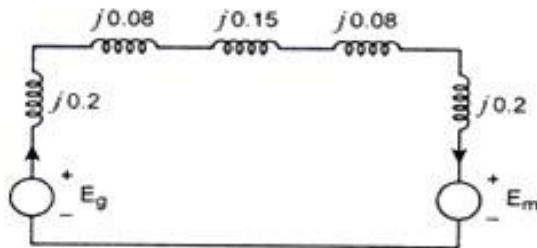
$$= (1)\begin{bmatrix} \overset{(1)}{j0.19} \end{bmatrix}$$

$$= \begin{matrix} (1) \\ (2) \end{matrix} \begin{bmatrix} \overset{(1)}{j0.19} & \overset{(2)}{0} \\ 0 & j0.08 \end{bmatrix}$$

$$= \begin{matrix} (1) \\ (2) \\ (3) \end{matrix} \begin{bmatrix} \overset{(1)}{j0.19} & \overset{(2)}{0} & \overset{(3)}{0} \\ 0 & j0.08 & j0.8 \\ 0 & j0.08 & j0.8 \end{bmatrix}$$

$$
= \begin{array}{c} (1) \\ (2) \\ (3) \\ (a) \end{array} \begin{bmatrix} \overset{(1)}{j0.9} & \overset{(2)}{0} & \overset{(3)}{0} & \overset{(a)}{0} \\ 0 & j0.08 & j0.08 & j0.08 \\ 0 & j0.08 & j0.58 & j0.08 \\ 0 & j0.08 & j0.58 & j0.66 \end{bmatrix}
$$

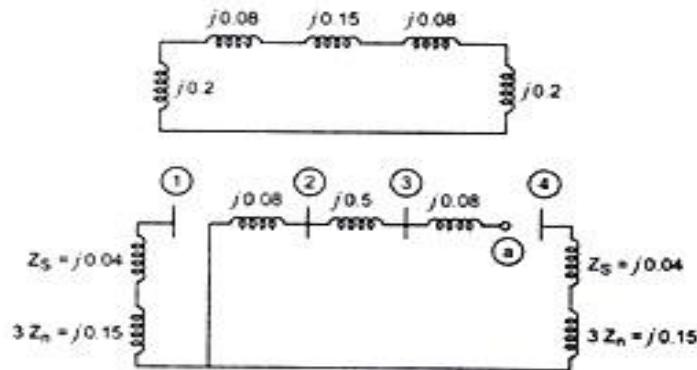Node a is eliminating using Kron reduction techniques, we get   **(8)**

$$
= \begin{bmatrix} j0.19 & 0 & 0 \\ 0 & j0.08 & j0.08 \\ 0 & j0.08 & j0.58 \end{bmatrix}
$$

Add branch j0.19 from bus (4) to the ref. we get,

$$
Z_{bus}^0 = \begin{array}{c} (1) \\ (2) \\ (3) \\ (4) \end{array} \begin{bmatrix} \overset{(1)}{j0.19} & \overset{(2)}{0} & \overset{(3)}{0} & \overset{(4)}{0} \\ 0 & j0.08 & j0.08 & 0 \\ 0 & j0.08 & j0.58 & 0 \\ 0 & 0 & 0 & j0.19 \end{bmatrix}
$$

Negative sequence network :

The Zeros in Z_{bus}^0 shows that the zero sequence current injected into bus (1) or bus (4)

Cannot cause voltage at the other buses because of the open circuits introduced by the Δ-Y transformers.

6. A single line diagram of power system is shown in figure, determine the fault current and fault MVA for a line to line fault occurs between phases b and c at bus 4 as shown in fig.

G1:G2 : 100 MVA, 20KV, $X^+=X^-$ 15 %

T!,T2 : 100 MVA, 20/345 KV, X_{leak}=9 %: Line : $X^+=X^-$ 5 %   **(M/J'13)(16)**
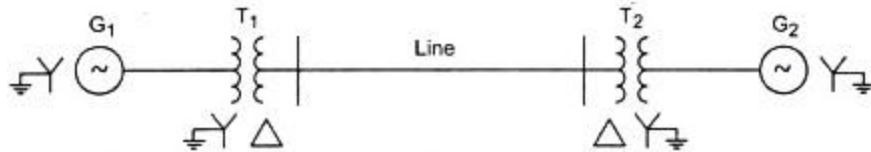
Solution:

Positive Sequence Thevenin equivalent: **(6)**
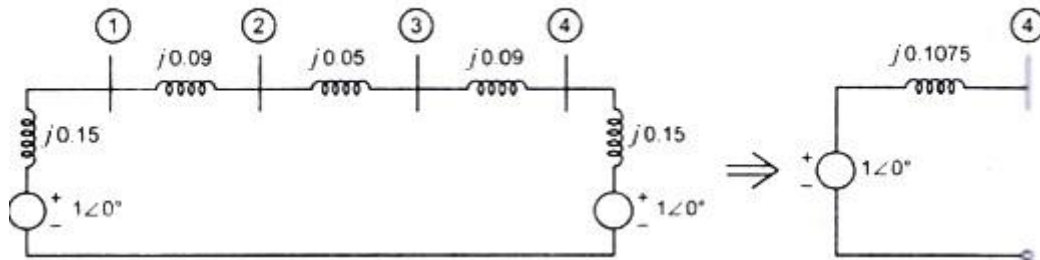
Positive sequence Thevenin equivalent :

Negative sequence thevenin equivalent:
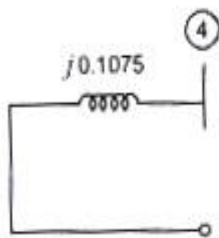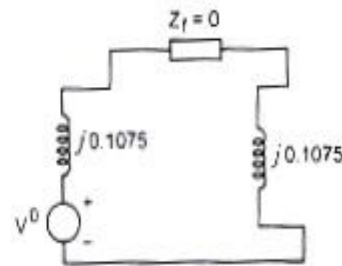
Current in phase do min *e* :

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j13.426 \\ j13.426 \end{bmatrix}$$

$$I_a = 0$$

$$I_b = 1 \times 0 + a^2(-j13.426) + a(j13.426) = -23.254\,KA$$

$$I_c = -I_b = 23.254\,KA$$

$$I_n = I_a + I_b + I_c = 0$$

$$Fault\,MVA = \sqrt{3} \times I_f(KA) \times KV = \sqrt{3} \times 23.254 \times 20$$
$$= 805.542$$

$$prefault\,voltage = E_a = V^0 = 1\angle 0°$$

$$I_a^+ = -I_a^- = \frac{1\angle 0°}{j0.1075 + j0.1075} = -j4.651\,p.u$$

(8) $$|I_a^+| = |I_a^-| = 4.651 \times \frac{100}{\sqrt{3} \times 20} = 13.426 \, KA$$

$$I_a^+ = -j13.426, I_a^- = j13.426 \, KA$$

Current in phase do $\min e$:

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j13.426 \\ j13.426 \end{bmatrix}$$

$$I_a = 0$$

$$I_b = 1 \times 0 + a^2(-j13.426) + a(j13.426) = -23.254 \, KA$$

$$I_c = -I_b = 23.254 \, KA$$

$$I_n = I_a + I_b + I_c = 0$$

$$Fault \, MVA = \sqrt{3} \times I_f (KA) \times KV = \sqrt{3} \times 23.254 \times 20$$

$$= 805.542$$

$$\boxed{\begin{aligned} I_a &= 0 \\ I_b &= -23.254 \, KA \\ I_c &= -I_b = 23.254 \, KA \\ I_n &= 0 \\ Fault \, MVA &= 805.542 \end{aligned}}$$

(2)

7. Discuss in detail about the sequence impedance and network of synchronous machine, transmission lines transformers and loads **(M/J'13) (16)**

Synchronous Generator

Consider the three phase synchronous generator with netural grounded through an impedance Z_n is as shown in figure

Let V_a, V_b, V_c be the phase voltages (line to neutral). **(4)**

Let I_a, I_b, I_c be the phase currents.

Line to neutral voltages are written as

$$V_a = E_a - Z_s I_a - Z_n I_n$$

$$V_b = E_b - Z_s I_b - Z_n I_n$$

$$V_c = E_c - Z_s I_c - Z_n I_n$$
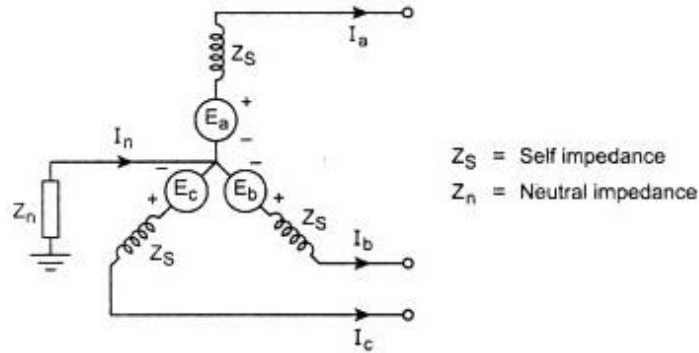
3 φ synchronous generator with neutral grounded through impedance

Z_S = Self impedance
Z_n = Neutral impedance

Substituting $I_n = I_a + I_b + I_c$ in (1), we get

$$V_a = E_a - Z_s I_a - Z_n I_a - Z_n I_b - Z_n I_c$$
$$V_b = E_b - Z_s I_b - Z_n I_a - Z_n I_b - Z_n I_c$$
$$V_c = E_c - Z_s I_c - Z_n I_a - Z_n I_b - Z_n I_c$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} E_a \\ E_b \\ E_c \end{bmatrix} = \begin{bmatrix} Z_s + Z_n & Z_n & Z_n \\ Z_n & Z_s + Z_n & Z_n \\ Z_n & Z_n & Z_s + Z_n \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$[V_P] = [E_P] - [Z^{abc}][I_P]$$
$$[V^{abc}] = [E^{abc}] - [Z^{abc}][I^{abc}]$$

where the sequence impedance $[Z^{012}] = [T]^{-1}[Z^{abc}][T]$

$$[Z^{012}] = \begin{bmatrix} Z_s + 3Z_n & 0 & 0 \\ 0 & Z_s & 0 \\ 0 & 0 & Z_s \end{bmatrix} = \begin{bmatrix} Z^0 & 0 & 0 \\ 0 & Z^+ & 0 \\ 0 & 0 & Z^- \end{bmatrix}$$

Since the generator emf is balance, threr is only positive – sequence voltage E_a

$$\therefore [Z^{012}] = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z^0 & 0 & 0 \\ 0 & Z^+ & 0 \\ 0 & 0 & Z^- \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix}$$

From equation (6), *we can write*

$$\therefore V_a^0 = -Z^0 I_a^+$$
$$V_a^+ = E_a - Z^+ I_a^+$$
$$V_a^- = -Z^- I_a^-$$

Sequence Impedance of Transmission Line   **(4)**
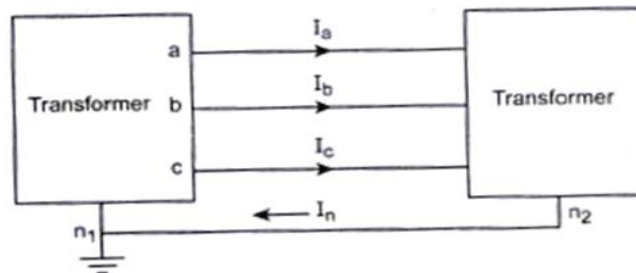
A 3Φ balanced transposed line shown in Figure. Impedance per phase is independent of the phase sequence of balance set of currents. Because the voltages and currents encounter the same geometry of the line. Thus the positive, negative impedance are equal.

For symmetrical line,

$$\begin{bmatrix} \Delta V_{an1} \\ \Delta V_{bn1} \\ \Delta V_{cn1} \end{bmatrix} = \begin{bmatrix} Z_1 & Z_2 & Z_2 \\ Z_2 & Z_1 & Z_2 \\ Z_2 & Z_2 & Z_1 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

WKT,

$$[Z_s] = [T]^{-1}[Z_P][T]$$

$$\begin{bmatrix} Z^0 \\ Z^+ \\ Z^- \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 3Z_1 + 6Z_2 & 0 & 0 \\ 0 & 3Z_1 + 3Z_2 & 0 \\ 0 & 0 & Z_1 + 3Z_2 \end{bmatrix}$$

$$= \begin{bmatrix} Z_1 + 2Z_2 & 0 & 0 \\ 0 & Z_1 + Z_2 & 0 \\ 0 & 0 & Z_1 + Z_2 \end{bmatrix}$$

Transformers:-

Y-Y connected (2)
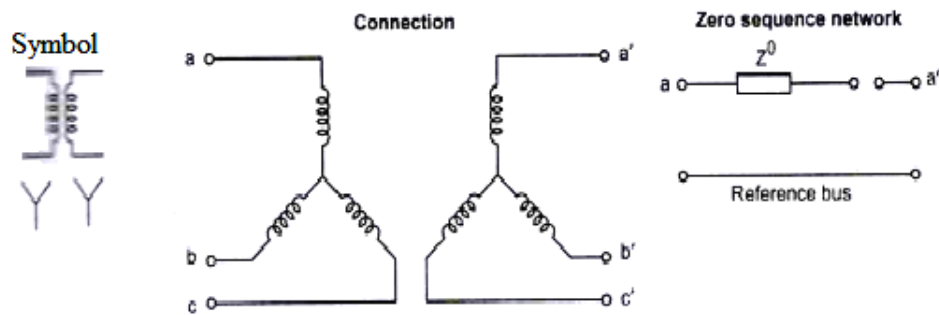
Y-Δ connected (2)
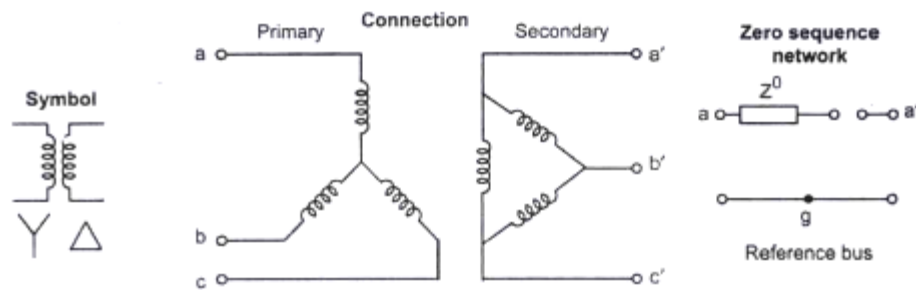
Transformer Y-Δ connected with isolated neutral
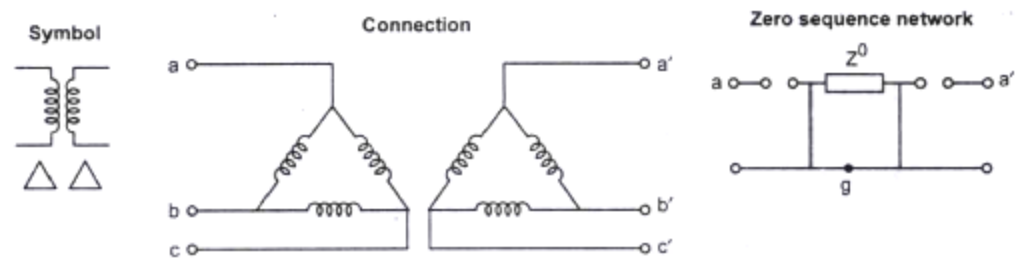
Δ-Δ connected (2)

Transformer Δ-Δ connected

Y − Δ connected, neutral solidly grounded (2)
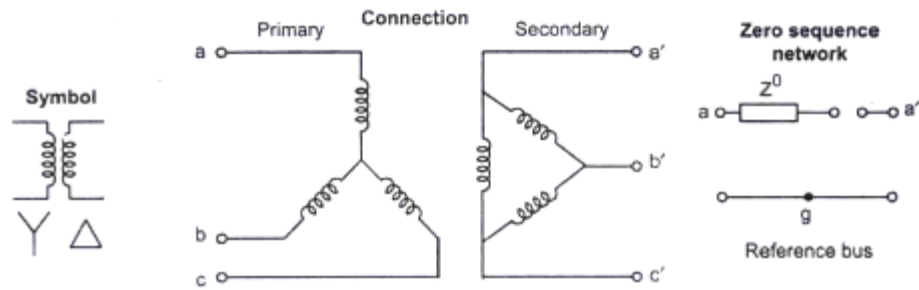
Transformer Y-Δ connected with isolated neutral

8.Draw the sequence network connection for a double line to ground fault at any point in a power system and from thaat obtain an expression for the fault current. (N/D'12) (16)

A three phase generator with a fault on phases b and c through an impedance Z_f to ground.

(3)
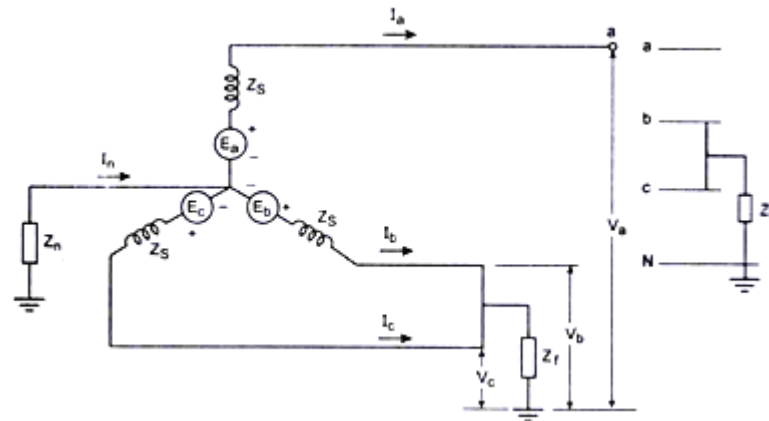
$I_a = 0$

$I_b + I_c = I_f$

$V_b = V_c = Z_f I_f = Z_f(I_b + I_c)$

(3)

The symmetrical components of voltage are :

$$\begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

Substitute $V_b = V_c$ in equation (2), we get

$$\begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_b \end{bmatrix}$$

$$V_a^0 = \frac{1}{3}(V_a + 2V_b)$$

$$V_a^+ = \frac{1}{3}(V_a - V_b)$$

$$V_a^- = \frac{1}{3}(V_a - V_b)$$

$$V_a^+ = V_a^-$$

(3)

The Phase current are given by

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix}$$

$$I_a = I_a^0 + I_a^+ + I_a^-$$
$$I_b = I_a^0 + a^2 I_a^+ + a I_a^-$$
$$I_c = I_a^0 + a I_a^+ + a^2 I_a^-$$
$$I_f = I_b + I_c = I_a^0 + a^2 I_a^+ + a I_a^- + I_a^0 + a I_a^+ + a^2 I_a^-$$
$$\qquad = 2I_a^0 + I_a^+(a^2 + a) + I_a^-(a + a^2)$$

$$(I_a^+ + I_a^-) = -I_a^0$$

substituting (5) *in* (6), *we get*

$$I_b + I_c = 3I_a^0$$

From the condition, $V_b = 3Z_f I_a^0$

The Phase voltage are

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix}$$

(3)

$$V_a = V_a^0 + V_a^+ + V_a^-$$
$$V_b = V_a^0 + a^2 V_a^+ + a V_a^+ \qquad \left[\because V_a^+ = V_a^- \right]$$
$$V_b = V_a^0 + V_a^+ \qquad \left[\therefore V_b = 3Z_f I_a^0 \right]$$

The symmetrical components voltage is given by

$$V_a^0 = -Z_{KK}^0 I_a^0$$
$$V_a^+ = E_a - Z_{KK}^+ I_a^+ \qquad\qquad \textbf{(4)}$$
$$V_a^- = -Z_{KK}^- I_a^-$$

Then $I_a^0 = \dfrac{-[E_a - Z_{KK}^+ I_a^+]}{Z_{KK}^0 + 3Z_f}$

$$I_a^- = \dfrac{-[E_a - Z_{KK}^+ I_a^+]}{Z_{KK}^-}$$

$$-I_a^0 = I_a^+ + I_a^-$$
$$I_a^+ = -I_a^- - I_a^0$$

on by solving we get

$$I_a^+ = \dfrac{E_a}{Z_{KK}^+ + \dfrac{Z_{KK}^-(Z_{KK}^0 + 3Z_f)}{Z_{KK}^0 + 3Z_f + Z_{KK}^-}}$$

$$\boxed{I_f = \dfrac{3}{Z_{KK}^0 + 3Z_f}\left[E_a - \dfrac{Z_{KK}^+ E_a}{Z_{KK}^+ + \dfrac{Z_{KK}^-(Z_{KK}^0 + 3Z_f)}{Z_{KK}^0 + Z_{KK}^+ + 3Z_f}}\right]}$$

9. i) Derive an expression for the total power in a three phase system in terms of sequence components of voltage and currents **(3)**

Ans :

Apparent power $S_{(3\phi)} = [V_P]^T [I_P]^*$

By u sin *g symmetrical component transformation,*

$$= [TV_S]^T [TI_S]^*$$

$$S_{(3\phi)} = V_S^T T^T T^* I_S^*$$

$$T^T T^* = TT^* = 3 \qquad \left[T^T = T\right]$$

$$\therefore S_{(3\phi)} = 3\left[V_S^T I_S^*\right]$$

$$= 3V_a^0 I_a^{0*} + 3V_a^+ I_a^{+*} + 3V_a^- I_a^{-*}$$

The Total unbalanced power can be obtained from the sum of the symmetrical components powers.

$$\boxed{S_{(3\phi)} = 3V_a^0 I_a^{0*} + 3V_a^+ I_a^{+*} + 3V_a^- I_a^{-*}}$$

ii) Discuss in detail about the sequence impedance of transmission lines **(3)**

A 3Φ balanced transposed line shown in Figure. Impedance per phase is independent of the phase sequence of balance set of currents. Because, the voltages and currents encounter the same geometry of the line. Thus the positive, negative impedance are equal.

For symmetrical line,

$$\begin{bmatrix} \Delta V_{an1} \\ \Delta V_{bn1} \\ \Delta V_{cn1} \end{bmatrix} = \begin{bmatrix} Z_1 & Z_2 & Z_2 \\ Z_2 & Z_1 & Z_2 \\ Z_2 & Z_2 & Z_1 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

we know that,

$$[Z_s] = [T]^{-1}[Z_P][T]$$

$$\begin{bmatrix} Z^0 \\ Z^+ \\ Z^- \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 3Z_1 + 6Z_2 & 0 & 0 \\ 0 & 3Z_1 + 3Z_2 & 0 \\ 0 & 0 & Z_1 + 3Z_2 \end{bmatrix}$$

$$= \begin{bmatrix} Z_1 + 2Z_2 & 0 & 0 \\ 0 & Z_1 + Z_2 & 0 \\ 0 & 0 & Z_1 + Z_2 \end{bmatrix}$$

iii) The bus impedance matrix of four bus system with values in p.u. is given by,

$$Z_{bus} = j \begin{bmatrix} 0.15 & 0.08 & 0.04 & 0.07 \\ 0.08 & 0.15 & 0.06 & 0.09 \\ 0.04 & 0.06 & 0.13 & 0.05 \\ 0.07 & 0.09 & 0.05 & 0.12 \end{bmatrix}$$

In this system generators are connected to buses 1 and 2 and their subtransient reactances were included when finding Zbus. If prefault current is neglected, find subtransient current in p.u. in the fault on a bus 4. Assume prefault voltage as 1 p.u. If the subtransient reactance of generator in bus 2 is 0.2 p.u. find the subtransient fault current supplied by generator. **(10)**

Solution:

Let $I_f{''}$ be the subtransient current in the fault on bus 4.

Now, $I_f{''} = \dfrac{V_{pf}}{Z_{44}}$

Where V_{pf} = prefault voltage at bus 4 = $1\angle 0°$ p.u.

$$\therefore I_f{''} = \dfrac{1\angle 0°}{j0.12} = -j8.333 = 8.333\angle -90° \text{ p.u.}$$

$$\boxed{I_f{''} = 8.333\angle -90° \text{ p.u.}}$$

The voltage at bus 2 when there is a 3 phase fault in bus 4 is given by,

$$V_2 = V_{pf} - I_f{''} Z_{pf}$$

$$\therefore V_2 = 1\angle 0° - 8.333\angle -90° \times j0.09 = 1 - 8.333\angle -90° \times 0.09\angle 90°$$

$$V_2 = 1 - 0.74997 = 0.25003 \approx 0.25\angle 0° \text{ p.u.}$$

Since there is no current in the system prior to the fault all the buses will be at same potential prior to fault. Also there won't be any potential drop in the synchronous reactance of the generator, because it does not deliver any current prior to fault. Hence the induced emef of the generator in bus – 2 is also 1 p.u. . The generator in bus – 2 can be represented as shown fig.

With reference to fig.

The subtransient fault current delivered by the generator at bus -2, $I_{g2}{''} = \dfrac{E_{g2}{''} - V_2}{jX_{d2}{''}}$

$$I_{g2}{''} = \dfrac{1\angle 0° - 0.25\angle 0°}{j0.2} = \dfrac{1 - 0.25}{0.2\angle 90°} = 3.75\angle -90° \text{ p.u.}$$

$$I_{g2}'' = 3.75\angle\text{-}90° \text{ p.u.}$$

Note: $I_f = I''_{g1} + I''_{g2}$

Result:

The subtransient fault current in the bus -4 = $I_f'' = 8.333\angle\text{-}90°$ p.u.

The voltage at bus – 2 when there is a 3 phase fault in bus – 4 = $V_2 = 0.75\angle 0°$ p.u.

The subtransient fault current delivered by the generator – 2 when there is a 3 – phase fault in bus – 4 $\Big\}$ $I_{g2}'' = 3.75\angle\text{-}90°$ p.u.

10. A 30 MVA . 11 KV generator has $Z_1 = Z_2 = j0.2$ p.u, $Z_0 = j0.05$ p.u. A line to ground fault occurs on the generator terminals. Find the fault current and line to line voltage during limit conditions. Assume that the generator neutral is solidly grounded and that the generator is operating at no load and at rated voltage at the occurrence of fault.

(N/D'11) (16)

Solution : - **(4)**
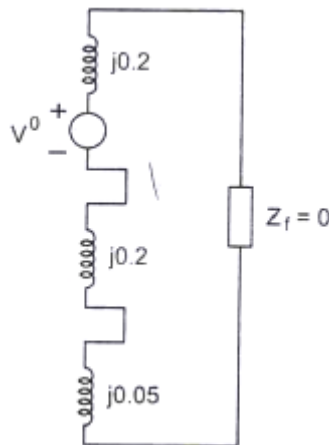
$Z^+ = j0.2 \ p.u$

$Z^- = j0.2 \ p.u$

$Z^0 = j0.05 \ p.u$

$prefault\ voltage, V^0 = 1\angle 0°$

Symmetrical components of fault current

$$I_a^+ = I_a^- = I_a^0 = \frac{V^0}{Z^+ + Z^- + Z^0}$$

$$= \frac{1\angle 0°}{j0.2 + j0.2 + j0.05}$$

$$= -j2.222 \; p.u$$

Fault current in p.u $= 3I_a^+$

$$= 3 \times -j2.222$$

$$= -j6.666 \; p.u$$

$$Base \; current = \frac{MVA_b \times 10^3}{\sqrt{3} \times KV_b} = \frac{30 \times 10^3}{\sqrt{3} \times 11} \qquad \textbf{(4)}$$

$$= 1574.6 \; Amp$$

Fault current in Amp $= -j6.666 \times 1574.6$

$$= 10496.3 \; Amp$$

Symmetrical components of fault current

$I_a^+ = -j2.222 \; p.u$

Fault current in p.u $= -j6.666 \; p.u$

Base current $= 1574.6 \; Amp$

Fault current in Amp $= 10496.3 \; Amp$

(4)

Line to line voltage during the fault :

$$V_a^0 = -Z^0 I_a^+$$

$$= -j0.05 \times -j2.222 = -0.1111$$

$$V_a^+ = V^0 - Z^+ I_a^+$$

$$= 1\angle 0° - j0.2 \times -j2.222$$

$$= 0.5555$$

$$V_a^- = -Z^- I_a^+$$

$$= -j0.2 \times -j2.222$$

$$= -0.4444$$

Line to line voltage during the fault :

$V_a^0 = -0.1111$

$V_a^+ = 0.5555$

$V_a^- = -0.4444$

Subtransient phase voltages

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix}$$

$V_a = V_a^0 + V_a^+ + V_a^-$

$\quad = -0.111 + 0.5555 - 0.4444 = 0$

$V_b = V_a^0 + a^2 V_a^+ + a V_a^-$ **(4)**

$\quad = -0.1111 + 1\angle -240° \times 0.5555 + 1\angle -120° \times -0.4444$

$\quad = -0.29 - j0.0267$

$V_c = V_a^0 + a V_a^+ + a^2 V_a^-$

$\quad = -0.29 + j0.267$

Subtransient phase voltages

$V_a = 0$

$V_b = -0.29 - j0.0267$

$V_c = -0.29 + j0.267$

Line to line Voltage

$V_{ab} = V_a - V_b = 0 - [-0.29 - j0.267]$

$\qquad = 0.29 + j0.267 \; p.u$

$V_{bc} = V_b - V_c = -0.29 - j0.267 - [-0.29 + j0.267]$

$\qquad = -j0.534 \; p.u$

$V_{ac} = V_a - V_c = 0 - [-0.29 + j0.267]$

$\qquad = 0.29 - j0.267$

Line to line Voltage

$V_{ab} = 0.29 + j0.267 \; p.u$

$V_{bc} = -j0.534 \; p.u$

$V_{ac} = 0.29 - j0.267$
---

11. A 50 MVA, 11KV, 3ph alternator was subjected to different types of faults. The fault current are 3-ph fault 1870 A, line to line fault 2590 A, single line to ground fault 4130 A. The alternator neutral is solidly grounded. Find the p.u values of three sequence reactance of the alternator. (M/J'12) (16)

Solution :- (10)

$$MVA_b = 50$$
$$KV_b = 11$$
$$I_f(3\phi \, fault) = 1870A$$
$$I_f(L-L \, fault) = 2590A$$
$$I_f(L-G \, fault) = 4130A$$
$$Z^0, Z^+, Z^- = ?$$
$$Base \, current = \frac{MVA_b \times 10^3}{\sqrt{3} \times KV_b}$$
$$= \frac{50 \times 10^3}{\sqrt{3} \times 11} = 2624.3A$$
$$Base \, current = 2624.3A$$
$$I_{f \, p.u(3\phi)} = \frac{V^0}{Z^+}$$

$$= \frac{1870}{2624.3} = -j0.713$$
$$\Rightarrow Z^+ = \frac{1\angle 0°}{-j0.713} = j1.4 \, p.u$$
$$I_{f \, p.u}(L.L) = \frac{-j\sqrt{3}V^0}{Z^+ + Z^-} = \frac{2590}{2624.3} = -j0.99$$

$$Z^+ + Z^- = \frac{-j\sqrt{3} \times 1\angle 0°}{-j0.99} = j1.75 \, p.u$$

$$Z^- = j1.75 - Z^+$$
$$= j1.75 - j1.4 = j0.35$$
$$I_{fp.u}(LG) = \frac{3V^0}{Z^+ + Z^- + Z^0}$$ **(6)**
$$= \frac{4130}{2624.3} = -j1.57$$

$$Z^+ + Z^- + Z^0 = \frac{3 \times 1\angle 0°}{-j1.57} = j1.91$$
$$Z^0 = j1.91 - j1.4 - j0.35$$
$$= j0.16 \, p.u$$

12. Derive the equation for the L-G fault under symmetrical analysis. The single line to ground fault is the most common type of fault, is caused by lightning or by conductors making contact with grounded structures. **(16)**

Suppose a line to ground fault is occurs on phase a connected to ground through impedance
Z_f. **(6)**
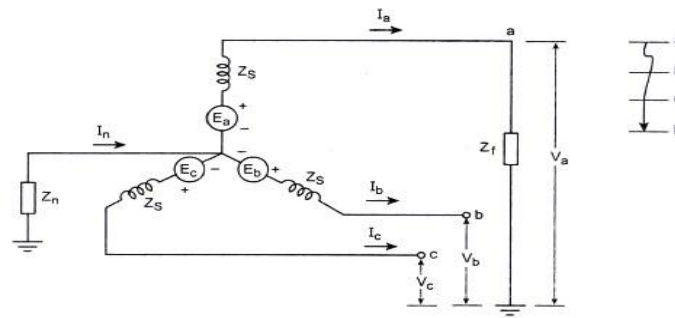
$$V_a = Z_f I_a$$

$$I_b = I_c = 0$$

$$I_f = I_a$$

Symmetrical componets ofcurrents are

$$\begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

Substitute for $I_b = I_c = 0$, symmetrical components of currents are

$$\begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix}$$

From eq(3), we get

$$I_a^0 = \frac{I_a}{3}$$

$$I_a^+ = \frac{I_a}{3}$$ **(5)**

$$I_a^- = \frac{I_a}{3} = \frac{I_f}{3}$$

$$I_a^+ = I_a^- = I_a^0 = \frac{I_a}{3}$$

From sequence network of generator, symmetrical voltages are given by

$$\begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_{KK}^0 & 0 & 0 \\ 0 & Z_{KK}^+ & 0 \\ 0 & 0 & Z_{KK}^- \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix}$$

$$V_a^0 = -Z_{KK}^0 I_a^0 = -Z_{KK}^0 I_a^+$$

$$V_a^+ = E_a - Z_{KK}^+ I_a^+$$

$$V_a^- = -Z_{KK}^- I_a^- = -Z_{KK}^- I_a^+$$

The phase voltage are given by

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix}$$

From eq.(6) we get,

$$V_a = V_a^0 + V_a^+ + V_a^-$$

From the condition $V_a = Z_f I_a$

$$\therefore V_a^0 + V_a^+ + V_a^- = Z_f I_a$$

Substituting the symmetrical components, we get

$$I_a^+ = \frac{E_a}{Z_{KK}^0 + Z_{KK}^+ + Z_{KK}^- + 3Z_f}$$

(5)

The fault current is

$$I_f = I_a = 3I_a^+ = \frac{3E_a}{Z_{KK}^0 + Z_{KK}^+ + Z_{KK}^- + 3Z_f}$$

$$\boxed{\text{The fault current, } I_f = I_a = 3I_a^+ = \frac{3E_a}{Z_{KK}^0 + Z_{KK}^+ + Z_{KK}^- + 3Z_f}}$$
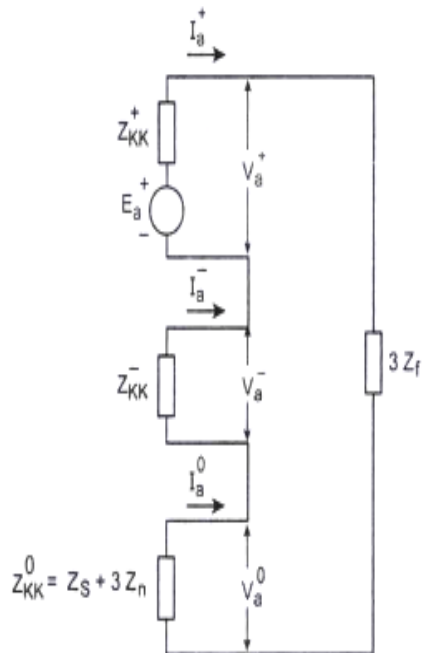
13. Two 11 KV, 20 MVA, three phase star connected generators operate in parallel as shown in figure. The positive, negative and zero sequence reactance's of each being . respectively, j0.18, j0.15,j0.10 p.u. The star point of one of the generators is isolated and that of other is earthed through a 2 ohms resistor. A single line to ground fault occurs at the terminals of one of the generators.

Estimate ,

i) The fault current,

ii) Current in grounding resistor and

iii) The voltage across grounding resistor,   **(M/J'11) (16)**
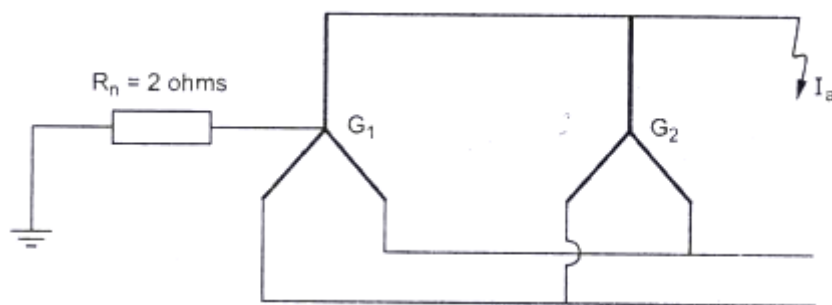
Solution :-

Positive sequence network : (Generator G1 & G2 are in parallel)   **(5)**

$$Z^+ = \frac{j0.18 \times j0.18}{j0.18 + j0.18} = j0.09$$

Negative Sequence network :

$$Z^- = \frac{j0.15 \times j0.15}{j0.15 + j0.15} = j0.075$$
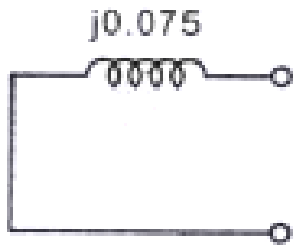
Zero Sequence network,

$$Z^0 = Z_s + 3Z_n$$

$$Z_{n\,p.u} = \frac{2 \times MVA_b}{KV_b^2}$$

$$= 2 \times \frac{20}{11^2} = 0.33\ p.u$$

$$Z^0 = Z_s + 3Z_n$$

$$= j0.1 + 3 \times j0.33$$

$$= 0.99 + j0.1$$
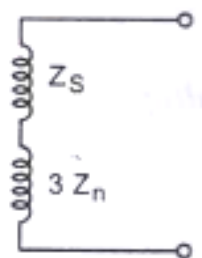
Sequence network for L-G fault: **(5)**

$$I_a^+ = I_a^- = I_a^0$$

$$= \frac{E_a}{Z^+ + Z^- + Z^0}$$

$$= \frac{1\angle 0°}{j0.09 + j0.075 + 0.99 + j0.1}$$

$$= \frac{1}{0.99 + j0.265}$$

Sequence network for L-G fault
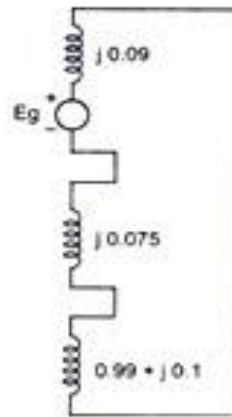
i) *Fault current I_f in p.u.* $= 3I_a^+$ **(6)**

$$= 3 \times \frac{1}{0.99 + j0.265}$$

$$= 2.827 - j0.756 \ p.u$$

$\boxed{Fault \ current \ I_f \ in \ p.u. = 2.827 - j0.756 \ p.u}$

ii)*Current in the grounding resistor I_r :*

$I_f = 2.827 - j0.756 \ p.u.$

$| I_f | = 2.926 \ p.u$

$$Base \ current = \frac{MVA_b}{\sqrt{3} \times KV_b} = \frac{20 \times 10^3}{\sqrt{3} \times 11} = 10497 \ A$$

$| I_r | \ in \ Amp = 2.926 \times 10497 = 3.07 \ KA$

$\boxed{Base \ current = 3.07 \ KA}$

iii)*Voltage across the grounding resistor :*

$$= | I_r | \ in \ Amp \times 2\Omega$$

$$= 3.07 \times 2 = 6.14 \ KV$$

$$\boxed{Voltage\ across\ the\ grounding\ resistor = 6.14\ KV}$$

14 .i) Explain how an unbalanced set of three phase can be represented by system of balance voltages. **(7)**

Let V_a, V_b, V_c be the phase voltages and V_a^0, V_a^+, V_a^- be the positive,
negative sequence voltages of phase 'a'.

$$V_a = V_a^0 + V_a^+ + V_a^-$$
$$V_b = V_a^0 + V_a^+ + V_a^-$$
$$V_b = V_a^0 + V_a^+ + V_a^-$$

Substituting the symmetrical components with respect to phase a.

$$V_a = V_a^0 + V_a^+ + V_a^-$$
$$V_b = V_a^0 + a^2 V_a^+ + a V_a^-$$
$$V_b = V_a^0 + a V_a^+ + a^2 V_a^-$$

In matrix form,

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix}$$

$$[V_P] = [T][V_s]$$
$$(or)$$
$$\left[V^{abc}\right] = [T]\left[V^{012}\right]$$

ii) Draw the Zero sequence network for **(3)**

1. **Y grounded-Y grounded transformer**
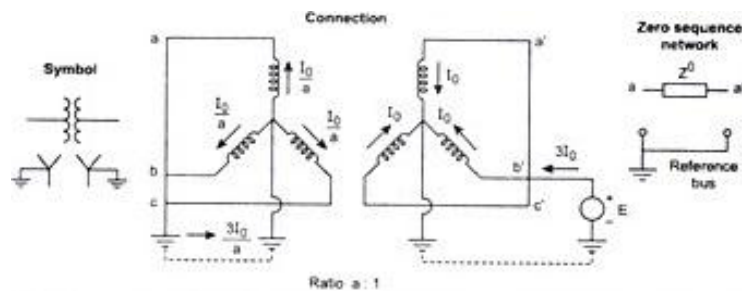
2. **Δ-Δ connected transformer** (5)
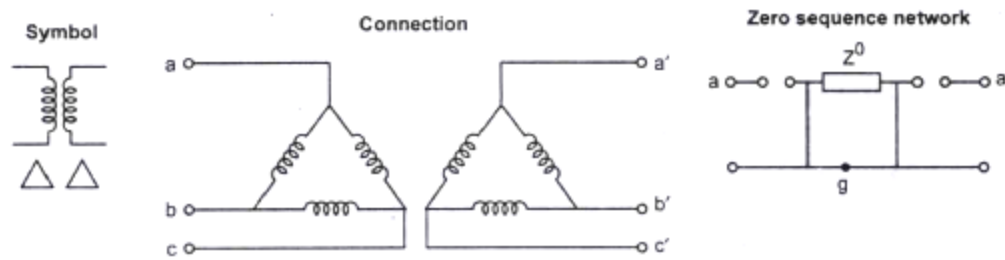
Transformer Δ-Δ connected

3. **Generator sequence diagram** (5)
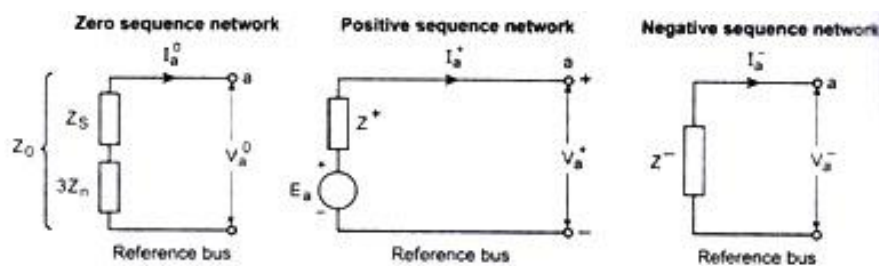
Zero, positive and negative sequence network

15. Determine the fault current and MVA at faulted bus for a line to ground (solid) fault at bus 4 as shown in fig.
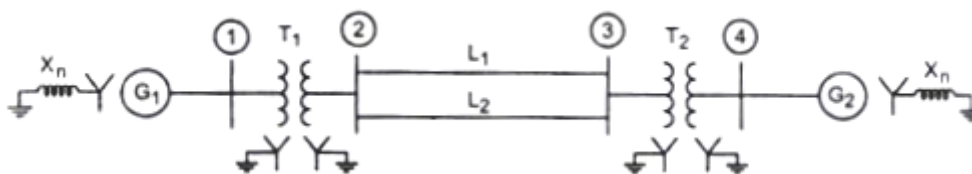
G1,G2: 100MVA, 11KV, $X^+ = X^- = 15\%$, $X^0 = 5\%$, $X^n = 6\%$

T1,T2: 100 MVA, 11 KV/220KV, $X_{leak} = 9\%$

L1,L2: $X^+ = X^- = 10\%$, $X^0 = 10\%$, on a base of 100 MVA.

Consider a fault at phase 'a' (16)

Solution :-

Step 1: Positive sequence Thevenin equivalent viewed from bus 4: (2)

Determination of Thevenin's
equivalent impedance

Thevenin's circuit
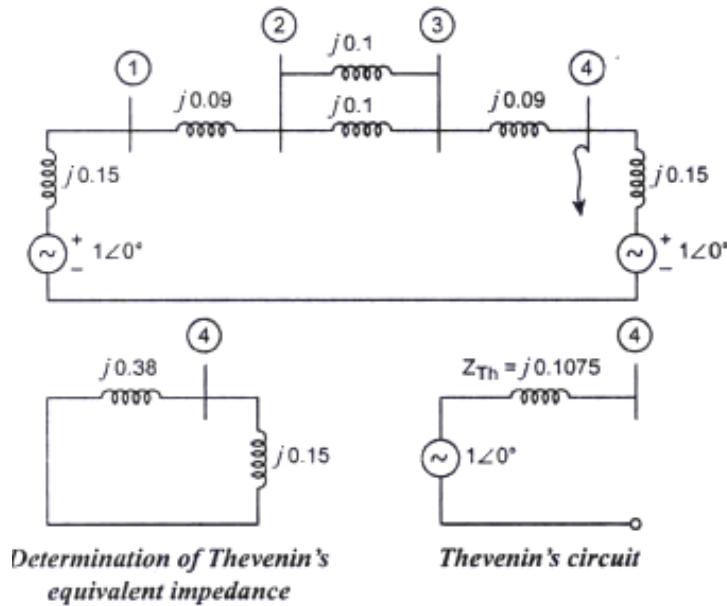
Step 2: Negative sequence Thevenin equivalent viewed from bus 4: (2)
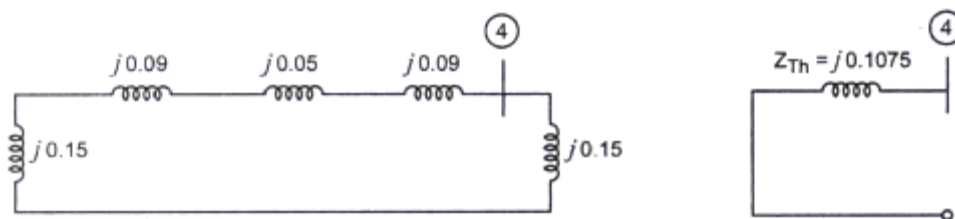
Step 3: Zero sequence Thevenin equivalent viewed from bus 4: (8)
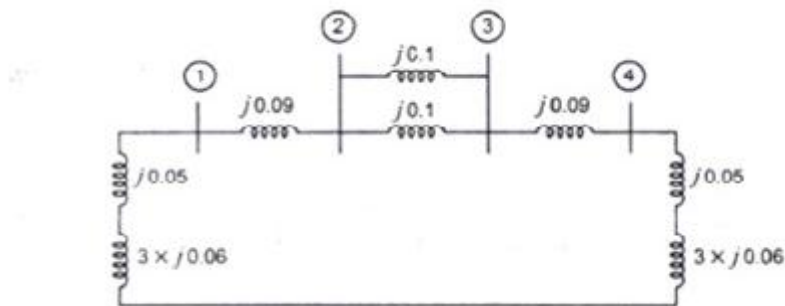
$$For\ transmission\ line\ Z_{p.u}^{new} = \frac{Actual\ value}{Base\ value} = \frac{j0.121}{Base\ KV^2} \times Base\ MVA$$

$$= \frac{j0.121}{11^2} \times 100 = j0.1$$

$j0.1\ and\ j0.1\ are\ in\ parallel.$

$$\frac{j0.1 \times j0.1}{j0.1 + j0.1} = j0.05$$
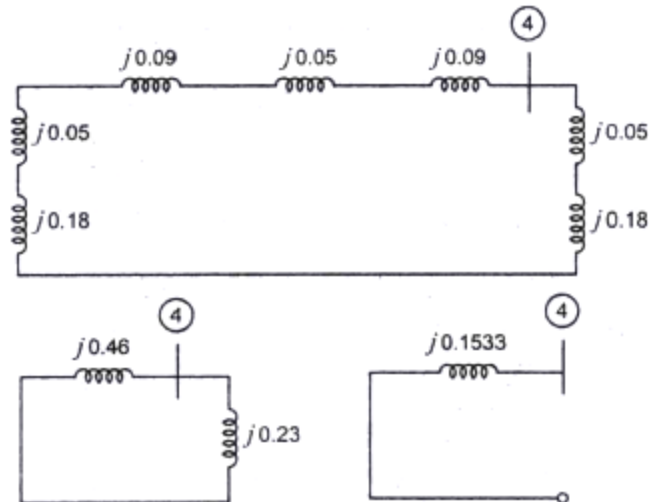
Step 4: Draw sequence network. (4)

$$prefault\ voltage\ E_a = V^0 = 1\angle 0°$$

$$I_a^+ = I_a^- = I_a^0$$

$$= \frac{V^0}{Z_{44}^+ + Z_{44}^- + Z_{44}^0 + Z_f}$$

$$= \frac{1\angle 0°}{j0.1075 + j0.1075 + j0.1533}$$

$$= -j2.7152$$

$$I_f = 3 \times I_a^+ = 3 \times -j2.7152$$

$$= -j8.1455\ p.u$$

$$Actual\ fault\ current\ (KA) = I_{f\ p.u} \times Base\ current$$

$$= 8.1455 \times \frac{100}{\sqrt{3} \times 11} = 42.75\ KA$$

$$Fault\ MVA = \sqrt{3} \times KV \times KA$$

$$= \sqrt{3} \times 11 \times 42.75 = 814.55$$

$$\boxed{Fault\ MVA = 814.55}$$

$$\boxed{\begin{array}{l} Current\ in\ phase\ domain: \\ I_a = I_f = 42.75\ KA \\ I_a = I_c = 0 \\ I_n = I_a + I_b + I_c = 42.75\ KA \end{array}}$$
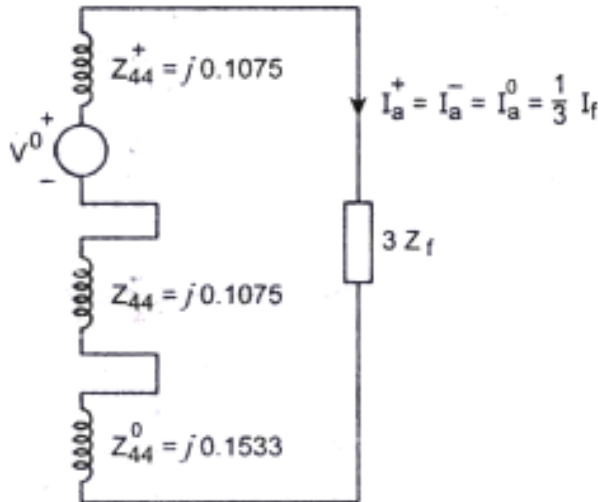
16 (i). What are the different steps involved in unsymmetrical fault analysis. **(6)**

The unsymmetrical fault analysis can be done by using the following steps.

- Assemble the Thevenin's equivalent positive, negative and zero sequence networks separately using the sequence impedance of various power system. Components like generators, motors, transformers and transmission lines.

- Compute the positive, negative and zero sequence impedance matrices Z^+, Z^- and Z^0 using bus building algorithm or short circuit fault impedance matrix $Z_{s,bus}$

- Select the type (L-L, L-G, L-L-G), location (bus number) and mathematical description of the fault.

- Determine the fault current at the bus using the sequence networks for a L-G, L-L and L-L-G fault.

- Determine the prefault sequence voltages and post fault sequence voltages.

- Compute the positive, negative and zero sequence line currents.

16 (ii).Determine the fault current when LLG fault occurs between phases b and c. Fault impedance is j0.15 p.u. **(10)**
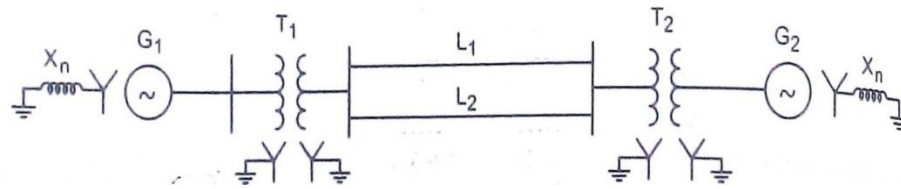
G_1, G_2: 100 MVA, 11 KV, $X^+ = X^- = 15\%$, $X^o = 5\%$, $X_n = 6\%$

T_1, T_2: 100 MVA, 11/220 KV, $X_{leak} = 9\%$

L_1, L_2: $X^+ = X^- = 10\%$, $X^o = 10\%$ on a base of 100 MVA.

Solution:

Positive sequence impedance $Z_{44}^+ = j0.1075$ p.u. **(5)**

Negative sequence impedance $Z_{44}^- = j0.1075$ p.u.

Zero sequence impedance $Z_{44}^0 = j0.1533$ p.u.

Fault impedance $Z_f = j0.15$ p.u.

Prefault voltage $= E_a = V^0 = 1\angle 0°$

Symmetrical components of currents are:

$$I_a^+ = \frac{V^0}{Z^+ + \frac{Z^-(Z^0+3Z_f)}{Z^-+Z^0+3Z_f}}$$

$$= \frac{1\angle 0°}{j0.1075 + \frac{j0.1075\ (0.1533+3\ X\ j0.15)}{j0.1533\ +3\ X\ j0.15+j0.1075}} = -j5.0317 \text{ p.u.}$$

$$\boxed{I_a^+ = -j5.0317 \text{ p.u.}}$$

$$I_a^- = - I_a^+ \left[\frac{3\ Z_f+Z_{44}^0}{Z_{44}^- +3Z_f+Z_{44}^0}\right] = - (-j5.0317) \left[\frac{3\ X\ j0.15\ +j0.1553}{j0.1075 +3\ X\ j0.15+j0.1533}\right]$$ **(5)**

$$\boxed{I_a^- = j4.2707 \text{ p.u.}}$$

$$I_a^0 = - I_a^+ \left[\frac{Z_{44}^-}{Z_{44}^- +3Z_f+Z_{44}^0}\right] = - (-j5.0317)\left[\frac{j0.1075}{j0.1075 +3\ X\ j0.1533}\right]$$

$$\boxed{I_a^- = j0.761 \text{ p.u.}}$$

Fault current $I_f = 3 I_a^0 = 3 \times j0.761 = j2.2829$ p.u.

Fault current $I_f = j2.2829$ p.u.

17. Determine the fault current in p.u., current in phase domain form for a double line to ground fault occurs between phases 'b' and 'c'. **(16)**
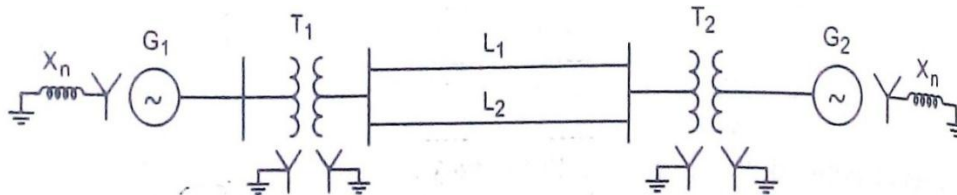
G_1, G_2: 100 MVA, 11 KV, $X^+ = X^- = 15\%$, $X^o = 5\%$, $X_n = 6\%$

T_1, T_2: 100 MVA, 11/220 KV, $X_{leak} = 9\%$

**L_1, L_2: $X^+ = X^- = 10\%$, $X^o = 10\%$ on a base of 100 MVA.**

Solution:

Step 1: Positive sequence Thevenin equivalent **(2)**
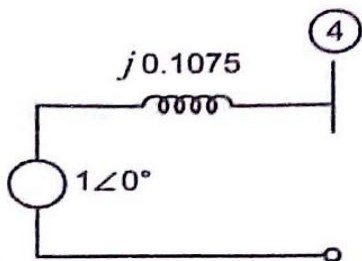
Step 2: Negative sequence Thevenin equivalent **(2)**
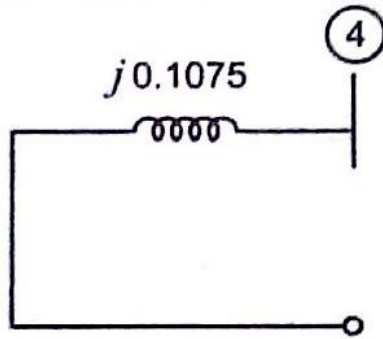
Step 3: Zero sequence Thevenin equivalent (2)
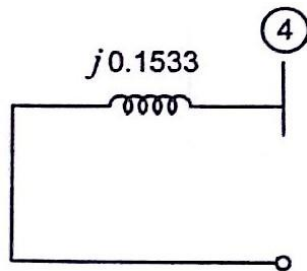
Step 4: Sequence Network. ` (2)
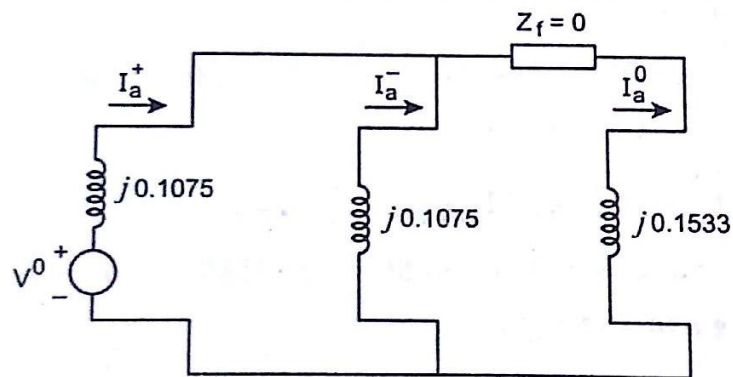
(4)

$$I_a^+ = \frac{V^0}{Z^+ + \frac{Z^-(Z^0 + sZ_f)}{Z^- + Z^0 + sZ_f}}$$

$$Zf = 0; \quad I_a^+ = \frac{V^0}{Z^+ + \frac{Z^- \cdot Z^0}{Z^- + Z^0}} = \frac{1\angle0°}{j0.1075 + \frac{j0.1075 \times j0.1533}{j0.1075 + j0.1533}}$$

$$I_a^+ = \frac{1\angle0°}{j0.1707} = \text{-j5.856 p.u.}$$

$$\boxed{I_a^+ = \text{-j5.856 p.u.}}$$

$$I_a^- = - \left[\frac{I_a^+ \times Z^0}{Z^- + Z^0}\right] = - \left[\frac{-j5.8586 \times j0.1533}{j0.1075 + j0.1533}\right] = \text{j3.4437 p.u.}$$

$$\boxed{I_a^- = \text{j3.4437 p.u.}}$$

$$I_a^0 = - \left[\frac{I_a^+ \times Z^-}{Z^- + Z^0}\right] = - \left[\frac{-j5.8586 \times j0.1075}{j0.1075 + j0.1533}\right] = \text{j2.4149 p.u.}$$

$$\boxed{I_a^0 = \text{j2.4149 p.u.}}$$

Current in the phase domain: **(4)**

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix}$$

$$I_a = I_a^+ + I_a^- + I_a^0$$

$$= \text{-j5.8586} + \text{j3.4437} + \text{j2.4149} = 0$$

$$I_b = I_a^0 + a^2\, I_a^+ + a\, I_a^-$$

$$= \text{j2.4149} + (-0.5 - \text{j0.866}) \times -\text{j5.8586} + (-0.5 + \text{j0.866}) \times \text{j3.4437}$$

$$= \text{-8.056} + \text{j3.6223 p.u.}$$

$$I_c = I_a^0 + a\, I_a^+ + a^2\, I_a^-$$

$$= \text{j2.4149} + (-0.5 + \text{j0.866}) \times \text{j5.8586} + (-0.5 - \text{j0.866}) \times 3.4437$$

$$= \text{8.056} + \text{j3.6223 p.u.}$$

$$I_n = I_a + I_b + I_c$$

$$= 0 - 8.056 + \text{j3.6223} + 8.056 + \text{j0.3.6223}$$

$$= j7.2446 \text{ p.u.}$$

Fault current $I_f = 3 \times I_a^0 = 3 \times j2.4149 = j7.2447$ p.u.

Fault current $I_f = j7.2447$ p.u.

18. Determine the fault current in p.u., current in phase domain form for a double line to ground fault occurs between phases 'b' and 'c'. **(16)**
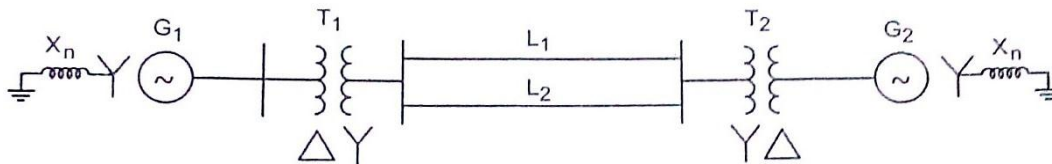
G_1, G_2: 100 MVA, 11 KV, $X^+ = X^- = 15\%$, $X^o = 5\%$, $X_n = 6\%$

T_1, T_2: 100 MVA, 11/220 KV, $X_{leak} = 9\%$

L_1, L_2: $X^+ = X^- = 10\%$, $X^o = 10\%$ on a base of 100 MVA.

Solution:

$$Z^+ = j0.1075, \ Z^- = j0.1075, \ Z^0 = j0.23 \qquad \textbf{(4)}$$
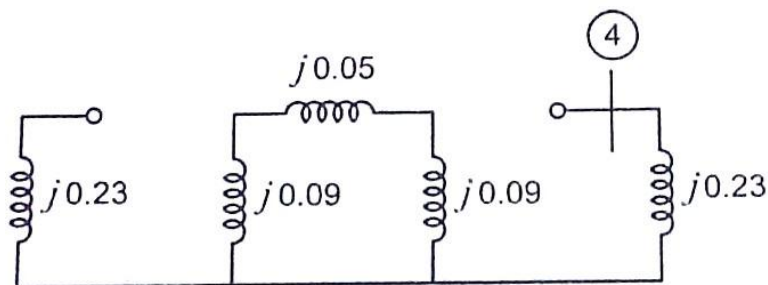
Sequence network. **(6)**
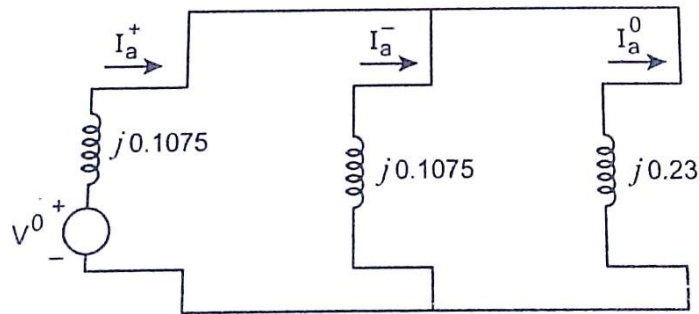
Prefault voltage = $E_a = V^0 = 1\angle 0°$

$$I_a^+ = \frac{V^0}{Z^+ + \frac{Z^- Z^0}{Z^- + Z^0}} = \frac{1\angle 0°}{j0.1075 + \frac{j0.1075 \times j0.23}{j0.1075 + j0.23}} = j5.5322 \text{ p.u.}$$

$$I_a^- = -\left[\frac{-j5.5322 \times j0.23}{j0.1075 + j0.23}\right] = j3.77 \text{ p.u.}$$

$$I_a^- = -\left[\frac{-j5.5322 \times j0.1075}{j0.1075 + j0.23}\right] = j1.7621 \text{ p.u.}$$

Fault current $I_f = 3I_a^0 = 3 \times j1.7621 = j5.2863$ p.u.

Fault current $I_f = j5.2863$ p.u.

Current in phase domain: **(6)**

$$I_a = I_a^+ + I_a^- + I_a^0 = 0$$

$$I_b = I_a^0 + a^2 I_a^+ + a\, I_a$$

$$= j1.7621 + (-0.5 - j0.866)(-j5.5322) + (-0.5 + j0.866) \times j3.77$$

$$= -8.0557 + j2.6431 \text{ p.u.}$$

$$I_c = I_a^0 + a\, I_a^+ + a^2 I_a^- = 8.0557 + j2.6431 \text{ p.u.}$$

$I_n = 5.286$ p.u.

19. Derive expression for bolted LLG fault (16)

Double Sequence Network: (4)

The positive, negative and zero sequence networks are connected in parallel as shown in figure.
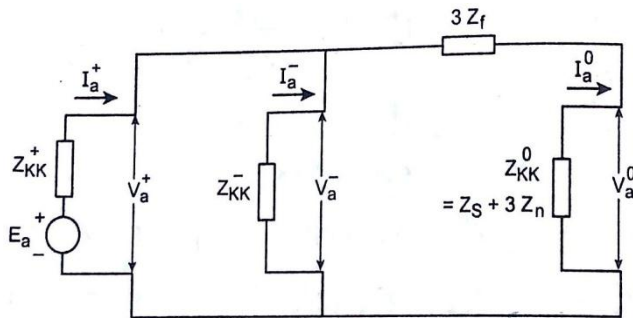
Direct short circuit or Bolted LLG Fault: (8)

Figure shows the direct short circuit or bolted double line to ground fault.
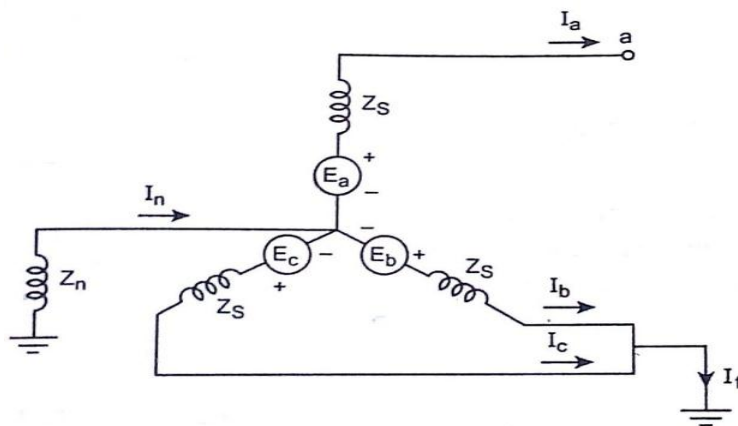
Fault impedance, $Z_f = 0$

The conditions of the fault at bus K are,

$$I_a = 0, V_b = 0, V_c = 0$$
$$I_f = I_b + I_c$$

The sequence network for short circuit LLG fault is as shown.
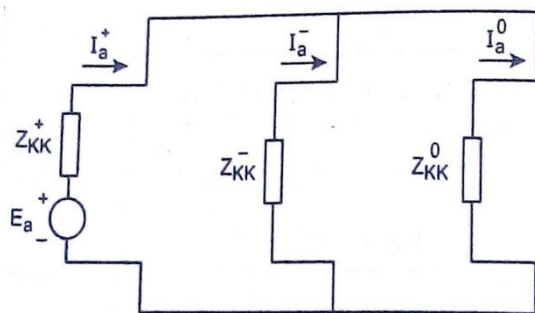
$$I_a^+ = \frac{E_a}{Z_{KK}^+ + \left[\frac{Z_{KK}^- \times Z_{KK}^0}{Z_{KK}^- + Z_{KK}^0}\right]}$$

$$I_a^- = -I_a^+ \times \frac{Z_{KK}^0}{Z_{KK}^- + Z_{KK}^0}$$

$$I_a^0 = -I_a^+ \times \frac{Z_{KK}^-}{Z_{KK}^- + Z_{KK}^0}$$

Double line to Ground fault when $Z_f = \alpha$ (4)

When $Z_f = \alpha$
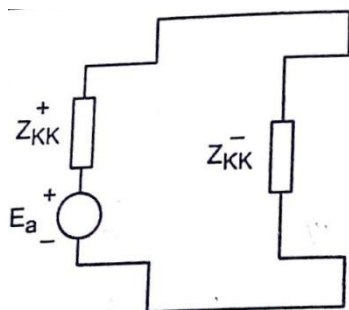
Zero sequence circuit becomes an open circuit. Therefore no zero sequence circuit can flow. The Sequence network is similar to that of bolted line to line fault.

20. Discuss unsymmetrical fault with a neat flowchart. (16)

Start

Read the bus data, line data, fault bus, positive, negative, zero sequence reactance of each machine, transformer, line

Assume E_a^0-prefault voltage, f represents as fault. Zero, positive, negative sequence values of voltages and currents are indicated by superscript 0, + and −. Assume a, b, c be the phase values.

Assume fault at phase a and bus k

Draw the positive, negative and zero sequence networks using the sequence impedances of the power system. Compute positive, negative, zero sequence bus impedance matrices $[Z^+, Z^- \& Z^0]$ using bus building algorithm.

Is LG fault — Yes / No

Is LL fault — Yes / No (LLG fault)

LG fault (Yes):

Symmetrical components of fault current at bus K is

$$I_a^0 = I_a^+ = I_a^- = \frac{E_a^0}{Z_{KK}^+ + Z_{KK}^- + Z_{KK}^0 + 3Z_f}$$

where Z_{KK}^+, Z_{KK}^-, Z_{KK}^0 are the diagonal elements in the K axis of $[Z_{bus}]$

Fault phase current

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix}$$

$I_f = I_a = 3I_a^+$

$I_b = I_c = 0$

Symmetrical voltages are

$V_a^0 = -Z_{KK}^0 I_a^+$

$V_a^+ = E_a^0 - Z_{KK}^+ I_a^+$

$V_a^- = -Z_{KK}^- I_a^+$

LL fault (Yes):

$I_a^0 = 0$

$I_a^+ = -I_a^+ = \dfrac{E_a^0}{Z_{KK}^+ + Z_{KK}^- + Z_f}$

$I_a = 0$

$I_b = -I_c = -j\sqrt{3}\, I_a^+$

$I_f = I_b = \dfrac{-j\sqrt{3}\cdot E_a^0}{Z_{KK}^+ + Z_{KK}^- + Z_f}$

$V_a^0 = 0$

$V_a^+ = E_a^0 - Z_{KK}^+ I_a^+$

$V_a^- = -Z_{KK}^- I_a^- = Z_{KK}^- I_a^+$

LLG fault (No):

$$I_a^+ = \frac{E_a^0}{Z_{KK}^+ + \dfrac{Z_{KK}^-(Z_{KK}^0 + Z_f)}{Z_{KK}^- + Z_{KK}^0 + 3Z_f}}$$

$$I_a^- = \frac{-I_a^+(Z_{KK}^0 + 3Z_f)}{Z_{KK}^- + Z_{KK}^0 + 3Z_f}$$

$$I_a^0 = \frac{-I_a^0 \times Z_{KK}^0}{Z_{KK}^- + Z_{KK}^0 + 3Z_f}$$

$I_a = 0$

$I_b + I_c = I_f$

$I_f = 3I_a^0$

$V_a^0 = -Z_{KK}^0 I_a^0$

$V_a^+ = V_a^- = E_a - Z_{KK}^+ I_a^+$

$$E_a = P.F.$$

Post fault zero sequence bus voltages

$$V_1^{f+} = V_{p.f} - Z_{1K}^+ I_K^{f+}$$
$$\vdots$$
$$V_K^{f+} = V_{p.f} - Z_{KK}^+ I_K^{f+}$$
$$\vdots$$
$$V_K^{f+} = V_{p.f} - Z_{nK}^+ I_K^{f+}$$

Post fault negative sequence bus voltages

$$V_1^{f-} = -Z_{1K}^- I_K^{f-}$$
$$\vdots$$
$$V_K^{f-} = -Z_{KK}^- I_K^{f-}$$
$$\vdots$$
$$V_n^{f-} = -Z_{nK}^- I_K^{f-}$$

Post fault zero sequence bus voltages

$$V_1^{f0} = V_{p.f} - Z_{1K}^0 I_K^{f0}$$
$$\vdots$$
$$V_K^{f0} = V_{p.f} - Z_{KK}^0 I_K^{f0}$$
$$\vdots$$
$$V_K^{f0} = V_{p.f} - Z_{nK}^0 I_K^{f0}$$

Positive sequence line current	Negative sequence line current	Zero sequence line current
$I_{ij}^{f+} = \dfrac{V_i^{f+} - V_j^{f+}}{Z_{ij}^+}$	$I_{ij}^{f-} = \dfrac{V_i^{f-} - V_j^{f-}}{Z_{ij}^-}$	$I_{ij}^{f0} = \dfrac{V_i^{f0} - V_j^{f0}}{Z_{ij}^0}$

Phase voltages

$$[V_p] = [T][V_s]$$

Phase current

$$[I_p] = [T][I_s]$$

Print all results

End

> **STABILITY ANALYSIS**
>
> Importance of stability analysis in power system planning and operation - classification of power system stability - angle and voltage stability – simple treatment of angle stability into small-signal and large-signal (transient) stability Single Machine Infinite Bus (SMIB) system: Development of swing equation - equal area criterion - determination of critical clearing angle and time by using modified Euler method and Runge-Kutta second order method. Algorithm and flow chart.

PART – A

Importance of stability analysis in power system planning and operation- classification of power system stability - angle and voltage stability

1. Define stability and power system stability.

The stability of a system is defined as the ability of power system to return to stable operation when it is subjected to a disturbance.

Power system stability (M/J 07)

Power system stability is the property of the system that enables it to remain in a state of operating equilibrium under normal operating conditions and to regain an acceptable state of equilibrium after being subjected to a disturbance.

2. How power system stability is classified?

- Angle stability
- Voltage stability.
- Small signal stability
- Large signal stability
- Mid term stability
- Long term stability
- Transient stability
- Oscillatory stability
- Non- Oscillatory stability

3. How the stability studies are classified, what are they?

Depending on the nature of disturbance the stability can be classified into the following three types,

i) Steady state stability

ii) Dynamic stability

iii) Transient stability

4. How do you classify steady state stability limit. Define them.

Depending on the nature of the disturbance, the steady state stability limit is classified into,

- Static stability limit
- Dynamic stability limit

Static stability limit refers to steady state stability limit that prevails without the aid of regulating devices.

Dynamic stability limit refers to steady state stability limit prevailing in an unstable system with the help of regulating devices such as speed governors, voltage regulators, etc.

5. Define steady state stability and Steady state stability limit . (A/M 10)

It is the ability of the power system to bring it to a stable condition after a small disturbance such as gradual infinitesimal variations in system variables like rotor angle, voltage, etc

Steady state stability limit (Nov/Dec- 14)

When the load on the system is increased gradually, maximum power that can be transmitted without losing synchronism is termed as steady state stability limit. In steady state, the power transferred by synchronous machine of a power system is always less than the steady state stability limit.

6. What is rotor angle stability and voltage stability?

Rotor angle stability

- Rotor angle stability is the ability of inter-connected synchronous machines of a power system to remain in synchronism
- Torque balance of synchronous machines (Input turbine and output generator)

Voltage stability

It is the ability of a power system to maintain steady acceptable voltages at all buses in the system under normal operating conditions and after being subjected to a disturbance.

7. State the causes of voltage instability.

A system enters a state of voltage instability when a disturbance, increase in load

demand, or change in system condition causes a progressive and uncontrollable drop in voltage

The main factor causing instability is the inability of the power system to meet the demand for reactive power.

8. What is small signal stability and how it is analyzed?

- It is concerned with the maintenance of stability of a synchronous machine or a group of synchronous machines when subjected to a small disturbance.
- Analyzed by – Linearizing the differential equation that describe the swing of the machines around an operating point determined by initial power flow voltage conditions.

9. Define transient stability and dynamic stability. (May/June -14)

Transient stability

It is the ability of the system to bring it to a stable condition after a large disturbance. Large disturbance can occur due to the occurrence of fault, sudden outage of a line, sudden loss of excitation, sudden application or removal of loads, etc.

Dynamic stability

It is the ability of a power system to remain in synchronism after the initial swing (transient stability period) until the system has settled down to the new steady state equilibrium condition

10. Define Transient stability limit: (M/J 12)

The maximum power which can be transmitted between the given pair of buses such that the system does not become unstable when it is subjected to a specified sudden large disturbances under specified initial condition.

(or)

When the load on the system is increased suddenly, maximum power that can be transmitted without losing synchronism is termed as transient state stability limit. Normally, steady state stability limit is greater than transient state stability limit.

11. Write any three assumptions upon transient stability.

The assumptions for transient stability are given as follows.

- Rotor speed is assumed to be synchronous. In fact, it varies insignificantly

during the course of the stability study.

- Shunt capacitances are not difficult to account for in a stability study.
- Loads are modeled as constant admittances.

12. How to improve the transient stability limit of power system?

The transient stability limit of power system can be improved by following methods.

- Increase of system voltages
- Use of high speed excitation systems.
- Reduction in system transfer reactance
- Use of high speed reclosing breakers.

13. What are the numerical integration methods of power system stability?

The numerical integration methods of power system stability are as follows.

- Point by point method or step by step method
- Euler method
- Modified Euler method
- Runge-Kutta method(R-K method)

14. What are the machine problems seen in the stability study?

The major problem seen in the machine during the stability study are given as follows

- Those having one machine of finite inertia machines swinging with respect to an infinite bus
- Those having two finite inertia machines swinging with respect to each other.

15. What are the causes of oscillatory and non-oscillatory instabilities in power systems? (A/M 10)

The causes of oscillatory and non-oscillatory instabilities in power systems are as follows.

Oscillatory :
- Due to insufficient damping torque
- Due to unstable control action

Non – oscillatory :
- Due to insufficient synchronous torque

16. Differentiate between voltage stability and rotor angle stability. (N/D 2013)

Voltage Stability	Rotor angle stability
Ability of power system to maintain steady acceptable voltages at all buses in power system under normal operating conditions and after being subjected to a disturbance.	Ability of inter – connected synchronous machines of a power system to remain in synchronism.
Reactive power balance	Torque balance of synchronous machines

Simple treatment of angle stability into small-signal and large-signal (transient) stability

Single Machine Infinite Bus (SMIB) system

17. State the assumptions made in stability studies.

The assumptions made for stability studies are listed as follows.

- Machines represents by classical model
- The losses in the system are neglected (all resistance are neglected)
- The voltage behind transient reactance is assumed to remain constant.
- Controllers are not considered (Shunt and series capacitor)
- Effect of damper winding is neglected.

18. Give the control schemes included in stability control techniques.

The control schemes included in the stability control techniques are:

- Excitation systems
- Turbine valve control
- Single pole operation of circuit breakers
- Faster fault clearing times

19. What are the assumptions that are made in order to simplify the computational task in stability studies?

The assumptions are,

- The D.C offset currents and harmonic components are neglected. The currents and voltages are assumed to have fundamental component alone.
- The symmetrical components are used for the representation of unbalanced faults.
- It is assumed that the machine speed variations will not affect the generated voltage.

20. Explain the concept synchronous speed.

The mechanical torque Tm and the electrical torque Te are considered positive for synchronous generator. Tm is the resultant shaft torque which tends to accelerate the rotor in the positive θ_m direction of rotation . Under steady-state operation of the generator Tm and Te are equal and the accelerating torque Ta is zero. Hence there is no acceleration of deceleration of the rotor, masses and the resultant constant speed is the synchronous speed.

21. Write the power angle equation of a synchronous machine connected to an infinite bus and also the expression for maximum power transferable to the bus.

$$P_1 = P_e = |E'|G_{11} + \frac{|E'||V|}{X_{12}} \sin \delta$$

$$P_e = P_C + P_{max} \sin \delta$$

This equation is called power angle equation

$$M_{eq} = \frac{M_1 S_1}{S_b} + \frac{M_2 S_2}{S_b}$$

Expression for Maximum Power transfer:-

$$P_{max} = \frac{|E'||V|}{X_{12}} \sin \delta$$

22.Define infinite bus in a power system. (M/J 13,08)

The substation bus voltage and frequency is assumed to remain constant. This is called as infinite bus, since its characterized do not change regardless of the power supplied or consumed by any device connected to it.

23. What is meant by power angle curve? (M/J 13)

$$P_e = \frac{|E'||V|}{X_{12}} \sin \delta = P_{max} \sin \delta$$

Power transmitted depends on the transfer reactance X_{12} and the angle between the voltages E' and V i.e., (δ). The curve P_C versus δ is known as power angle curve.
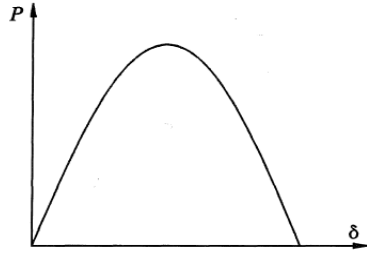
24. Define small disturbance and large disturbance.

Small disturbance

The disturbances that cause small and gradual infinitesimal variation in system variables such as rotor angle, voltage, etc., are classified as small disturbance.

Large disturbance

- Any disturbance that causes a change in the admittance matrix encountered in stability is classified as large disturbance

  Eg – Fault occurring, removal of line

25. What is meant by synchronism and damping torques?

The component of electrical torque proportional to rotor angle deviation from initial value is referred to as synchronizing torque and that proportional to speed deviation from initial value is called damping torque.

Assume initially, the machine is in steady state, then

Machine speed = Rated speed = synchronous speed

Then Initial speed deviation = 0

26. State the causes of voltage instability.

The various causes for voltage instability are listed as follows.

- At the time of disturbance occurs
- Increase in load demand
- Inability of power system to meet the demand for reactive power

- Voltage drop occurs when active power and reactive power flow through inductive load

27. State the assumption made in stability studies .

The assumptions made in stability studies are stated as follows.

- Machine represented by classical model
- Controllers are not considered
- Loads are constants
- Voltage and current are sinusoidal.

28. Write the expression for maximum power transfer.

$$P_{\max} = \frac{|E'||V|}{X_{12}} \sin \delta$$

$where\ X_{12} = Transient\ reac \tan ce$

$\qquad E' = Transient\ int\ ernal\ source\ voltage.$

$\qquad V = Infinite\ bus\ voltage.$

29. What are the methods to improve steady state stability.

- Reduce the reactance. Steady state stability limit can be improved by using two parallel lines which increases reliability of the system.
- Increase either of both |E| and |V|. Series capacitors are included in lines to get better voltage regulation by decreasing X.
- Higher excitation voltages and quick excitation system are also employed.

30. What is synchronizing power coefficient?

The quantity $P_S = P_{\max} \cos\delta_0$ is the slope of the power angle at δ_0

$P_{e=} P_{\max} \sin \delta_0$

$$P_s = \frac{dP_e}{d\delta}\bigg|_{\delta_0} = P_{\max} \cos \delta_0 = \frac{E'V \cos \delta_0}{X}$$

$where\ P_s\ is\ known\ as\ synchroni \sin g\ power.$

Co-efficient or stiffness of synchronous machine.

31. What are the assumptions made to simplify the transient stability problem?

- Neglecting the saliency of synchronous machine

  $X_d = X_q$

  $X'_d = X'_q$
- Synchronous machine are represented by constant terminal voltage
- Neglecting governor action for turbine
- Resistances are neglected
- Damping is neglected
- Loads are represented by common admittance.

32. What are the assumptions made to solve swing equation?

- Mechanical power input P_m is constant during the period of electromechanical transient
- Rotor speed changes are insignificant
- The generated machine e.m.f remains constant.
- Effect of voltage regulation loop is neglected.

33. Difference between steady state and transient state

Steady State	Transient State
• A power system is in steady state means all the measured quantities are in operating condition • Steady state – when it occurs disturbance, and it returns to same steady state condition. • Analysed by linear equation, non linear equation are replaced by linear equation.	• A power system is in transient state if the measured quantities are not in constant. • Transient state – large disturbance occurs change in operating condition occurs. • Analysed by using non linear equation

34. What is the importance of stability analysis in power system planning and operation?

- Transient stability studies give the information of magnitude of voltage and frequency
- It deals with the stability of the system
- Transient stability steady is needed when the new generating station and transmission facilities are planned
- Stability system is need to determine the nature of relaying system, critical clearing time of circuit breaker.
- More helpful in determining power transfer capability between two difference systems.

35. What are the different mode of small signal stability?

- Local plant mode
- Inter area mode
- Control mode
- Torsional mode

36. What is meant by local modes and Inter-area mode ?

local modes

Local modes are associated with swinging of unit at a generating station with respect to the rest of the power system. The term local is used because the oscillation are localized at one station or a small part of the power system.

Inter-area mode

Inter area mode is associated with the swinging of many machines in one part of the system against machines in other parts. They are caused by two or more groups of closely coupled machines being interconnected by weak ties.

37. What is meant by control modes and Torsional mode?

Control modes

Control modes are associated with generating units and other controls. Poorly tuned exciters, speed governor, HVDC converters and statics VAR compensators are the usual causes of instability of these modes.

Torsional mode

Torsional modes are associated with the turbine – generator shaft system rotational components. Instability of torsional modes may be caused by interaction with excitation controls, speed governors, HVDC controls, and series capacitor-compensated lines.

38. Write down the units of inertia constants M and H and their interrelationship.

The unit of M is MJ-s/elec.rad or MJ-s/mech-rad.

The unit of H is MJ/MVA or MW-s/MVA

The M and H are related by the equation,

$$M = \frac{HS}{\pi f}$$

Where, S = MVA rating of Machine

F = Frequency in Hz

39. If two machines are swinging coherently with inertia M_1 and M_2 what will be the inertia of the equivalent machine?

The equivalent moment of inertia,

$$M_{eq} = \frac{M_1 S_1}{S_b} + \frac{M_2 S_2}{S_b}$$

Where, S_1 & S_2 = MVA rating of machine 1 & 2 respectively

S_b = Base MVA or MVA rating of system.

40. What are the systems design strategies aimed at lowering system reactance?

The system design strategies aimed at lowering system reactance are:

- Minimum transformer reactance
- Series capacitor compensation of lines
- Additional transmission lines.

41. List the method of improving the transient stability limit of a power system.

The various methods to improve transient stability limit are given as follows.

- Increase of the system voltage and use of AVR

- Use of high speed excitation systems.

- Reduction in system transfer reactance

- Use of high speed reclosing

Swing equation

42. What is Multimachine stability?

If a system has any number of machines, then each machine is listed for stability by advancing the angular position, δ of its internal voltage and noting whether the electric power output of the machine increases (or) decreases. If it increases,

i.e if ∂Pn / ∂δn > 0

then machine n is stable. If every machine is stable, then the system having any number of machine is stable.

43. List the assumptions made in multimachine stability studies.

The assumptions made are,

- The mechanical power input to each machine remains constant during the entire period of the swing curve computation

- Damping power is negligible

- Each machine may be represented by a constant transient reactance in series with a constant transient voltage.

- The mechanical rotor angle of each machine coincides with δ , the electrical phase angle of the transient internal voltage.

44. Define swing curve. What is the use of this curve? (N/D 2013)

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_{m(p.u)} - P_{e(p.u)}$$

The graphical display of δ versus *t* is called the swing curve. The plot of swing curves of all machines tells us whether machines will remain in synchronism after a disturbance.

The swing curve is the plot or graph between the power angle δ, and time, t. It is usually plotted for a transient state to study the nature of variation in δ for a sudden large disturbance.

From the nature of variations of δ, the stability of a system for any disturbance can be determined.

45. Give an example for swing equation. Explain each term along with their units. (N/D 11)

Eg : Turbo generator, water wheel generator, etc

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2}=P_{m(p.u)}-P_{e(p.u)}$$

$$Where \quad H = p.u.\, inertia\, cons\tan t$$
$$f = Frequency$$
$$\delta = Power\, angle$$

46. Write swing equation. (A/M 11)

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2}=P_{m(p.u)}-P_{e(p.u)}$$

$$Where \quad H = p.u.\, inertia\, cons\tan t$$
$$f = Frequency$$
$$\delta = Power\, angle$$

47. Plot the swing curve.
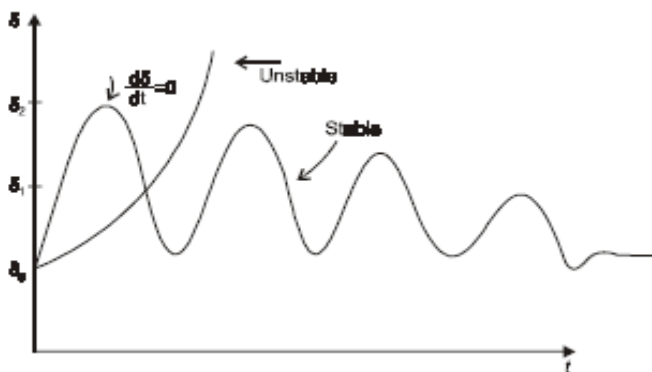
48. What are the assumptions made in solving swing equation?

- Mechanical power input to the machine remains constant during the period of electromechanical transient of interest.
- Rotor speed changes are insignificant that had already been ignored in formulating the swing equations.
- Effect of voltage regulating loop during the transient.

49. Write the swing equation and explain the terms involved in it. (N/D 07)

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_{m(p.u)} - P_{e(p.u)}$$

$Where \quad H = p.u.\,inertia\,cons\tan t$

$\qquad f = Frequency$

$\qquad \delta = Power\,angle$

Equal area criterion

50. State equal area criterion. (N/D 11) (M/J 09, 07)

- The equal area criterion for stability states that the system is stable if the area under $P_a - \delta$ curve reduced to zero at some value of δ.

- This is possible if the positive (acceleration) are under $P_a - \delta$ curve is equal to the negative (deceleration) area under $P_a - \delta$ curve for a finite change in δ. Hence the stability criteria is called equal area criterion.

51. Draw the figure for equal area criterion
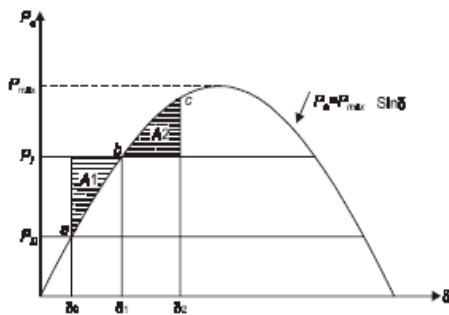
52. What are various faults that increase severity of equal area criterion?

The various faults that increases severity of equal area criterion are,

- Single line to ground fault
- Line to line fault
- Double line to ground fault
- Three phase fault

53. State the application of equal area criterion.

We apply the equal area criterion to two different systems of operation

i) Sustained line fault

ii) line fault cleared after sometime by the simultaneous tripping of the breakers at both the end

54. List the types of disturbances that may occur in a single machine infinite bus bar system of the equal area criterion stability

The types of disturbances that may occur are,

- Sudden change in mechanical input
- Effect of clearing time on stability
- Sudden loss of one of parallel lines
- Sudden short circuit on one of parallel lines
    - ➤ Short circuit at one end of line
    - ➤ Short circuit away from line ends
    - ➤ Reclosure of lines.

Determination of critical clearing angle and time

55. Define critical clearing angle. (A/M 11)

The critical clearing angle, is the maximum allowable change in the power angle δ before clearing the fault, without loss of synchronism. The time corresponding to this angle is called critical clearing time, t_{cc}. It can be defined as the maximum time delay that can be allowed to clear a fault without loss of synchronism.

56. Define critical clearing time and critical clearing angle. (N/D- 14,12,08) (M/J – 12)

Critical clearing angle :

For any given initial load in the case of a fault clearance on a synchronous machine connected to an infinite bus bar, there is a critical clearing angle. If the actual clearing angle is greater than the critical value, the system is unstable, other wise the system is stable. Maximum allowable angle for a system to remain stable.

Critical clearing time :

Maximum allowable time for a system to remain stable are known as critical clearing time.

57. Give the expression for critical clearing time. (N/D 07)

Critical time margin = critical clearing time – clearing time specified

$$= t_{cr(critical)} - t_{spec}$$
$$where$$
$$t_{spec} = Specified\ clearing\ time$$

58. On what basis do you conclude that a given synchronous machine has lost stability? (A/M 08)

Unstable System : If the system is unstable, δ continues to increase with time and the machine loses synchronism.

From this we can easily able to conclude that the given synchronous machine has lost its stability.

59. What are coherent machines? (APR/MAY 2004)

Machines which swing together are called coherent machines. When both ωs and δ are expressed in electrical degrees or radians, the swing equations for coherent machines can be combined together even though the rated speeds are different. This is used in stability studies involving many machines.

60. What will happen if there a loss of excitation?

- It operates as an induction generator running above synchronous speed.
- The excitation is supplied from the power system and hence the machine draws reactive power from the system.
- It may cause severe system voltage reductions.
- The stator current may 2 to 3 times full load current causing excessive stator heating.

61. Define inertia constant (M).

M-Constant or inertia constant is defined as the angular momentum at synchronous speed. If energy is measured in Joules and speed in mechanical radians per second. Unit of M is Joule-sec/Mechanical radian.

62. What happen if there exist of load rejection?

When there is a load rejection in the system, the speed of the generators will increase suddenly and hence the system frequency will rise. The speed governing systems will respond by reducing the mechanical power generated by the turbines.

63. Draw the equivalent circuit model of SMIB.

(a) Generator model   (b) Equivalent circuit
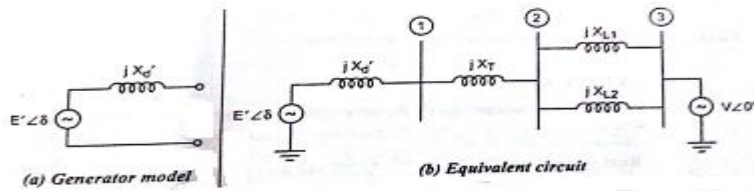
64. Draw the synchronous machine represented in classical model.

Part – B

1.Explain the equal area criteria for the following application : (16) (N/D'14)

    i.   **Sustained fault**

    ii.   **Fault with subsequent clearing.**

A three phase fault is occurred at point F of the outgoing radial line at bus 1 is shown in figure. The accelerating area A_1 begins to increase and point moves along *bc*. At time t_c (clearing time) corresponding 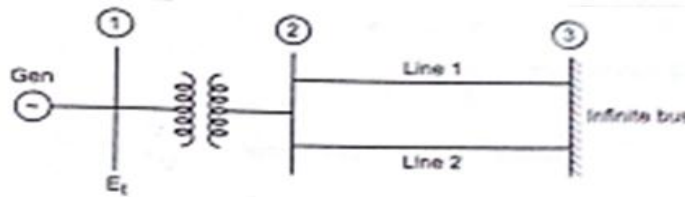to angle δc (clearing angle), the faulted line is cleared by opening of the circuit breaker. The rotor is now decelerated and the decelerating area A_2 begins, while the point moves along *de* and the path is retraced along the curve.

(6)
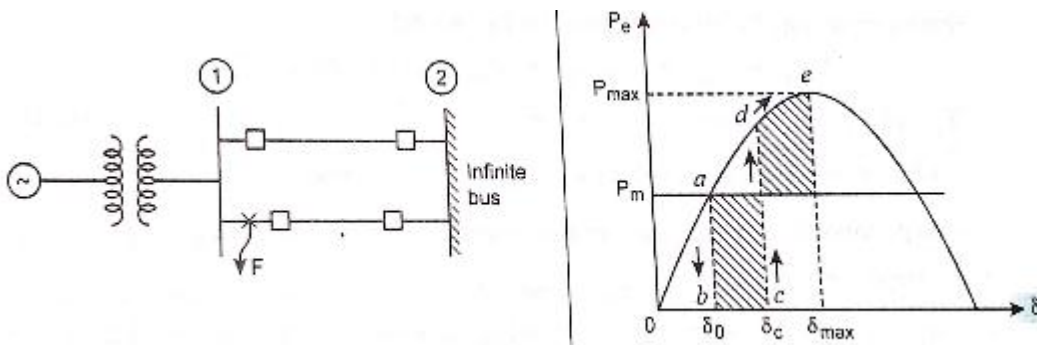
If an angle $δ_1$ can be found that area A_1 = Area A_2 the system is found to be stable. The system finally settles down to be the steady state operating point at *a* in an oscillatory manner because of inherent damping. At point

Prefault condition: (2)

Power angle equation is given by

$$P_{e1} = \frac{|E'||V|}{X_d' + \left[\dfrac{X_1 X_2}{X_1 + X_2}\right]} \sin\delta = P_{max1}\sin\delta$$

During Fault condition :

The generator gets isolated from power system for purpose of power flow as shown in figure.
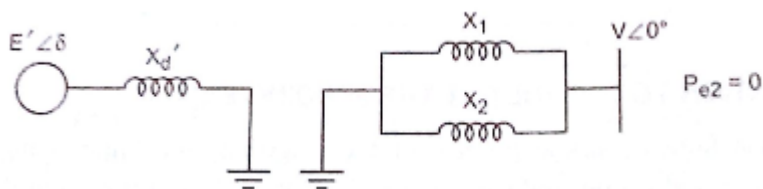
(4)

Post fault condition:

The circuit breaker at two ends of the faulted line open at time t_{cr} disconnecting the faulted line. The circuit is as shown in figure.

Power angle equation is given by

(4)
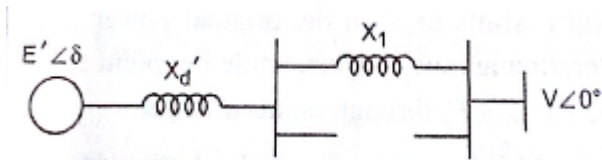
$$P_{e3} = \frac{|E'||V|}{X_d' + X_1} \sin\delta = P_{max3}\sin\delta$$

2.Derive the swing equation from the basic principles. Why it is non-linear?

(16)(N/D'14) (M/J'07,14)

Let T_m be the driving mechanical torque (6)

T_e be the electrical torque

The motor action and generator action is shown in figure.

For generator action, T_m and T_e are positive

θ_m is positive

(a) Motor action | (b) Generator action
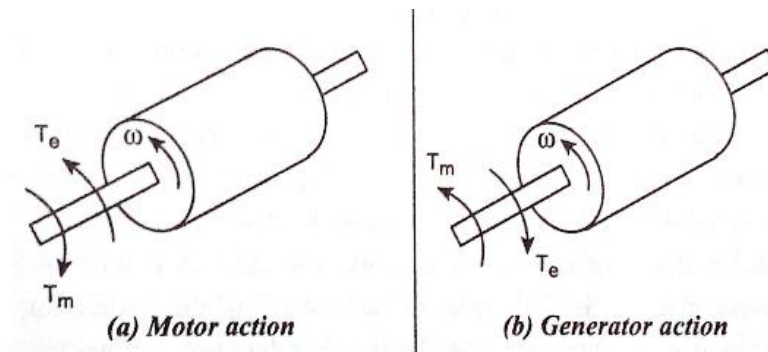
Under steady state with losses neglected.

$$T_m = T_e$$

Acceleration torque $T_a = T_m - T_e = 0$

i.e. no accelerating torque

$$T_a = T_m - T_e$$

Let J be the moment of inertia of the prime mover and generator.

From Law's of rotation,

$$Acceleration\ \alpha = \frac{d^2\theta_m}{dt^2}$$

$$Acceleration\ torque\ T_a = J\alpha$$

$$\therefore J\frac{d^2\theta_m}{dt^2} = T_m - T_e \qquad (1)$$

Where θ_m is the angular displacement of the rotor with respect to the stationary reference axis on stator. (5)

θ_m increase with time even at constant synchronous speed.

$$\therefore \theta_m = \omega_{sm}t + \delta_m \qquad (2)$$

$$where\ \delta_m = Angular\ displacement\ of\ rotor$$

$$before\ disturbance\ in\ mechancal\ radians.$$

$$\omega_{sm}t = Cons\tan t\ angular\ velocity$$

$$Diff.eq.(2), wrt\ t, we\ get$$

$$\omega_m = \frac{d\theta_m}{dt} = \omega_{sm}t + \frac{d\delta_m}{dt} \qquad (3)$$

Diff .eq.(3), wrt t, rotor acceleration is

$$\frac{d^2\theta_m}{dt^2} = \frac{d^2\delta_m}{dt^2}$$

Substituting in eq(1) we get,

$$J\frac{d^2\delta_m}{dt^2} = T_m - T_e$$

Multiplying by ω_m on both side,

$$J\omega_m\frac{d^2\delta_m}{dt^2} = \omega_m T_m - \omega_m T_e$$

Inertia constant

$M = J\omega_m$ *is the inertia cons*tan*t*

i.e.Angular momentum of the rotor at synchronous speed.

$$M\frac{d^2\delta_m}{dt^2} = P_m - P_e \quad (P = \omega T) \qquad (4)$$

p.u. Inertia Contant

Kinetic Energy of rotating masses $W_K = \frac{1}{2}J\omega_m^2$

Stored kinetic energy in mega joules of turbine,

$$p.u.of\ H = \frac{alternator\ and\ exciter\ rotor\ at\ synchronous\ speed}{Machine\ rating\ in\ MVA}$$

$$H = \frac{\frac{1}{2}J\omega_{sm}^2}{S_B}\sec$$

$$J\omega_{sm}^2 = \frac{2HS_B}{\omega_{sm}} = M \qquad\qquad (6)$$

Substituting in eq(4)

$$\frac{2HS_B}{\omega_{sm}}\frac{d^2\delta_m}{dt^2} = P_m - P_e \qquad (5)$$

by solving we get

$$\frac{HS_B}{\pi f}\frac{d^2\delta_m}{dt^2} = P_m - P_e \qquad (6)$$

Dividing by MVA rating S_B on both side we get,

$$\frac{H}{\pi f} \frac{d^2 \delta_m}{dt^2} = \frac{P_m}{S_B} - \frac{P_e}{S_B}$$

$$\frac{P_m}{S_B} = p.u. mechanical\ power$$

$$\frac{P_e}{S_B} = p.u. electrical\ power$$

$$\frac{H}{\pi f} \frac{d^2 \delta_m}{dt^2} = P_{m(p.u)} - P_{e(p.u)} = P_{m(p.u)} - P_{max} \sin \delta \quad (7)$$

$$M_{(p.u)} \frac{d^2 \delta}{dt^2} = P_{m(p.u)} - P_{e(p.u)}$$

where $M_{(p.u)} = \dfrac{H}{\pi f}$ *,* δ *in radians*

If δ *expressed in electrical degrees,*

$$\frac{H}{180 f} \frac{d^2 \delta}{dt^2} = P_{m(p.u)} - P_{e(p.u)}$$

These equation are called as **Swing Equation**

3. Describe the algorithm for modified Euler method of finding solution for power system stability problem studies. (16) (M/J'14)

Numerical integration techniques can be applied to obtain approximate solutions of non-linear differential equations.   (5)

Consider a generator connected to an infinite bus through two parallel lines and a 3ϕ fault occurs at the middle of line 2 as shown in figure.

Let P_m be the input power which is a constant.

Prefault Condition: Under steady state operation,

Power transfer from generator to an infinite bus,

    $P_e = P_m$

$$\frac{E'V}{X_1}\sin\delta_0 = P_{max1}\sin\delta_0 = P_m$$

$$\sin\delta_0 = \frac{P_m}{P_{max1}} \Rightarrow \delta_0 = \sin^{-1}\left[\frac{P_m}{P_{max1}}\right] \qquad (1)$$

$$where\ P_{max1} = \frac{E'V}{X_1}$$

$X_1 = Transfer\ reac\tan ce\ for\ the\ prefault\ condition$

The rotor is running at synchronous speed,

$$\omega_0 = 2\pi f$$

Change in angular velocity is zero.

$$i.e., \Delta\omega_0 = 0$$

$\qquad(5)$

During the fault : Consider a 3ϕ fault occurs at the middle of one line 2 as shown in fig.

$$P_{e2} = \frac{|E'||V|}{X_{II}}\sin\delta_1 = P_{max2}\sin\delta$$

$$where\ P_{max2} = \frac{|E'||V|}{X_{II}}$$

$X_{II} = Transef\ reac\tan ce\ during\ the\ fault$

the swing eq.is given by,

$$\frac{d^2\delta_m}{dt^2} = \frac{\pi f}{H}\left[P_m - P_{max2}\sin\delta\right] = \frac{\pi f}{H}P_a$$

the above eq.can be transformed into the state variable form,

$$\frac{d\delta^{(1)}}{dt} = \Delta\omega \qquad (2)$$

$$\frac{d^2\delta_m}{dt^2} = \frac{d\Delta\omega^{(1)}}{dt} = \frac{\pi f}{H}P_a \quad (3)$$

Compute the first estimate at $t_1 = t_0 + \Delta t.$

$$\delta_{i+1}^P = \delta_i + \left.\frac{d\delta^{(1)}}{dt}\right|_{\Delta\omega_i}.\Delta t \qquad (4) \qquad\qquad (6)$$

$$\Delta\omega_{i+1}^P = \Delta\omega_i + \left.\frac{d\Delta\omega^{(1)}}{dt}\right|_{\delta_i}.\Delta t \qquad (5)$$

Compute the derivatives : Using the predicted value, determine the derivatives at the end of iteration.

$$\left.\frac{d\delta^{(2)}}{dt}\right|_{\Delta\omega_{i+}^P} = \Delta\omega_{i+1}^P \qquad (6)$$

$$\left.\frac{d\Delta\omega^{(2)}}{dt}\right|_{\delta_{i+1}^P} = \frac{\pi f}{H} P_a\Big|_{\delta_{i+1}^P} \qquad (7)$$

Computing the final estimated corrected value,

$$\delta_{i+1}^C = \delta_i + \left[\frac{\left.\frac{d\delta}{dt}\right|_{\Delta\omega_i} + \left.\frac{d\delta}{dt}\right|_{\Delta\omega_{i+1}^P}}{2}\right]\Delta t \qquad (8)$$

$$\Delta\omega_{i+1}^C = \Delta\omega + \left[\frac{\left.\frac{d\Delta\omega}{dt}\right|_{\delta_i} + \left.\frac{d\Delta\omega}{dt}\right|_{\delta_{i+1}^P}}{2}\right]\Delta t \qquad (9)$$

4. Explain the methods of improving power system stability. (16) (N/D'13,11)

Methods of Improving transient Stabiltiy:-   (4)

- Reducing in the disturbing influence by minimizing the fault severity and duration

- Increasing the restorating synchronizing forces.

- Reduction of acceleration torque through control of prime mover mechanical power.

- Reduction of accelerating torque by applying artificial load.

Traditional approach to transient stability problems.   (6)

- Increasing system voltage by using automatic voltage regulator.
- Using high speed excitation system to increase the voltage profile.
- Reducing the transfer reactance.
- Using high speed reclosing breakers. (employing single-pole operation of reclosing circuit breakers).
- Reducing inertia constant.
- Single pole operation of reclosing circuit breakers
- Use of bundled conductors
- High speed fault clearing
- Increasing no .of parallel lines between points
- Regulated shunt compensation
- Dynamic breaking
- Single pole switching
- Generator tripping.

Increasing system voltage by AVR

When a fault occurs, the bus voltages are reduced. In generator terminals, the terminal voltage is maintained by the automatic voltage regulators or by using high speed excitation system.

Reducing the transfer reactance: (3)

Maximum power transfer, $P_{max} = \dfrac{\left|E^{'}\right|\left|V\right|}{X}$

By reducing reactance, system voltage profile increases and P_{max} increases.

$$\text{Inductance } L = 0.2 \ln\left[\frac{D}{r^{'}}\right]$$

Where D = spacing

r' = Geometric mean radius

Reactance $X = \omega L$

Reactance can be decreased by reducing conductor spacing or by increasing conductor diameter.

Series reactance may be reduced by using bundled conductors.

For long transmission lines, series capacitors are added to the line for compensation is used to reduce reactance and increase the stability limit.

Switched series capacitors decreases load voltage fluctuations and raise the transient stability limit almost equal to steady state stability limit.

Transfer reactance is reduced by increasing the number of parallel lines.

Using High speed reclosing breakers: (3)

Most of the faults are transient in nature. Rapid switching and isolation of unhealthy lines followed by reclosing is used to improve the stability margin.

Most occurring fault like L.C. fault, the use of single pole opening and reclosing improves stability limit.

There will be a definite power transfer because one line is opened during the fault. Power transfer takes place in other two lines. But using these poles switching, the power transfer becomes zero.

Recent trends :-

- HVDC links

- Use of breaking resistors
- Short circuit current limiters
- Turbine fast valving of bypassing valve
- Full load rejection technique.

5. Explain the term critical clearing angle and critical clearing time in connection with the transient stability of a power system. (16) (A/M'11)(N/D'07,13)

Obviously $P_{max2} < P_{max1}$ (8)

       The critical clearing angle is reached when any further increase in δ_c causes the Area A_2 < Area A_1. This occurs when δ_{max} or point *e* is at the intersection of line P_m and curve P_e as shown in figure.
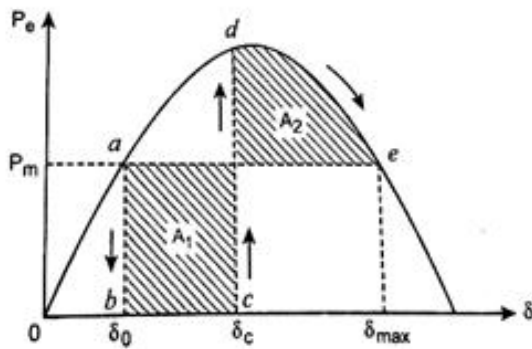
       Apply equal area criterion. Area A_1 = Area A_2

$$\int_{\delta_0}^{\delta_c} P_m d\delta = \int_{\delta_c}^{\delta_{max}} (P_{max} \sin \delta - P_m) d\delta$$

$$P_m \left[\delta\right]_{\delta_0}^{\delta_c} = P_m(-\cos \delta) - P_m \delta]_{\delta_c}^{\delta_{max}}$$

solving for δ_c, *we get*

$$P_{max} \cos \delta_c = P_m(\delta_{max} - \delta_0) + P_m \cos \delta_{max}$$

Dividing by P_{max},

$$\cos \delta_c = \frac{P_m}{P_{max}}(\delta_{max} - \delta_0) + \cos \delta_{max}$$

For a stable system,

$$\cos \delta_c = \frac{P_m}{P_{max}}(\delta_{max} - \delta_0) + \cos \delta_{max} \qquad (1)$$

During a 3φ fault, $P_e = 0$, therefore the swing equation becomes $\qquad$ (8)

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_m$$

$$\frac{d^2\delta}{dt^2} = \frac{\pi f}{H}P_m$$

Intergrating both side,

$$\frac{d^2\delta}{dt^2} = \frac{\pi f}{H}P_m\int_0^t dt = \frac{\pi f\, P_m t}{H}$$

$$At\ \delta = \frac{\pi f\, P_m}{H}\int_0^t t\,dt = \frac{\pi f\, P_m t^2}{2H} + \delta_0$$

$$\delta = \delta_{cr}, t = t_{cr}$$

$$\therefore \delta_{cr} = \frac{\pi f\, P_m t_{cr}^2}{2H} + \delta_0 \qquad (2)$$

$$t_{cr}^2 = \frac{2H}{\pi f P_m}(\delta_{cr} - \delta_0)$$

$$t_{cr} = \sqrt{\frac{2H}{\pi f\, P_m}(\delta_{cr} - \delta_0)} \qquad (3)$$

Where H= p.u. inertia constant.

$\qquad$ f= frequency

$\qquad$ P_m= Mechanical Power

$\qquad$ δ_{cr} = Critical clearing angle

$\qquad$ δ_0 = Rotor angle

6. A generator is operating at 50 Hz delivers 1 p.u. power to an infinite bus through a transmission circuit in which resistance is ignored. A fault takes place reducing the maximum power transferable to 0.4 p.u., whereas before the fault, this power was 1.6 p.u. and after the clearance of the fault, it is 1.2 p.u. By the use of equal area criterion, determine the critical clearing angle. (16) (N/D'11)

Solution:

The power angle curve is as shown in fig. (6)

P_{e1}= 1.6 sin δ (5)

P_{max1}= 1.6 p.u.

P_{e2}= 0.4 sin δ

P_{max2}= 0.4 p.u.

P_{e3}= 1.2 sin δ

P_{max3}= 1.2 p.u.

Initial loading P_m = sin $δ_0$ = 1/ 1.6

$$\delta_0 = \sin^{-1}\left[\frac{1}{1.6}\right] = 0.675\, rad$$

$$\delta_{max} = \pi - \sin^{-1}\left[\frac{P_m}{P_{max3}}\right]$$

$$= \pi - \sin^{-1}\left[\frac{1}{1.2}\right] = 2.156\, rad$$

Applying equal area criterion, (5)

Area A_1 = Area A_2

$$P_m(\delta_{cr} - \delta_0) - \int_{\delta_0}^{\delta_{cr}} P_{e2}\, d\delta = \int_{\delta_{cr}}^{\delta_{max}} P_{e3}\, d\delta - P_m(\delta_{max} - \delta_{cr})$$

$$\Rightarrow \cos\delta_{cr} = \frac{P_m(\delta_{max} - \delta_{cr}) - P_{max2}\cos\delta_0 + P_{max3}\cos\delta_{max}}{P_{max3} - P_{max2}}$$

$$= \frac{1.0(2.156 - 0.675) - 0.4\cos 0.675 + 1.2\cos 2.156}{1.2 - 0.4}$$

$$\cos\delta_{cr} = 0.632\, rad$$

$$\delta_{cr} = \cos^{-1} 0.632 = 0.887\, rad = 50.82°$$

7. Derive the swing equation of a single machine connected to an infinite bus system and explain the steps of solution by Runge-Kutta method. (16) (N/D '08,11)

Runge – Kutta method

The following steps involved in Runge-Kutta method to determine stability.

I estimates :

$$K_1 = \frac{d\delta}{dt}\bigg|_{\Delta\omega_i} \times \Delta t = \Delta\omega_i \times \Delta t \qquad (1)$$

$$l_1 = \frac{d\Delta\omega}{dt}\bigg|_{\delta i} \times \Delta t = \frac{\pi f}{H}\Big[P_m' - P_{e(\delta i)}\Big] \times \Delta t \qquad (2)$$

II estimates :

$$K_2 = \left[\Delta\omega_i + \frac{l_1}{2}\right]\Delta t \qquad (3)$$

$$l_2 = \frac{\pi f}{H}\Big[P'_m - P_{e(\delta_i + (K_1/2))}\Big] \times \Delta t \qquad (4)$$

III estimates :

$$K_3 = \left[\Delta\omega_i + \frac{l_2}{2}\right]\Delta t \qquad (5)$$

$$l_3 = \frac{\pi f}{H}\Big[P'_m - P_{e(\delta_i + (K_2/2))}\Big] \times \Delta t \qquad (6)$$

IV estimates :

$$K_4 = (\Delta\omega_i + l_3) \times \Delta t \qquad (7)$$

$$l_4 = \frac{\pi f}{H}\Big[P'_m - P_{e(\delta_i + (K_3))}\Big] \times \Delta t \qquad (8)$$

Final estimates at $t = t_1$

$$\delta_{i+1} = \delta_i + \frac{1}{6}\Big[K_1 + 2K_2 + 2K_3 + K_4\Big] \qquad (9)$$

$$\Delta\omega_{i+1} = \Delta\omega_i + \frac{1}{6}\Big[l_1 + 2l_2 + 2l_3 + l_4\Big] \qquad (10)$$

8.(i). Write the swing equation describing the rotor dynamic of a synchronous machine connected to infinite bus through a double circuit transmission line. (10)

Consider a generator connected to an infinite bus through a double transmission line as shown in fig.
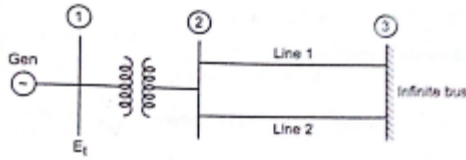
(5)

Infinite Bus

The substation bus voltage and frequency is assumed to remain constant. This is called as infinite bus, since its characteristics do not change regardless of the power supplied or consumed by any device connected to it.

The generator model is shown below and the equivalent circuit diagram also represented a classical model and all resistance are neglected is as shown in figure.

(a) Generator model   (b) Equivalent circuit
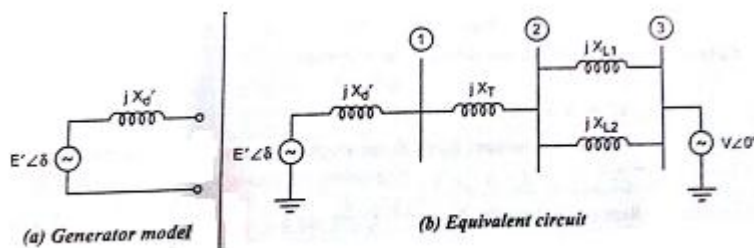
The simplified equivalent circuit is as shown in Fig.
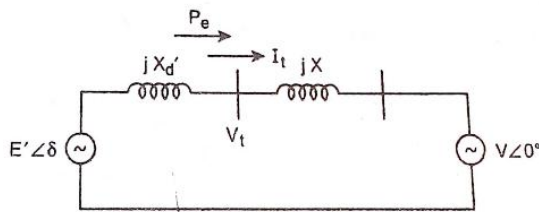
$$Now\ X = \frac{X_{L1} \times X_{L2}}{X_{L1} + X_{L2}} + X_T$$

Let E'_t be the ter min *al voltage magnitude.*

Let P_e, Q_e be the real and reactive power output power (5)

Case (i): Assume V_t as reference.

i.e., $V_t = |V_t| \angle 0°$

*Voltage behind transient reac*tan*ce E'.*

$$E' = V_t + jX'_d I_t \qquad (1)$$

where $I_t = Stator\ current = \dfrac{S*}{V_i*} = \dfrac{P_e - jQ_e}{|V_t| \angle 0°}$

$$= \frac{P_e}{|V_t|} - j\frac{Q_e}{|V_t|} = I_{Re} - j I_{Im} \qquad (2)$$

Sub.eq.(2) *in* (1), *we get*

$$\therefore E' = V_t + jX'_d \left[I_{Re} - j I_{Im} \right] = |E'| \angle \beta \qquad (3)$$

Voltage of inf*inite bus*,

$$V = V_t - jXI_t$$
$$= V_t - jX \left[I_{Re} - j I_{Im} \right] = |V| \angle \gamma \qquad (4)$$

angle between E' and V, $\delta = \beta - \gamma$ \qquad (5)

Voltage at bus (2) *or voltage at high voltage side of transformer.*

$$E_{HV} = V_t - jX_T \times I_t$$

Case (ii) : Assume infinite bus voltage V as reference.

$$V = |V| \angle 0°$$

$$E' = V + jXI_t; E' = |E'| \angle \delta$$

Where δ = Rotor angle with respect to synchronous rotating reference phasor $V \angle 0°$ E'

leads V by δ

Re*al Power transfer* $P_e = \dfrac{|E'||V|}{X} \sin \delta$ \qquad (6)

$$= P_{max} \sin \delta$$

8.(i). Sketch the classifications of Power system stability. (6)
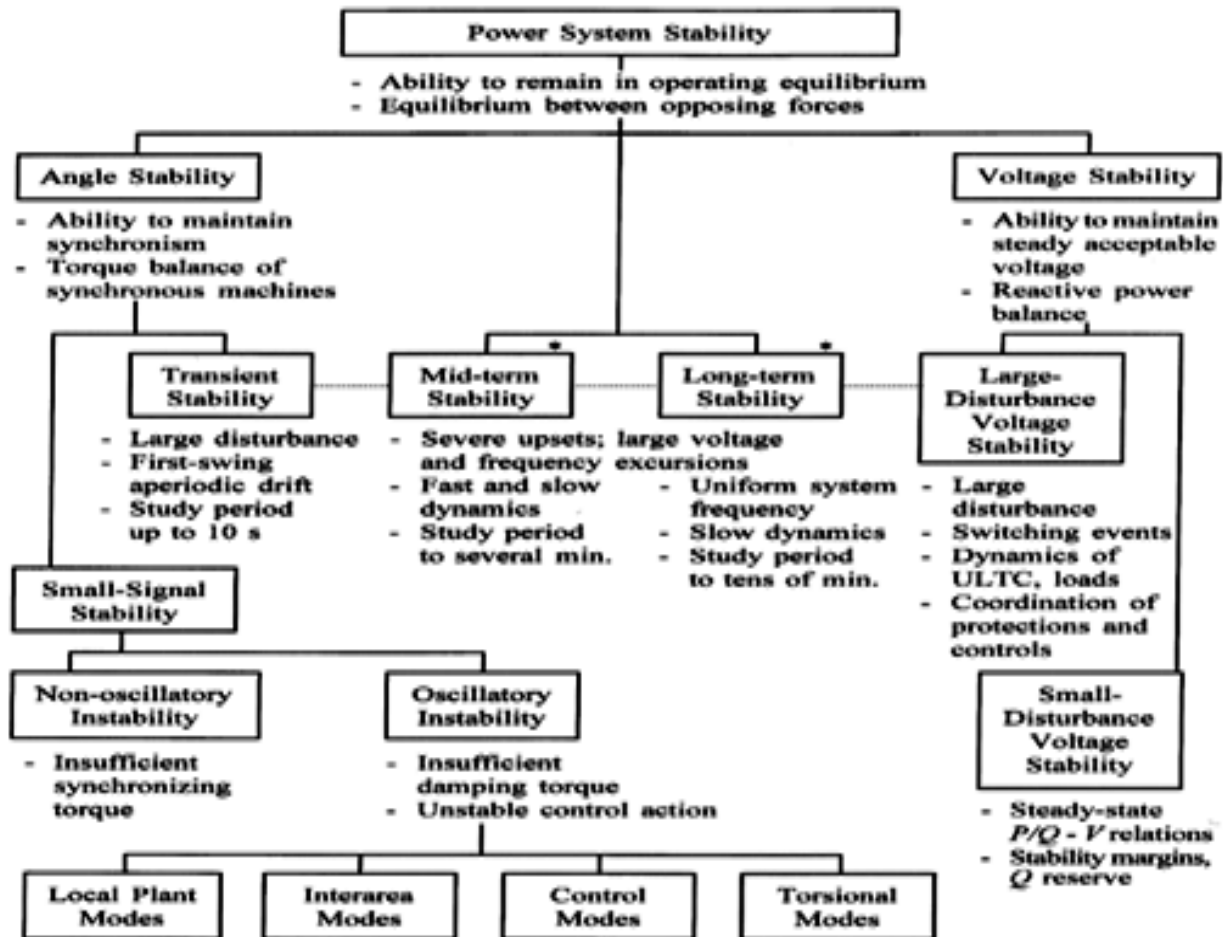
9. Explain the step-wise procedure of determining the swing curve of the above system using modified Euler's method. (16) (N/D'08)

Numerical integration techniques can be applied to obtain approximate solutions of non-linear differential equations.   (5)

Consider a generator connected to an infinite bus through two parallel lines and a 3ϕ fault occurs at the middle of line 2 as shown in figure.

Let P_m be the input power which is a constant.

Prefault Condition: Under steady state operation,

Power transfer from generator to an infinite bus,

$$P_e = P_m$$

$$\frac{E'V}{X_1}\sin\delta_0 = P_{max1}\sin\delta_0 = P_m$$

$$\sin\delta_0 = \frac{P_m}{P_{max1}} \Rightarrow \delta_0 = \sin^{-1}\left[\frac{P_m}{P_{max1}}\right] \qquad (1)$$

$where\ P_{max1} = \dfrac{E'V}{X_1}$

(5)

$X_1 = Transfer\ reac\tan ce\ for\ the\ prefault\ condition$

$The\ rotor\ is\ running\ at\ synchronous\ speed,$

$\omega_0 = 2\pi f$

$Change\ in\ angular\ velocity\ is\ zero.$

$i.e.,\Delta\omega_0 = 0$

During the fault : Consider a 3ϕ fault occurs at the middle of one line 2 as shown in fig.

$$P_{e2} = \frac{|E'||V|}{X_{II}}\sin\delta_1 = P_{max2}\sin\delta$$

$where\ P_{max2} = \dfrac{|E'||V|}{X_{II}}$

$X_{II} = Transef\ reac\tan ce\ during\ the\ fault$

$the\ swing\ eq.is\ given\ by,$

$$\frac{d^2\delta_m}{dt^2} = \frac{\pi f}{H}\left[P_m - P_{max2}\sin\delta\right] = \frac{\pi f}{H}P_a$$

$the\ above\ eq.can\ be\ transformed\ int o\ the\ state\ var iable\ form,$

$$\frac{d\delta^{(1)}}{dt} = \Delta\omega \qquad (2)$$

$$\frac{d^2\delta_m}{dt^2} = \frac{d\Delta\omega^{(1)}}{dt} = \frac{\pi f}{H}P_a \quad (3)$$

$Compute\ the\ first\ estimate\ at\ t_1 = t_0 + \Delta t.$

$$\delta_{i+1}^P = \delta_i + \left.\frac{d\delta^{(1)}}{dt}\right|_{\Delta\omega_i}.\Delta t \qquad (4)$$

(6)

$$\Delta\omega_{i+1}^P = \Delta\omega_i + \left.\frac{d\Delta\omega^{(1)}}{dt}\right|_{\delta_i}.\Delta t \qquad (5)$$

Compute the derivatives : Using the predicted value, determine the derivatives at the end of iteration.

$$\left.\frac{d\delta^{(2)}}{dt}\right|_{\Delta\omega_{i+}^{P}} = \Delta\omega_{i+1}^{P} \qquad (6)$$

$$\left.\frac{d\Delta\omega^{(2)}}{dt}\right|_{\delta_{i+1}^{P}} = \left.\frac{\pi f}{H} P_a\right|_{\delta_{i+1}^{P}} \qquad (7)$$

Computing the final estimated corrected value,

$$\delta_{i+1}^{C} = \delta_i + \left[\frac{\left.\frac{d\delta}{dt}\right|_{\Delta\omega_i} + \left.\frac{d\delta}{dt}\right|_{\Delta\omega_{i+1}^{P}}}{2}\right]\Delta t \qquad (8)$$

$$\Delta\omega_{i+1}^{C} = \Delta\omega + \left[\frac{\left.\frac{d\Delta\omega}{dt}\right|_{\delta_i} + \left.\frac{d\Delta\omega}{dt}\right|_{\delta_{i+1}^{P}}}{2}\right]\Delta t \qquad (9)$$

10. State and explain equal area criterion and discuss how you will apply it to find the maximum additional load that can be suddenly added. (16) (M/J'13) (N/D'12)

The equal area criteria for the stability states that the system is stable if the area under P_a-δ Curve reduces to Zero at some values of δ.   (5)
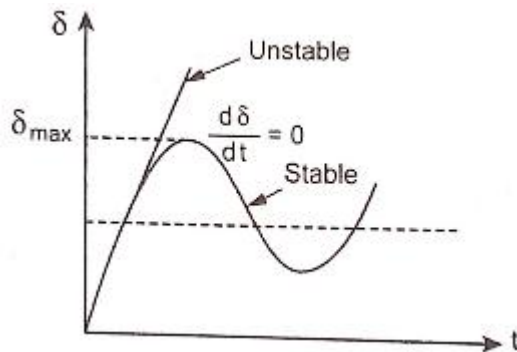
This is possible if the positive (accelerating) area under P_a-δc curve is equal to negative (decelerating) area under P_a-δ curve for a finite change in δ. Hence the stability criterion is called equal area criterion.

This method is only applicable to a one machine connected to an infinite bus or two machine system.

Stability Criterion :

Stable system: If the system is stable, $\delta(t)$ perform oscillations whose amplitude decreases in actual practice.

At some time $\dfrac{d\delta}{dt} = 0$

δ reaches maximum and will starts to reduce.

Unstable System : (5)

If the system is unstable, δ continues to increase with time and the machine loses synchronism.

$\dfrac{d\delta}{dt} > 0$ for a sufficiently long time.

Consider a synchronous machine connected to an infinite bus as shown in fig.
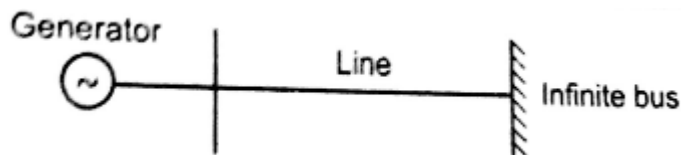
The swing eq. is given by

$$\frac{H}{180\,f}\frac{d^2\delta}{dt^2} = P_{m(p.u)} - P_{e(p.u)}$$

$$\frac{d^2\delta}{dt^2} = \frac{\pi f}{H}\left[P_m - P_e\right] \qquad (1)$$

Multiplying eq.(1) by $2\dfrac{d\delta}{dt}$ *on both sides, we get*

(6)

$$2\frac{d\delta}{dt}\frac{d^2\delta}{dt^2} = 2\frac{d\delta}{dt}\frac{\pi f}{H}\left[P_m - P_e\right]$$

This may be written as,

$$d\left[\frac{d\delta}{dt}\right]^2 = \frac{2\pi f}{H}\left[P_m - P_e\right]d\delta \qquad (2)$$

Integrating equation (2) on both sides, we get,

$$\left[\frac{d\delta}{dt}\right]^2 = \frac{2\pi f}{H} \int_{\delta_0}^{\delta} [P_m - P_e] d\delta$$

Relative speed of machine with respect to sync.revolving ref.frame,

$$\frac{d\delta}{dt} = \sqrt{\frac{2\pi f}{H} \int_{\delta_0}^{\delta} [P_m - P_e] d\delta} \qquad (3)$$

For stable system, this speed must become zero at some time after the disturbance.

$$\frac{d\delta}{dt} = 0, \int_{\delta_0}^{\delta} (P_m - P_e) d\delta = 0$$

$$\int_{\delta_0}^{\delta} P_a \, d\delta = 0 \qquad (4)$$

Where P_a = Accelerating power

The condition of stability can be stated as the positive (accelerating) area under P_a Vs δ curve must be equal to the negative (decelerating) area and hence the name equal area criterion of stability.

11. With a neat flowchart, explain how the transient stability can be made by modified Euler method. (16) (N/D'12)

Numerical integration techniques can be applied to obtain approximate solutions of non-linear differential equations. (5)

Consider a generator connected to an infinite bus through two parallel lines and a 3ϕ fault occurs at the middle of line 2 as shown in figure.

Let P_m be the input power which is a constant.

Prefault Condition: Under steady state operation,

Power transfer from generator to an infinite bus,

$P_e = P_m$

$$\frac{E'V}{X_1}\sin\delta_0 = P_{max1}\sin\delta_0 = P_m$$

$$\sin\delta_0 = \frac{P_m}{P_{max1}} \Rightarrow \delta_0 = \sin^{-1}\left[\frac{P_m}{P_{max1}}\right] \qquad (1)$$

$where\ P_{max1} = \dfrac{E'V}{X_1}$

$X_1 = Transfer\ reac\tan ce\ for\ the\ prefault\ condition$

$The\ rotor\ is\ running\ at\ synchronous\ speed,$

$\omega_0 = 2\pi f$

$Change\ in\ angular\ velocity\ is\ zero.$

$i.e., \Delta\omega_0 = 0$

During the fault : Consider a 3φ fault occurs at the middle of one line 2 as shown in fig.

$$(5)$$

$$P_{e2} = \frac{|E'||V|}{X_{II}}\sin\delta_1 = P_{max2}\sin\delta$$

$where\ P_{max2} = \dfrac{|E'||V|}{X_{II}}$

$X_{II} = Transef\ reac\tan ce\ during\ the\ fault$

$the\ swing\ eq.is\ given by,$

$$\frac{d^2\delta_m}{dt^2} = \frac{\pi f}{H}\left[P_m - P_{max2}\sin\delta\right] = \frac{\pi f}{H}P_a$$

$the\ above\ eq.can\ be\ transformed\ into\ the\ state\ variable\ form,$

$$\frac{d\delta^{(1)}}{dt} = \Delta\omega \qquad (2)$$

$$\frac{d^2\delta_m}{dt^2} = \frac{d\Delta\omega^{(1)}}{dt} = \frac{\pi f}{H}P_a \quad (3)$$

$Compute\ the\ first\ estimate\ at\ t_1 = t_0 + \Delta t.$

$$\delta_{i+1}^P = \delta_i + \left.\frac{d\delta^{(1)}}{dt}\right|_{\Delta\omega_i}.\Delta t \qquad (4)$$

$$\Delta\omega_{i+1}^P = \Delta\omega_i + \left.\frac{d\Delta\omega^{(1)}}{dt}\right|_{\delta_i}.\Delta t \qquad (5)$$

Compute the derivatives : Using the predicted value, determine the derivatives at the end of iteration.

$$\left. \frac{d\delta^{(2)}}{dt} \right|_{\Delta\omega_{i+}^{P}} = \Delta\omega_{i+1}^{P} \qquad (6)$$

$$\left. \frac{d\Delta\omega^{(2)}}{dt} \right|_{\delta_{i+1}^{P}} = \frac{\pi f}{H} P_a \Big|_{\delta_{i+1}^{P}} \qquad (7)$$

Computing the final estimated corrected value,

$$\delta_{i+1}^{C} = \delta_i + \left[\frac{\left. \frac{d\delta}{dt} \right|_{\Delta\omega_i} + \left. \frac{d\delta}{dt} \right|_{\Delta\omega_{i+1}^{P}}}{2} \right] \Delta t \qquad (8)$$

$$\Delta\omega_{i+1}^{C} = \Delta\omega + \left[\frac{\left. \frac{d\Delta\omega}{dt} \right|_{\delta_i} + \left. \frac{d\Delta\omega}{dt} \right|_{\delta_{i+1}^{P}}}{2} \right] \Delta t \qquad (9)$$

Flow Chart :- (6)

State variable form

$$\frac{d\delta^{(1)}}{dt} = \Delta\omega$$

$$\frac{d^2\delta}{dt^2} = \frac{d\Delta\omega^{(1)}}{dt} = \frac{\pi f P_a}{H}$$

Compute the first estimate $t_1 = t_0 + \Delta t$,

$$\delta_{i+1}^P = \delta_i + \frac{d\delta^{(1)}}{dt}\bigg|_{\Delta\omega_i} \cdot \Delta t$$

$$\Delta\omega_{i+1}^P = \Delta\omega_i + \frac{d\Delta\omega^{(1)}}{dt}\bigg|_{\delta_i} \cdot \Delta t$$

Compute the derivatives: Using the predicted values δ_{i+1}^P and $\Delta\omega_{i+1}^P$, determine the derivatives at the end of iteration

$$\frac{d\delta^{(1)}}{dt}\bigg|_{\Delta\omega_{i+1}^P} = \Delta\omega_{i+1}^P$$

$$\frac{d\Delta\omega^{(2)}}{dt}\bigg|_{\delta_{i+1}^P} = \frac{\pi f}{H}\bigg|_{\delta_{i+1}^P}$$

Compute the average derivatives

$$\frac{d\delta}{dt_{ave}} = \frac{\dfrac{d\delta^{(1)}}{dt}\bigg|_{\Delta\omega_i} + \dfrac{d\delta^{(2)}}{dt}\bigg|_{\Delta\omega_{i+1}^P}}{2}$$

$$\frac{d\Delta\omega}{dt_{ave}} = \frac{\dfrac{d\Delta\omega^{(1)}}{dt}\bigg|_{\delta_i} + \dfrac{d\Delta\omega^{(2)}}{dt}\bigg|_{\delta_{i+1}^P}}{2}$$
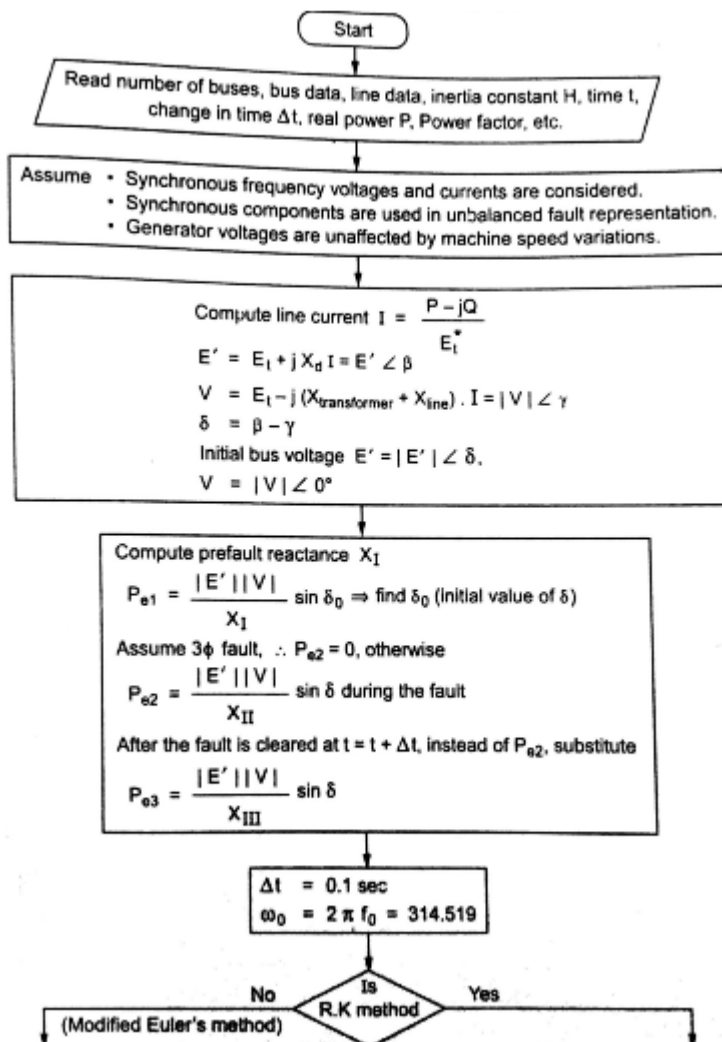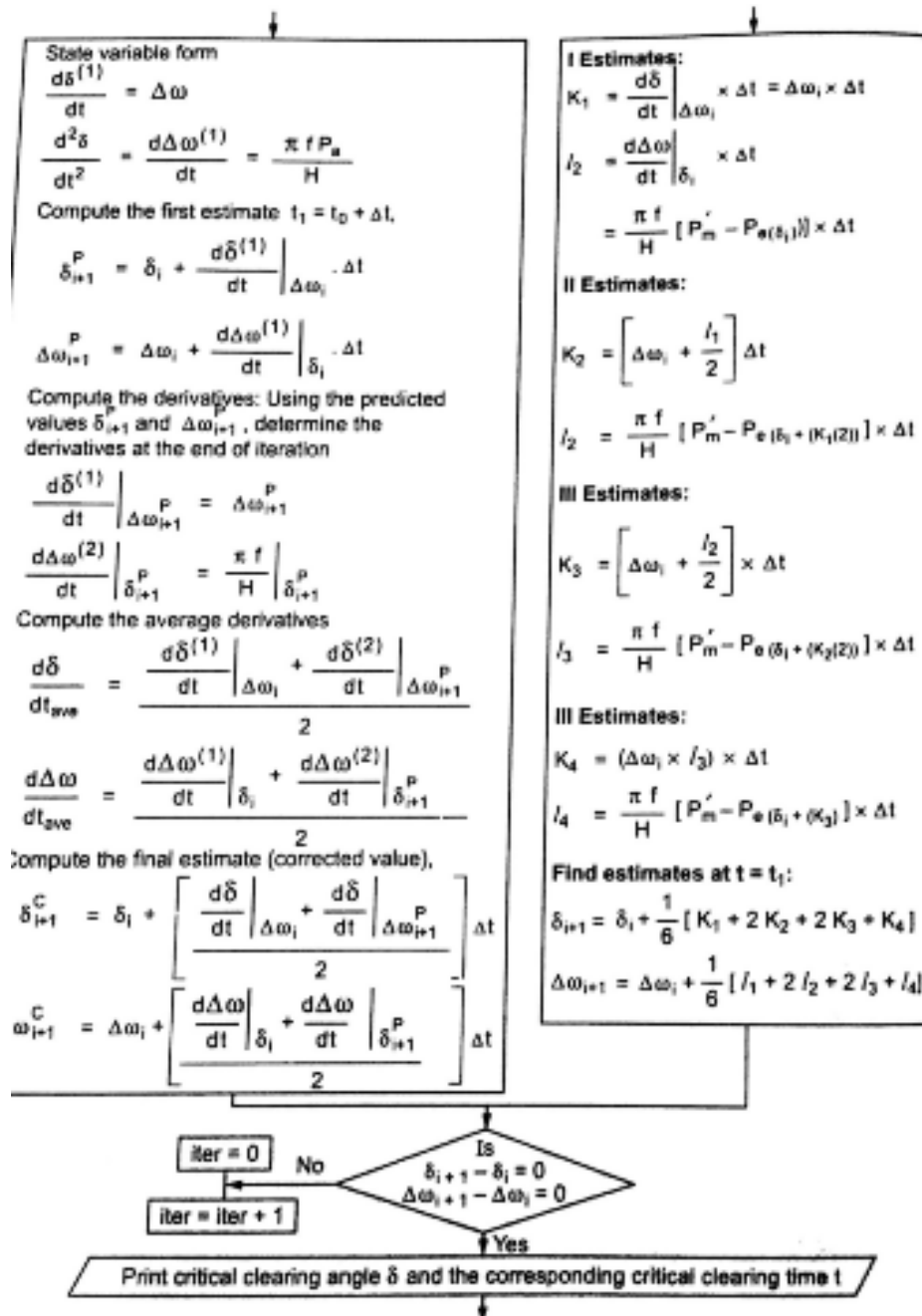
Compute the final estimate (corrected value),

$$\delta_{i+1}^C = \delta_i + \left[\frac{\dfrac{d\delta}{dt}\bigg|_{\Delta\omega_i} + \dfrac{d\delta}{dt}\bigg|_{\Delta\omega_{i+1}^P}}{2}\right]\Delta t$$

$$\omega_{i+1}^C = \Delta\omega_i + \left[\frac{\dfrac{d\Delta\omega}{dt}\bigg|_{\delta_i} + \dfrac{d\Delta\omega}{dt}\bigg|_{\delta_{i+1}^P}}{2}\right]\Delta t$$

I Estimates:

$$K_1 = \frac{d\delta}{dt}\bigg|_{\Delta\omega_i} \times \Delta t = \Delta\omega_i \times \Delta t$$

$$l_2 = \frac{d\Delta\omega}{dt}\bigg|_{\delta_i} \times \Delta t$$

$$= \frac{\pi f}{H}[P_m' - P_{e(\delta_i)}] \times \Delta t$$

II Estimates:

$$K_2 = \left[\Delta\omega_i + \frac{l_1}{2}\right]\Delta t$$

$$l_2 = \frac{\pi f}{H}[P_m' - P_{e(\delta_i + (K_1(2))}] \times \Delta t$$

III Estimates:

$$K_3 = \left[\Delta\omega_i + \frac{l_2}{2}\right] \times \Delta t$$

$$l_3 = \frac{\pi f}{H}[P_m' - P_{e(\delta_i + (K_2(2))}] \times \Delta t$$

III Estimates:

$$K_4 = (\Delta\omega_i \times l_3) \times \Delta t$$

$$l_4 = \frac{\pi f}{H}[P_m' - P_{e(\delta_i + (K_3)}] \times \Delta t$$

Find estimates at $t = t_1$:

$$\delta_{i+1} = \delta_i + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$$

$$\Delta\omega_{i+1} = \Delta\omega_i + \frac{1}{6}[l_1 + 2l_2 + 2l_3 + l_4]$$

iter = 0

No ← Is $\delta_{i+1} - \delta_i = 0$ $\Delta\omega_{i+1} - \Delta\omega_i = 0$

iter = iter + 1

↓ Yes

Print critical clearing angle δ and the corresponding critical clearing time t

12. Derive a power angle equation for a

    i. SMIB system. Also draw the power-angle curve. (16)

The equation relating the electrical power generated (P_e) to the angular displacement of the rotor (δ) is called power angle equation. Here the synchronous machine represented by a constant voltage E' behind the direct axis transient reactance X'_d as shown in fig.   (5)
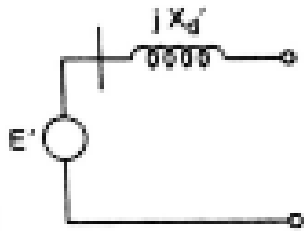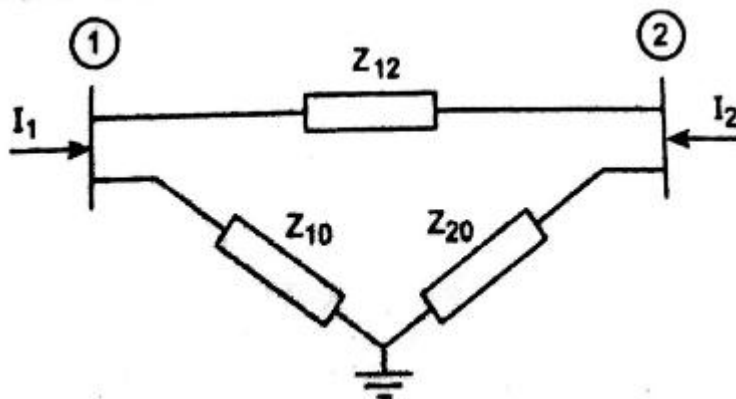

Consider a generator connected to a major substation of a very large system (Infinite bus) through a transmission line as shown in fig.
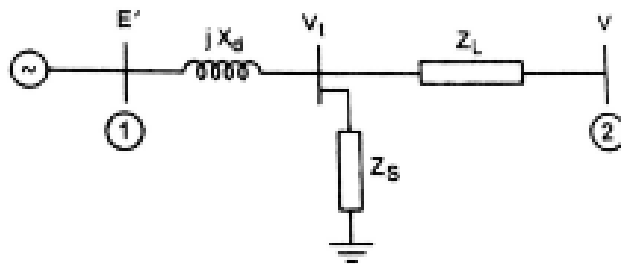
    (5)

Eliminate the generator terminal voltage (V_t) node by using Y-Δ transformation as shown in fig.

$$z_{12} = \frac{jX'_d\, z_L + jX'_d\, z_L + z_L z_S}{z_S} \; ; z_{10} = \frac{jX'_d\, z_L + jX'_d\, z_S + z_L z_S}{z_L}$$

$$z_{20} = \frac{jX'_d\, z_L + jX'_d\, z_S + z_L z_S}{jX_d}$$

Nodal Equations : (6)

$$Node\,1:I_1 = \left[\frac{1}{z_{12}} + \frac{1}{z_{10}}\right]E' - \frac{1}{z_{12}}V$$

$$Node\,2:I_2 = -\frac{1}{z_{12}}E' + \left[\frac{1}{z_{12}} + \frac{1}{z_{20}}\right]V$$

$$\therefore \overrightarrow{I_1} = \overrightarrow{Y_{11}}\overrightarrow{E'} + \overrightarrow{Y_{12}}\overrightarrow{V} \qquad (1)$$

$$\overrightarrow{I_2} = \overrightarrow{Y_{21}}\overrightarrow{E'} + \overrightarrow{Y_{22}}\overrightarrow{V} \qquad (2)$$

$Power\,injected\,at\,bus\,1,$

$$P_1 + jQ_1 = E'I*$$

$$= \overrightarrow{E'}\left[\overrightarrow{Y_{11}}\overrightarrow{E'}\right]* + \overrightarrow{E'}\left[\overrightarrow{Y_{12}}\overrightarrow{V}\right]*$$

$$= |E'|\angle\delta\left[|Y_{11}|\angle-\theta_{11}|E'|\angle-\delta\right] + |E'|\angle\delta\times\left[|Y_{12}|\angle-\theta_{12}|V|\angle 0°\right]$$

$$= |E'|^2|Y_{11}|\angle-\theta_{11} + |E'||V||Y_{12}|\angle\delta-\theta_{12}$$

$$P_1 = \mathrm{Re}\{P_1 + jQ_1\}$$

$$P_1 = |E'|^2|G_{11}| + |E'||V||Y_{12}|\cos(\delta-\theta_{12}) \qquad (3)$$

$Mostly\,z_L\,and\,z_s\,are\,inductive.\,so\,resis\,\tan ce\,are\,neglected.$

$$\theta_{12} = 90°, |Y_{12}| = \frac{1}{|X_{12}|}$$

$$P_1 = P_e = |E'|G_{11} + \frac{|E'||V|}{X_{12}}\sin\delta \qquad (4)$$
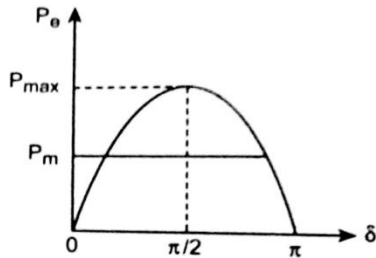
$$P_e = P_C + P_{\max}\sin\delta$$

This equation is called as **Power angle Equation.**

Power angle curve:

All the elements are susceptance, then $G_{11} = 0$.

$$\boxed{\therefore P_e = \frac{|E'||V|}{X_{12}}\sin\delta = P_{\max}\sin\delta}$$

Power transmitted depends on the transfer reactance X_{12} and the angle between the voltages E' and V i.e., (δ). The curve P_e versus δ is known as power angle curve. The Power angle curve is as shown in fig.
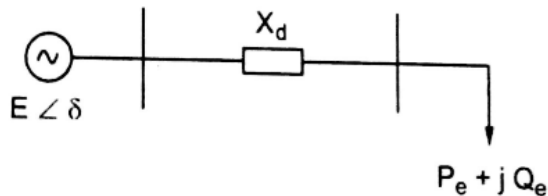

13.(i). A generator having X_d= 0.7 p.u deliver rated load at a power factor of 0.8 lagging. Find P_e, Q_e and E and δ. (8) (M/J'12)

$X_d = 0.7$ p.u

P.f $= 0.8$ lag

P_e, Q_e E and δ.= ?

$$S = \frac{1\angle 36.87}{0.8} = 1.25\angle 36.87$$

$$I_a = S*/V*$$
$$= 1.25\angle -36.87$$

$$E' = V + jX'_d \times I_a = 1.0 + j0.7 \times 1.25\angle -36.87$$
$$= 0.35 + j0.59 = 0.69\angle 1.03$$

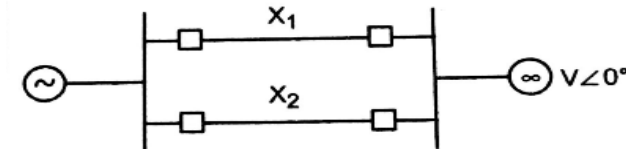$$|E'| = 0.69, \delta = 1.03$$

$$P_e = \frac{|E'||V|\sin\delta}{X'_d}$$

$$= \frac{0.69 \times 1\sin 1.03}{0.7} = 0.845$$

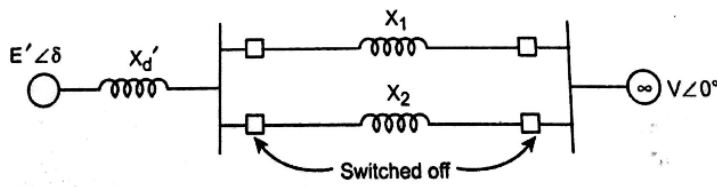$$Q_e = \frac{|E'||V|\cos\delta}{X'_d} - \frac{|V|^2}{X'_d}$$

$$= \frac{0.69 \times 1\cos 1.03}{0.7} - \frac{1}{0.7} = -0.92 \, p.u$$

13.ii. Using equal area criteria, derive an expression for critical clearing angle for a system having a generator feeding a large system through a double circuit line (8)

  Consider a single machine connected to infinite bus through two parallel lines as shown in fig.

The equivalent circuit is shown in Fig.

Consider one of the line is suddenly switched off with system operating at a steady load.

  Prefault condition (Before Switching off)

Power angel curve is given by

$$P_{e1} = \frac{|E'||V|}{X'_d + \left[\dfrac{X_1 X_2}{X_1 + X_2}\right]} \sin\delta = P_{max1} \sin\delta \quad (1)$$
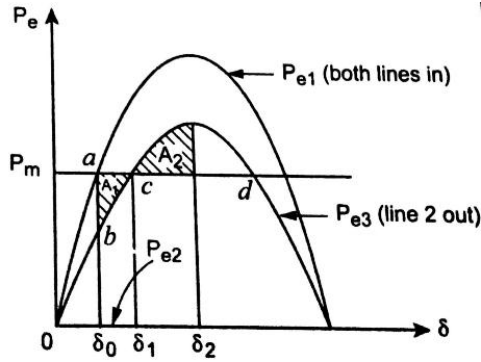
During fault $P_{e2} = 0$

  The rotor therefore accelerate and angle δ increases. Synchronism will be lost unless the fault is cleared in time.

Post fault (Immediately on switching off line 2):

Power angle curve is given by,

$$P_{e3} = \frac{|E'||V|}{X'_d + X_1} \sin\delta = P_{max2} \sin\delta \quad (2)$$

The power angle curves are drawn as shown in fig.

$$As \ X'_d + X_1 > X'_d + \left[\frac{X_1 X_2}{X_1 + X_2} \right]$$

$$\therefore P_{e2} < P_{e1}$$

The system is operating initially with steady power transfer P_m at a torque angle δ_0 on curve 1. Immediately on switching off line 2, the electrical operating point shifts to curve 2 (point b).

Accelerating energy corresponding to area A_1 is followed by decelerating energy for $\delta > \delta_1$, Apply equal area criterion, for stable system
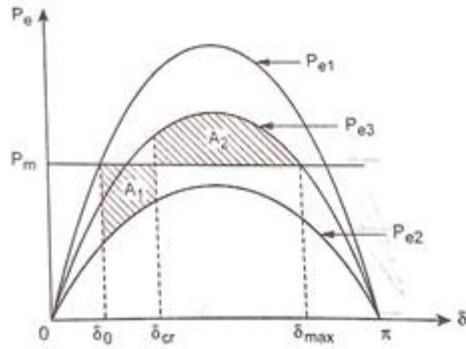
Area A_1 = Area A_2

The system will finally operate at *c* corresponding to a new rotor angle $\delta > \delta_0$. This is because a single line offers larger reactance and larger rotor angel to transfer the same steady power.

(i.e., $\delta_1 = \delta_{max} = \pi - \delta_0$)

14. A 3 ph generator delivers 1.0 p.u. power to an infinite bus through a transmission network when a fault occurs. The maximum power which can be transferred during prefault, during fault and post fault conditions is 1.75 p.u., 0.4 p.u, 1.25 p.u. Find critical clearing angle. (16) (M/J'12)

The power angle curve is as shown in fig. (6)

$P_{e1} = 1.75 \sin \delta$

$P_{max1} = 1.75$ p.u.

$P_{e2} = 0.4 \sin \delta$

$P_{max2} = 0.4$ p.u.

$P_{e3} = 1.25 \sin \delta$

$P_{max3} = 1.25$ p.u.

Initial loading $P_m = 1.0$ p.u.   (5)

$$1.75 \sin \delta_0 = P_m \Rightarrow \sin \delta_0 = \frac{1}{1.75}$$

$$\delta_0 = \sin^{-1} \frac{1}{1.75} = 0.608 \ rad$$

$$\delta_{max} = \pi - \sin^{-1}\left[\frac{P_m}{P_{max3}}\right]$$

$$= \pi - \sin^{-1}\left[\frac{1}{1.25}\right] = 2.214 \ rad$$

Applying equal area criterion,   (5)

Area A_1 = Area A_2

$$P_m(\delta_{cr} - \delta_0) - \int_{\delta_0}^{\delta_{cr}} P_{e2} d\delta = \int_{\delta_{cr}}^{\delta_{max}} P_{e3} d\delta - P_m(\delta_{max} - \delta_{cr})$$

$$\Rightarrow \cos \delta_{cr} = \frac{P_m(\delta_{max} - \delta_{cr}) - P_{max2} \cos \delta_0 + P_{max3} \cos \delta_{max}}{P_{max3} - P_{max2}}$$

$$= \frac{1.0(2.214 - 0.608) - 0.4 \cos 0.608 + 1.25 \cos 2.214}{1.25 - 0.4}$$

$$\cos \delta_{cr} = 0.6212 \ rad$$

$$\delta_{cr} = \cos^{-1} 0.6212 = 0.9 \ rad = 51.57°$$

$$\boxed{\delta_{cr} = 0.9 \ rad = 51.57°}$$

15. Derive the Runge-Kutta method of solution of swing equation for multi-machine systems. (16) (A/M'11'08)

In this method, the accuracy is of the order of (Δt). Swing equation of one machine connected to infinite bus.

$$\frac{d\delta}{dt} = \Delta\omega$$

$$\frac{d\Delta\omega}{dt} = \frac{\pi f_0}{H}(p_m - p_e) = \frac{\pi f_0}{H}(p_m - p_{\max}\sin\delta) \qquad (5)$$

Value of $p_e = p_{\max}\sin\delta$

Initial value of $\delta_0 = \sin^{-1}\left[\dfrac{p_m}{p_{\max}}\right]$

I estimates:

$$K_1 = \left.\frac{d\delta}{dt}\right|_{\Delta\omega_i} \times \Delta t = \Delta\omega_i \times \Delta t$$

$$l_1 = \left.\frac{d\Delta\omega}{dt}\right|_{\delta_i} \times \Delta t = \frac{\pi f}{H}[P_m' - P_{e(\delta_i)}]\times\Delta t$$

II estimates: (5)

$$K_2 = \left[\Delta\omega_i + \frac{l_1}{2}\right]\Delta t$$

$$l_2 = \frac{\pi f}{H}[P_m' - P_{e(\delta_i+(K_1/2))}]\times\Delta t$$

III estimates:

$$K_3 = \left[\Delta\omega_i + \frac{l_2}{2}\right]\Delta t$$

$$l_3 = \frac{\pi f}{H}[P_m' - P_{e(\delta_i+(K_2/2))}]\times\Delta t$$

IV estimates:

$$K_4 = \left[\Delta\omega_i + l_3\right]\Delta t$$

$$l_4 = \frac{\pi f}{H}[P_m' - P_{e(\delta_i+K_3)}]\times\Delta t$$

Final estimates at t=t$_1$:

$$\delta_{i+1} = \delta_i + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$$

$$\Delta\omega_{i+1} = \Delta\omega_i + \frac{1}{6}[l_1 + 2l_2 + 2l_3 + l_4]$$

(6)

In the final estimates, the value of δ' and Δω' for the first iterations are updated.

Replace δ° and Δω° by δ' and Δω'recalculate the values of $k_1,k_2,k_3,k_4, l_1,l_2,l_3,l_4$.

Compute

$$\delta_{i+1} = \delta_i + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$\Delta\omega_{i+1} = \Delta\omega_i + \frac{1}{6}[l_1 + 2l_2 + 2l_3 + l_4]$$

Where i=1,2,……………….,n (i.e., number of generators)

Check for convergence: If δ_{i+1} - δ_I =0 and $\Delta\omega_{i+1}$ - $\Delta\omega_I$ =0 are satisfied, then note down critical clearing angle δ and the critical clearing time t.

Otherwise repeat the process and do it for each and every machine.

16.As shown in given figure the three phase fault is applied at point 'p'. Find the critical clearing angle for clearing the fault with simultaneous opening of the breaker 1 and 2.The reactance values of various components are indicated on the diagram. The generator is delivering 1.0 p.u power at the instant proceeding the fault. (16) (M/J'09)
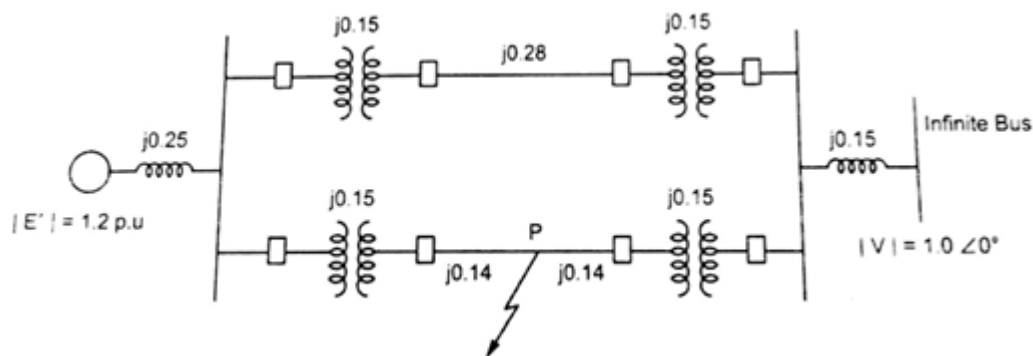
Normal operation (Prefault)   (5)

$$X_1 = 0.25 + \frac{0.5 \times 0.4}{0.5 + 0.4} + 0.05$$

$$= 0.522\ p.u.$$

$$P_{e1} = \frac{|E'||V|}{X_1}\sin\delta = \frac{1.2 \times 1}{0.522}\sin\delta$$

$$= 2.3\sin\delta$$

Prefault operating power angel is given by

$$1.0 = 2.3 \sin \delta_0$$

$$\delta_0 = 25.8^o = 0.45 \text{ radians}$$

(ii) During Fault: No power is transferred during fault   (5)

$$P_{e2} = 0$$

(iii) Post fault operational (fault cleared by opening the faulted line)

$$X_{III} = 0.25 + 0.5 + 0.05 = 0.8$$

$$P_{e3} = \frac{1.2 \times 1}{0.8} \sin \delta = 1.5 \sin \delta$$

$$\delta_{\max} = \pi - \sin^{-1}\left[\frac{P_m}{P_{\max 3}}\right]$$

$$= \pi - \sin^{-1}\left[\frac{1}{0.5}\right] = 2.41\, rad$$

$$\boxed{\delta_{\max} = 2.41\, rad}$$

Applying equal area criterion for critical clearing angle δ_C (6)

$$A_1 = P_m(\delta_{cr} - \delta_0)$$

$$= 1.0(\delta_{cr} - 0.45) = \delta_{cr} - 0.45$$

$$A_2 = \int_{\delta_{cr}}^{\delta_{\max}} (P_{e3} - P_m)d\delta$$

$$= \int_{\delta_{cr}}^{2.41} (1.5 \sin \delta - 1)d\delta$$

$$= \int_{\delta_{cr}}^{2.41} (1.5 \sin \delta - 1)d\delta$$

$$= -1.5\cos \delta - \delta \Big|_{\delta_{cr}}^{2.41}$$

$$= 1.5\cos \delta_{cr} + \delta_{cr} - 1.293$$

Setting $A_1 = A_2$ and solving

$$\delta_{cr} - 0.45 = 1.5\cos \delta_{cr} - 1.293$$

$$\delta_{cr} = 55.8°$$

$$\boxed{\delta_{cr} = 55.8°}$$

17. State and explain 'equal area criteria' in connection with transient stability analysis. What are the advantages and limitations of this method? (16) (A/M'08,12)

Swing equation for a single synchronous machine connected to infinite bus is given by

$$\frac{H}{\pi f}\frac{d^2\delta_m}{dt^2} = P_m - P_e$$
$$where\ P_e = P_{max}\sin\delta$$

If $P_m = 0$, then swing equation can be solved easily.

For small disturbance, the equation can be liberalized using steady state stability concept.

For large disturbance, numerical methods are used to solve transient stability problem.

Numerical solution of the swing equation is obtained, giving a plot of δ Vs t is called swing curve as shown in Fig.
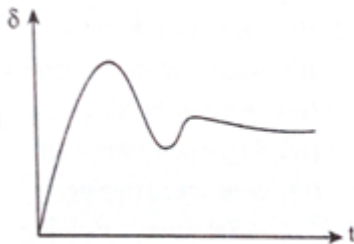
If δ value decreased after reaching a maximum value, then the system is stable otherwise the system is unstable.

Most of the line faults are transient in nature and get cleared immediately an opening the line. Auto reclose breaker are used for automatically close after the fault is cleared. If the fault is severe, the circuit breaker opens and lock permanently till the fault is cleared manually. Mostly the first reclosure will be sufficient, the system stability can be maintained by auto reclose breakers.

For a single machine connected to infinite bus system, stability can be determined by the equal area criterion.

18.Explain the modified Euler method of analyzing multi machine power stability, with neat flow chart. (16) (M/J'07,14)

Step by step procedure:

1. Perform load flow study for prefault condition and determine initial bus voltage magnitudes and angles.   (5)

2. Calculate prefault generator current,

$$I_i^\circ = \frac{S_i^o}{\left|V_i^o\right|^2}$$

3. Compute $E_i^{'}$

$$E_i^{'} = V_i^o + j(X_d^{'} + X_L)I_i$$

$$E_i^{'} = \left|E_i^{'}\right| \angle \delta_i$$

Define initial rotor angle $\overset{o}{X}$.

4. Compute Y-bus matrix during the fault and post fault condition.

5. Set time count r=0.

6. Calculate generator power output P_{ei}.   (5)

$$P_{ei}^r \Big|_{t=t^r} = \sum_{j=1}^{m} \left|E_i^{'}\right|\left|E_j^{'}\right|\left|Y_{ij}\right| \cos(\theta_{ij} + \delta_j - \delta_i)$$

7. Assume uniform discrete time interval Δt.

    Solve swing equation during the fault upto the fault clearing time and repeat the steps for post fault condition.

8. Compute $\left[\left(\overset{\bullet r}{x_{1i}}, \overset{\bullet r}{x_{2i}}\right), i=1,2,.............,m\right]$

    Using $\overset{\bullet}{x_{1i}} = x_{2i}$

    $$\overset{\bullet}{x_{2i}} = \frac{\pi f_0}{H_i}(P_m - P_{ei}), i=1,2,......,m$$

9. Compute the first sta E^{r+1} as

$$x_{1i}^{r+1} = x_{1i}^{r} + \overset{\bullet r}{x_{1i}} \cdot \Delta t; i = 1, 2, \ldots\ldots, m$$

$$x_{2i}^{r+1} = x_{2i}^{r} + \overset{\bullet r}{x_{2i}} \cdot \Delta t$$

10. Compute the first estimates of E_i^{r+1}. (6)

$$E_i^{r+1} = E_i^o (\cos x_{1i}^{r+1} + j \sin x_{1i}^{r+1})$$

11. Compute $P_{ei}^{r+1} and \delta_n$, $\qquad \Delta\delta_n = \Delta\delta_{n-1} + \dfrac{(\Delta t)^2}{M} P_{a(n-1)}$

$$\delta_n = \delta_{n-1} + \Delta\delta_n$$

12. Compute $\left(\overset{\bullet r+1}{x_{1i}}, \overset{\bullet r+1}{x_{2i}} \right), i = 1, 2, \ldots\ldots, m$

$$\overset{\bullet r+1}{x_{1i}} = \overset{\bullet r+1}{x_{2i}}$$

$$\overset{\bullet r+1}{x_{2i}} = \frac{\pi f_o}{H_i} (P_m - P_{ei}^{r+1}); i = 1, 2, \ldots\ldots, m$$

13. Compute the average values of state derivatives.

$$\overset{\bullet r}{x_{1i}}_{\text{ average}} = \frac{1}{2}\left[\overset{\bullet r}{x_{1i}} + \overset{\bullet r+1}{x_{1i}} \right]$$

$$\overset{\bullet r}{x_{2i}}_{\text{ average}} = \frac{1}{2}\left[\overset{\bullet r}{x_{2i}} + \overset{\bullet r+1}{x_{2i}} \right]$$

14. Compute the final state estimates for $t = t^{r+1}$.

$$x_{1i}^{r+1} = x_{1i}^{r} + \overset{\bullet r}{x_{1i}}_{\text{ average}} \cdot \Delta t$$

$$x_{2i}^{r+1} = x_{2i}^{r} + \overset{\bullet r}{x_{2i}}_{\text{ average}} \cdot \Delta t$$

15. Compute the final estimate for E_i at $t = t^{r+1}$.

$$E_i^{r+1} = \left| E_i^o \right| \cos x_{1i}^{r+1} + j \sin x_{1i}^{r+1}$$

16. Print $x_{1i}^{r+1}, x_{2i}^{r+1}$.

17. If $r > r_{final}$, stop.

   Otherwise $r = r+1$ (Increment r) and repeat from step(6).

18. Examine δ Vs t plot (swing curve) to determine stability of the system.

19. A 150 MVA generator transformer unit having an overall reactance of 0.3 p.u. is delivering 150 MW to infinite bus bar over a double circuit 220 KV line having reactance per phase per circuit of 100 ohms. A 4th Frohase fagdl vee.comidway along one of the
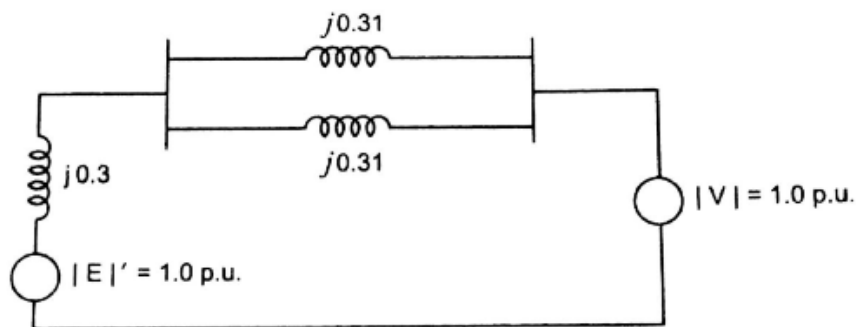
transmission lines. Calculate the maximum angle of swing that the generator may achieve before the fault is cleared without loss of stability. (16) (N/D '07)
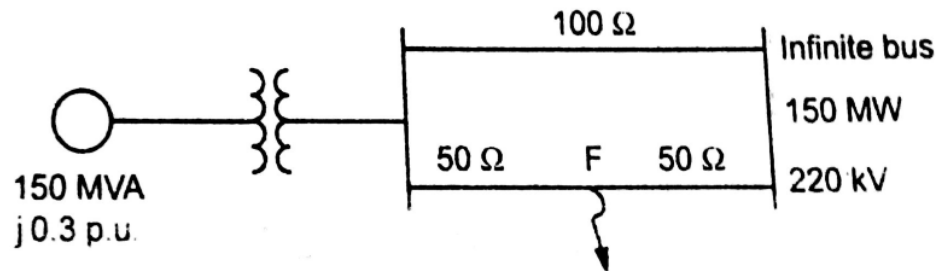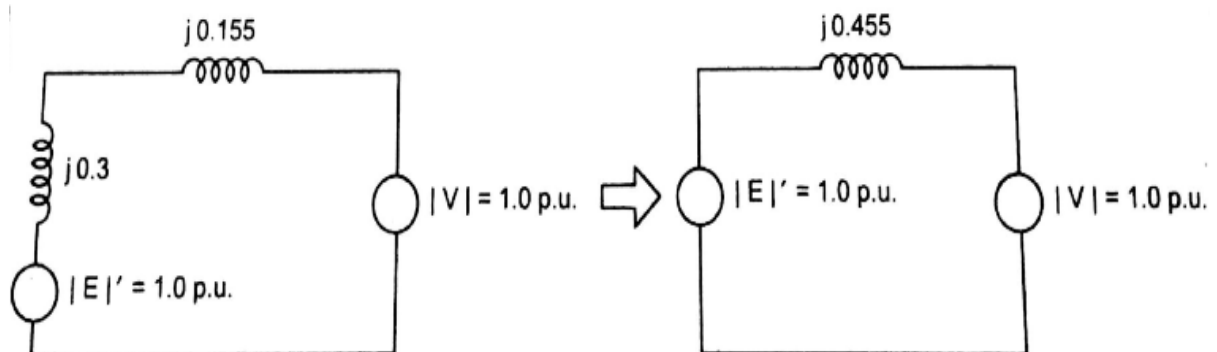

Solution

$$\text{Re} \, ac \tan ce \, of \, line \, X_{p.u.} = \frac{X_{actual}}{(KV_b)^2} \times MVA_b$$

(5)

$$= \frac{100}{220^2} \times 150 = j0.31 p.u.$$

Prefault condition : Impedance diagram for prefault condition is as shown in fig.

j0.31 is in parallel with j0.31

$$P_{e1} = \frac{|E'||V|}{X_1} \sin\delta = \frac{1.0 \times 1.0}{0.455} \sin\delta = 2.198 \sin\delta$$

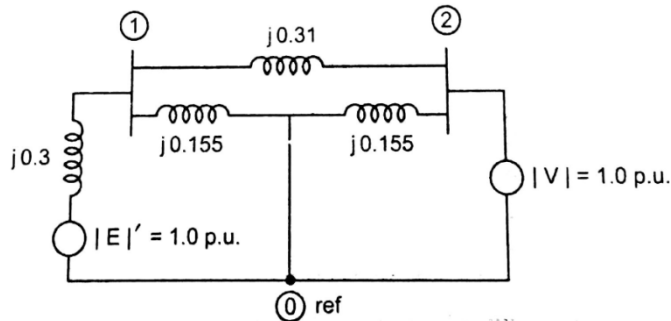*operating power angle* δ_0 *is given by*

$$2.198 \sin\delta_0 = 1.0$$

$$\delta_0 = \sin^{-1}\left[\frac{1.0}{2.198}\right] = 27.06° = 0.472 \, rad$$

$$\boxed{\delta_0 = 27.06° = 0.472\,rad}$$

During the fault : Positive sequence reactance diagram
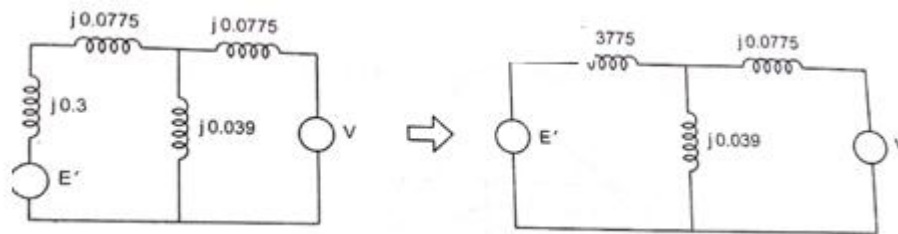

Using Delta-star conversion, the circuit becomes   (5)

$$Z_{IN} = \frac{Z_{12} \times Z_{10}}{Z_{12} + Z_{10} + Z_{20}} = \frac{j0.31 \times j0.155}{j0.31 + j0.155 + j0.155} = j0.0775\,p.u.$$
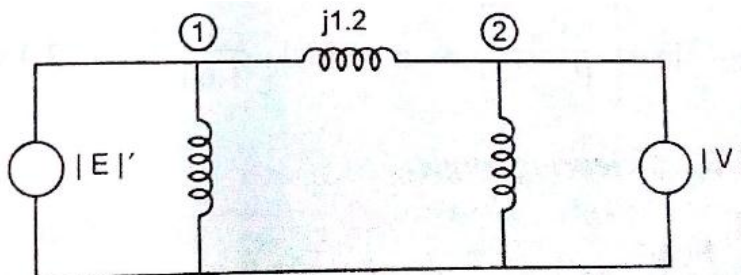
$$Z_{0N} = \frac{Z_{10} \times Z_{20}}{Z_{12} + Z_{10} + Z_{20}} = \frac{j0.155 \times j0.155}{j0.31 + j0.155 + j0.155} = j0.039\,p.u$$

Using star – Delta conversion,

$$Z_{12} = \frac{Z_{IN}Z_{N0} + Z_{N0}Z_{2N} + Z_{IN}Z_{N2}}{Z_{N0}}$$

$$= \frac{j0.3775 \times j0.039 + j0.039 \times j0.0775 + j0.3775 \times j0.0775}{j0.039} = j1.2$$
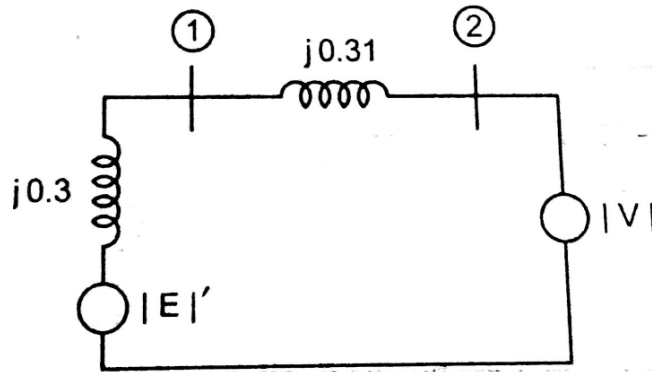

Power transfer during fault

$$P_{e2} = \frac{|E'||V|}{X_{12}} \sin\delta$$

$$= \frac{1.0 \times 1.0}{1.2} \sin\delta = 0.833 \sin\delta \, p.u.$$

Post fault condition: Faulted line is removed by opening the circuit breaker at ends.
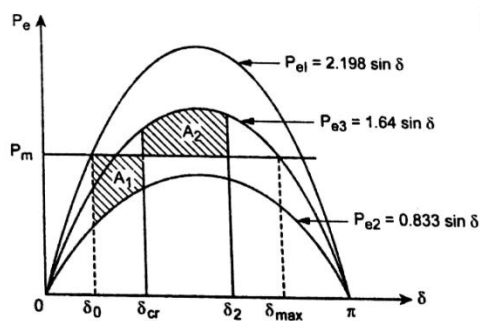
Impedance diagram for postfault is as shown in fig. (6)
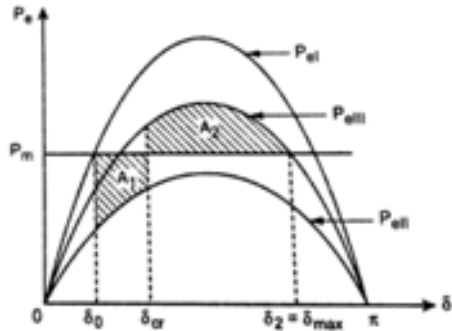

$$Z = X_{III} = 0.3 + 0.31 = 0.61 \, p.u$$

$$P_{e3} = \frac{|E'||V|}{X_{III}} \sin\delta = \frac{1.0 \times 1.0}{0.61} \sin\delta = 1.64 \sin\delta \, p.u.$$

Power angle curve is as shown in fig.

$$\delta_{max} = \pi - \sin^{-1}\left[\frac{P_m}{P_{max\,3}}\right] = \pi - \sin^{-1}\left[\frac{1}{1.64}\right] = 3.14 - 0.656 = 2.48 \, rad$$

Determining of critical clearing angle:

$$Area\ A_1 = 1.0[\delta_{cr} - \delta_0] - \int_{\delta_0}^{\delta_{cr}} P_{e2}\, d\delta$$

$$Area\ A_2 = \int_{\delta_{cr}}^{\delta_{max}} P_{e3}\, d\delta - P_m[\delta_{max} - \delta_{cr}]$$

$$= \int_{\delta_{cr}}^{\delta_{max}} 1.5 \sin \delta\, d\delta - 1.0 \times [2.4 - \delta_{cr}]$$

Applying equal area criteria $A_1 = A_2$

$$\delta_{cr} - 0.472 + \int_{0.472}^{\delta_{cr}} 0.833 \sin \delta = \int_{\delta_{cr}}^{2.48} 1.64 \sin \delta - (2.48 - \delta_{cr})$$

$$-0.472 + 0.833 \cos \delta\big]_{0.472}^{\delta_{cr}} = -1.64 \cos \delta\big]_{\delta_{cr}}^{2.48} - 2.48$$

$$-0.472 + 0.833 \cos \delta_{cr} - 0.393 = 1.294 + 1.64 \cos \delta_{cr} - 2.48$$

$$\cos \delta_{cr}(0.833 - 1.64) = 1.294 - 2.48 + 0.472 + 0.393$$

$$-0.807 \cos \delta_{cr} = -0.321$$

$$\cos \delta_{cr} = 0.398$$

$$\delta_{cr} = 1.16\, rad$$

$$\boxed{\delta_{cr} = 1.16\, rad}$$

20. A 50 Hz, 500 MVA, 400 KV generator (with transformer) is connected to a 400 KV infinite bus bar through an interconnector. The generator has H=2.5 MJ/MVA , voltage behind transients reactance of 450 KV and is loaded 460 MW. The transfer reactance between generator and bus under various conditions are:

Prefault : 0.5 p.u; During fault: 1.0 p.u; Post fault: 0.75 p.u

Calculate the swing curve using intervals of 0.05 sec and assuming that the fault is cleared at 0.15 sec. (16) (N/D'07)

Solution:

KVb = 400   (5)

$V = \dfrac{400}{400} = 1$ p.u.

$E' = \dfrac{450}{400} = 1.125$ p.u.

$P_{e1} = \dfrac{460}{400}$

Prefault, $X_1 = 0.5$ p.u.

$P_{e1} = \dfrac{\left|E'\right|\left|V\right|}{X_1}\sin\delta_0 = 0.92$

$\dfrac{1.125\times1}{0.5}\sin\delta_0 = 0.92$

$\delta_o = 0.42\,rad$

Assume 3ϕ fault occurs, $P_{e2} = 0$

Post fault condition, $P_{e3} = \dfrac{\left|E'\right|\left|V\right|}{X_{111}}\sin\delta$

$P_{e3} = \dfrac{1.125\times1}{0.75}\sin\delta = 1.5\sin\delta$

Using modified Euler's method:

$\omega_0 = 2\,\pi\,f = 2\,\pi\times50 = 314.159$

$\Delta t = 0.05$ sec

Iteration 1:   t = 0,   (5)

$$\left.\frac{d\delta}{dt}\right|_{\Delta\omega_o} = \Delta\omega_o = \omega_o - 2\pi f = 0$$

$$\left.\frac{d\Delta\omega}{dt}\right|_{\delta_o} = \frac{\pi f}{H}\left[P_m' - P_{e(\delta_o)}\right]$$

$$\left.\frac{d\Delta\omega}{dt}\right|_{\delta_o} = \frac{\pi \times 50}{2.5}[0.92 - 0] = 57.8$$

End of the first step at t=0.05 sec

Predicted values are

$$\delta_{0.05}^P = \delta_0 + \left.\frac{d\delta}{dt}\right|_{\Delta\omega_o} \times \Delta t$$

$$\delta_{0.05}^P = 0.42 + (0 \times 0.05) = 0.42$$

$$\Delta\omega_{0.05}^P = \Delta\omega_0 + \left.\frac{d\Delta\omega}{dt}\right|_{\delta_o} \times \Delta t$$

$$\Delta\omega_{0.05}^P = 0 + (57.8 \times 0.05) = 2.89 rad/\sec$$

$$\boxed{\Delta\omega_{0.05}^P \, 2.89 rad/\sec}$$

Derivation at the end of t =0.05.

$$\left.\frac{d\delta}{dt}\right|_{\Delta\omega_{0.05}^P} = \Delta\omega_{0.05}^P = 2.89 rad/\sec$$

$$\left.\frac{d\Delta\omega}{dt}\right|_{\delta_{0.05}^P} = \frac{\pi f}{H}[P_m' - P_{e(\delta_{0.05}^P)}]$$

$$\left.\frac{d\Delta\omega}{dt}\right|_{\delta_{0.05}^P} = \frac{\pi \times 50}{2.5}[0.92 - 0] = 57.8$$

Corrected values,

$$\delta_{0.05}^C = \delta_0 + \frac{\Delta t}{2}\left[\left.\frac{d\delta}{dt}\right|_{\Delta\omega_0} + \left.\frac{d\delta}{dt}\right|_{\Delta\omega_{0.05}^P}\right]$$

$$\delta_{0.05}^C = 0.42 + \frac{0.05}{2}[0 + 2.89]$$

$$\delta_{0.05}^C = 0.492 rad$$

$$\Delta\omega_{0.05}^C = \Delta\omega_0 + \frac{\Delta t}{2}\left[\left.\frac{d\Delta\omega}{dt}\right|_{\delta_0} + \left.\frac{d\Delta\omega}{dt}\right|_{\delta_{0.05}^P}\right]$$

$$\Delta\omega_{0.05}^C = 0 + \frac{0.05}{2}[57.8 + 57.8]$$

$$\Delta\omega_{0.05}^C = 2.89 rad/\sec$$

$$\boxed{\Delta\omega_{0.05}^{C} = 2.89 \, rad/\sec}$$

Iteration 2: (6)

$$\frac{d\delta}{dt}\bigg|_{\Delta\omega_{0.05}^{C}} = \Delta\omega_{0.05}^{C} = 2.89$$

$$\frac{d\Delta\omega}{dt}\bigg|_{\delta_{0.05}^{C}} = \frac{\pi \times 50}{2.5}[0.92 - 0] = 57.8$$

At t=0.1, predicted values are

$$\delta_{0.1}^{P} = \delta_{0.05}^{C} + \frac{d\delta}{dt}\bigg|_{\Delta\omega_{0.05}^{C}} \times \Delta t$$

$$\delta_{0.1}^{P} = 0.492 + 2.89 \times 0.05$$

$$\delta_{0.1}^{P} = 0.637 \, rad$$

$$\Delta\omega_{0.1}^{P} = \Delta\omega_{0.05}^{C} + \frac{d\Delta\omega}{dt}\bigg|_{\delta_{0.05}^{C}} \times \Delta t$$

$$\Delta\omega_{0.1}^{P} = 2.89 + 57.8 \times 0.05 = 5.78$$

$$\frac{d\delta}{dt}\bigg|_{\Delta\omega_{0.1}^{P}} = \Delta\omega_{0.1}^{P} = 5.78 \, rad/\sec$$

$$\frac{d\Delta\omega}{dt}\bigg|_{\delta_{0.1}^{P}} = \frac{\pi \times 50}{2.5}[0.92 - 0] = 57.8$$

$$\delta_{0.1}^{C} = \delta_{0.05}^{C} + \frac{\Delta t}{2}\left[\frac{d\delta}{dt}\bigg|_{\Delta\omega_{0.05}^{C}} + \frac{d\delta}{dt}\bigg|_{\Delta\omega_{0.1}^{P}}\right]$$

$$\delta_{0.1}^{C} = 0.492 + \frac{0.05}{2}[2.89 + 5.78] = 0.709 \, rad$$

$$\Delta\omega_{0.1}^{C} = \Delta\omega_{0.05}^{C} + \frac{\Delta t}{2}\left[\frac{d\Delta\omega}{dt}\bigg|_{\delta_{0.05}^{C}} + \frac{d\Delta\omega}{dt}\bigg|_{\delta_{0.1}^{P}}\right]$$

$$\Delta\omega_{0.1}^{C} = 2.89 + \frac{0.05}{2}[57.8 + 57.8] = 5.78 \, rad/\sec$$

$$\boxed{\Delta\omega_{0.1}^{C} = 5.78 \, rad/\sec}$$

Iteration 3:   t=0.1 sec

$$\left.\frac{d\delta}{dt}\right|_{\Delta\omega_{0.1}^{C}} = \Delta\omega_{0.1}^{C} = 5.78$$

$$\left.\frac{d\Delta\omega}{dt}\right|_{\delta_{0.1}^{C}} = \frac{\pi \times 50}{2.5}[0.92 - 0] = 57.8$$

End of the third step at t=0.15 , predicted values are,

$$\delta_{0.15}^{P} = \delta_{0.1}^{C} + \left.\frac{d\delta}{dt}\right|_{\Delta\omega_{0.1}^{C}} \times \Delta t$$

$$\delta_{0.15}^{P} = 0.709 + 5.78 \times 0.05 = 0.998 rad$$

$$\Delta\omega_{0.15}^{P} = \Delta\omega_{0.1}^{C} + \left.\frac{d\Delta\omega}{dt}\right|_{\delta_{0.1}^{C}} \times \Delta t$$

$$\Delta\omega_{0.15}^{P} = 5.78 + 57.8 \times 0.05 = 8.67 rad/\sec$$

Derivation at the end of t=0.15 sec

$$\left.\frac{d\delta}{dt}\right|_{\Delta\omega_{0.15}^{P}} = \Delta\omega_{0.15}^{P} = 8.67 rad/\sec$$

$$\left.\frac{d\Delta\omega}{dt}\right|_{\delta_{0.15}^{P}} = \frac{\pi \times 50}{2.5}[0.92 - 0] = 57.8$$

Corrected values:

$$\delta_{0.15}^{C} = \delta_{0.1}^{C} + \frac{\Delta t}{2}\left[\left.\frac{d\delta}{dt}\right|_{\Delta\omega_{0.1}^{C}} + \left.\frac{d\delta}{dt}\right|_{\Delta\omega_{0.15}^{P}}\right]$$

$$\delta_{0.15}^{C} = 0.709 + \frac{0.05}{2}[5.78 + 8.67]$$

$$\delta_{0.15}^{C} = 1.07 rad$$

$$\Delta\omega_{0.15}^{C} = \Delta\omega_{0.1}^{C} + \frac{\Delta t}{2}\left[\left.\frac{d\Delta\omega}{dt}\right|_{\delta_{0.1}^{C}} + \left.\frac{d\Delta\omega}{dt}\right|_{\delta_{0.15}^{P}}\right]$$

$$\Delta\omega_{0.15}^{C} = 5.78 + \frac{0.05}{2}[57.8 + 57.8]$$

$$\Delta\omega_{0.15}^{C} = 8.67 rad/\sec$$

Post fault condition, $P_e = 1.5 \sin \delta$

Calculate $\delta_{0.2}^{C}$ and $\Delta\omega_{0.2}^{C}$ using $P_e = 1.5 \sin \delta$