CONCEPT OF WORKING STRESS METHOD, ULTIMATE LOAD DESIGN AND LIMIT STATE DESIGN METHODS FOR RCC

Various methods used for the design of R.C.C. structures are as follows:

(i) Working stress method.
(ii) Load factor or ultimate load method.
(iii) Limit state method.

Working Stress method

This method of design was the oldest one. It is based on the elastic theory and assumes that both steel and concrete and elastic and obey Hook's law. It means that the stress is directly proportional to strain up to the point of collapse. Based on the elastic theory, and assuming that the bond between steel and concrete is perfect, permissible stresses of the materials are obtained. The basis of this method is that the permissible stresses are not exceeded anywhere in the structure when it is subjected to worst combination of working loads.

In this method, the ultimate strength of concrete and yield strength or 0.2% proof stress of steel are divided by factors of safety to obtain permissible stresses. These factors of safety take into account the uncertainties in manufacturing of these materials. As per IS456, a factor of safety of 3 is to be used for bending compressive stresses in concrete and 1.78 for yield/proof strength of steel.

The main drawbacks of the working stress method of design are as follows :

(i) It assumes that concrete is elastic which is not true as the concrete behaves in-elastically even on low level of stresses.

(ii) It uses factors of safety for stresses only and not for loads. Hence, this method does not give true margin of safety with respect to loads because we do not know the failure load.

(iii) It does not use any factor of safety with respect to loads. It means, there is no provision for the uncertainties associated with the estimation of loads.

(iv) It does not account for shrinkage and creep, which are time dependent and plastic in nature.

(v) This method gives uneconomical sections.

(vi) It pays no attention to the conditions that arise at the time of collapse.

The working stress method is very simple and reliable but as per IS 456:2000 the working stress method is to be used only if it is not possible to use limit state method of design. Working stress method is the basic method and its knowledge is essential for understanding the concepts of design.

Load Factor Method or Ultimate Load Method

In this method, ultimate or collapse load is used as design load. The ultimate loads are obtained by increasing the working/service loads suitably by some factors. These factors, which are multiplied by the working loads to obtain ultimate loads, are called as load factors. These load factors give the exact margins of safety in terms of load. This method used the real stress-strain curve of concrete and steel and takes into account the plastic behavior of these materials.

Many designers feel that the load factor provides a clear margin of safety and one can easily tell the load at which the structure fails, which is not clear from the working stress concept of permissible stresses. This method was given in detail in IS 456-1964,

The advantages of Ultimate load method are listed below:

(i) The method is more realistic as compared to working stress method because ultimate load method taken into account the non-linear behavior of the concrete.

(ii) This method gives exact margin of safety in terms of load unlike working stress method which is based on the permissible stresses which do not give any idea about the failure/collapse load.

(iii) The sections designed by ultimate load method are thinner and require less reinforcement. Hence the method is economical as compared to WSM.

The main limitations of the ultimate load method are following:

(i) This method gives very thin sections which leads to excessive deformations and cracking, thus making the structure unserviceable.

(ii) No factors of safety are used for material stresses.

As the serviceability requirements are not satisfied at all in this method, the code replaced this method by limit state method which takes into account the strength as well as serviceability requirements.

Limit State Method

This is the most rational method which takes into account the ultimate strength of the structure and also the serviceability requirements. It is a judicious combination of working stress and ultimate load methods of design. The acceptable limits of safety and serviceability requirements before failure occurs is called a limit state. This method is based on the concept of safety at ultimate loads (ultimate load method) and serviceability at working loads (working stress method).

The two important limit states to be considered in design are :

(i) Limit state of collapse.

(ii) Limit state of serviceability.

Limit State of Collapse

This limit state corresponds to the strength of the structure and categorized into following types :

(a) Limit state of collapse: Flexure.
(b) Limit state of collapse: Shear and bond.
(c) Limit State of collapse: Torsion.
(d) Limit state of collapse: Compression.
(2) Limit State of Serviceability

This limit state corresponds to the serviceability requirements i.e., deformation, cracking etc. It is categorized into following types:

(a) Limit state of deflection.
(b) Limit state of cracking
(c) Limit state of vibration.

This method is based upon the probabilities variation in the loads and material properties. Limit state method takes into account the uncertainties associated with loads and material properties, thus uses partial factors of safety to obtain design loads and design stresses.

The limit state method is based on predictions unlike working stress method, which is deterministic in nature, assumes that the loads, factors of safety and material stresses are known accurately. In the limit state method, the partial safety factors are derived using probability and statistics and are different for different load combinations, hence giving a more rational and scientific design procedure.

DESIGN OF SINGLY REINFORCED RECTANGULAR BEAMS BY WORKING STRESS METHOD

1. **Design a R.C beam to carry a load of 6 kN/m inclusive of its own weight on an effect span of 6m keep the breath to be 2/3 rd of the effective depth .the permissible stressed in the concrete and steel are not to exceed 5N/mm^2 and 140 N/mm^2.take m=18**

Step 1: Design constants.

Modular ratio, m =18.

A Coefficient n $=\frac{m\sigma_{cbc}}{m\sigma_{cbc}+\sigma_{st}} = 0.39$

Lever arm Coefficient, j=1-(n/3) = 0.87

Moment of resistance Coefficient Q $=\frac{\sigma_{cbc}}{2}$. n. j = 0.84 N/mm^2

Step 2: Moment on the beam.

M = (w.l^2)/8

= (6x6^2)/8

= 27kNm

M = Qbd^2

d^2 = M/Qb

= (27x10^6)/ (0.84x2/3xd)

d = 245mm.

Step 3: Balanced Moment.

M_{bal} = Qbd^2

= 0.84x245x365^2

= 27.41kNm. > M.

It can be designed as singly reinforced section.

Step 4: Area of steel.

Ast = M_{bal} / (σ_{st}.j.d)

= 616.72mm^2

Use 20mm dia bars ast = π/4 (20^2) = 314.15mm^2

No. of bars =Ast/ast

= 616.72/314.15

= 1.96 say 2nos.

Provide 2#20mm dia bars at the tension side

2. Design a beam subjected to a bending moment of 40kNm by working stress design. Adopt width of beam equal to half the effective depth. Assume the permissible stressed in the concrete and steel are not to exceed 5N/mm^2 and 140 N/mm^2.take m=18.

Step 1: Design constants.

Modular ratio, m =18.

A Coefficient n $= \frac{m\sigma_{cbc}}{m\sigma_{cbc}+\sigma_{st}} = 0.39$

Lever arm Coefficient, j=1-(n/3) = 0.87

Moment of resistance Coefficient Q $= \frac{\sigma_{cbc}}{2}$. n. j = 0.84 N/mm^2

Step 2: Moment on the beam.

M = 40kNm

M = Qbd^2

d^2 = M/Qb

= (40x10^6)/ (0.84x1/2xd)

d = 456.2 say 460 mm.

b= 0.5 ,d = 0.5x460

= 230mm

Step 3: Balanced Moment.

M_{bal} = Qbd^2

= 0.84x230x460^2

= 40.88kNm. > M.

It can be designed as singly reinforced section.

Step 4: Area of steel.

Ast =M_{bal} / (σ_{st}.j.d)

= (40.88x10^6)/(140x0.87x460)

= 729.64mm^2

Use 20mm dia bars ast π/4 (20^2) = 314.15mm^2

No. of bars = Ast/ast

= 729.64/314.15

= 2.96 say 3nos.

Provide 3#20mm dia bars at the tension side.

3 Determine the moment of resistance of a singly reinforced beam 160X300mm effective section, if the stress in steel and concrete are not to exceed 140N/mm^2 and 5N/mm^2.effectve span of the beam is 5m and the beam carries 4 nos of 16mm dia bars. Take m=18.find also the minimum load the bam can carry. Use WSD method.

Step 1: Actual NA.

$b\ xa^2/2 = m.Ast.(d- xa)$

$160.\ xa^2/2 = 18 \times 804.24(300 - xa)$

$Xa = 159.42mm$

Step 2: Critical NA.

$xc = \sigma_{bc}.d/(\sigma_{st}/.m + \sigma_{cbc})$

$= 117.39mm$

$xc < Xa = 159.42mm$

it is Over reinforced Section.

Step 3: Moment of Resistance

$M = (b.\frac{x_a}{2}.\sigma_{cbc})(d- xa/3)$

$= (160x159.42/2x5)(300-159.42/3)$

$= 15.74kNm$

Step 4: Safe load.

$M = (w.l^2)/8$

$W = (8 \times 15.74)/5^2$

$= 5.03\ kN/m$

4. A reinforced concrete rectangular section 300 mm wide and 600 mm overall depth is reinforced with 4 bars of 25 mm diameter at an effective cover of 50 mm on the tension side. The beam is designed with M 20 grade concrete and Fe 415 grade steel. Determine the allowable bending moment and the stresses developed in steel and concrete under this moment. Use working stress method.

Step 1: Actual NA.

$b\ xa^2/2 = m.Ast.(d- xa)$

$300.\ xa^2/2 = 18 \times 1963.50(550 - xa)$

Xa = 117.81mm

Step 2: Critical NA.

Xc = σ_{bc}.d/(σ_{st}/.m + σ_{cbc})

= 194.66mm >Xa

= 117.81mm

it is Under reinforced Section.

Step 3: Moment of Resistance For steel:

M = (Ast.σ_{st})(d- xa/3)

= (1963.5x230)(550-117.81/3)

= 230.64kNm

For concrete:

M = (b.$\frac{x_a}{2}$. σ_{cbc})(d- x_a/3)

= (300x117.81/2x7)(550-117.81/3)

= 63.17kNm

Limit State philosophy as detailed in IS code

DESIGN BASED ON LIMIT STATE METHOD:

Types of limit states:

Two categories of limit states are considered in design.

Limit states of collapse:

- Limit state of collapse in flexure
- Limit state of collapse in compression
- Limit state of collapse in compression and uniaxial bending.
- Limit state of collapse in compression and biaxial bending.
- Limit state of collapse in shear
- Limit state of collapse in bond
- Limit state of collapse in torsion
- Limit state of collapse in tension

Limit state of serviceability:

- Limit state of deflection
- Limit state of cracking
- Other limit states, such as vibration, fire resistance, durability etc.

1. **Limit state of collapse**:

The limit state of collapse of the structure or part of the structure could be assesses from rupture of one or more critical sections and from buckling due to elastic or plastic instability or overturning. The resistance to bending, shear, torsion and axial loads at every section shall not be less than the appropriate value at that section produced by the probable most unfavorable combination of loads on the structure using the appropriate partial safety factors.

2. **Limit state of serviceability**:

The limit state of serviceability relate to the performance or behavior of structure at working loads. Normally, design is based on the considerations of limit states of collapse on ultimate loads and on serviceability limit states of deflection and cracking under service loads. Durability is taken care of by prescribing appropriate grade of concrete, nominal cover for various exposure condition, cement content etc.

ADVANTAGES OF LIMIT STATE METHOD OVER OTHER METHODS

The advantages of limit state method over the other methods are the following

a) In the limit state method of analysis, the principles of both elastic as well as plastic theories used and hence suitable for concrete structures

b) The structure designed by limit state method is safe and serviceable under design loads and at the same time it is ensured that the structure does not collapse even under the worst possible loading conditions

c) The process of stress redistribution, moment redistribution etc., are considered in the analysis and more realistic factor of safety values are used in the design

d) Hence the design by limit state method is found to be more economical.

e) The overall sizes of flexural members (depth requirements) arrived by limit state method are less and hence they provide better appearance to the structures.

Working Stress Method	Limit State Method
The stress in a component is derived from the working load and compared with the permissible stress.	The stresses are derived from the design load and are compared with the design strength.
This method can also be referred to as the deterministic method as a result of the method assumes that the actual load, permissible stress and safety factors are identified.	This method can also be referred to as non-deterministic because the method is based on a probabilistic approach that relies on real data or experience.
The work stress method is based on elastic theory which assumes that concrete and steel are elastic and the stress-strain curve for both is linear.	The limit state method is based on the actual stress-strain curves of steel and concrete, The stress-strain curve for concrete is non-linear.
Physical capabilities are largely underestimated, Safety factors are used in the work stress method.	The capabilities of the material are not underestimated as much as they are in the working stress method. Partial protection factors

	are used in the limit state method.
The ultimate load-carrying capacity cannot be precisely predicted.	Ultimate stresses of the material themselves are used as allowable stresses.
Within the work stress method, the material follows Hooke's rule because the stress is not allowed to exceed the yield limit.	In the limit state method, stress is allowed to exceed the yield limit.
In working stress method, a section which is plane before bending remains plane after bending.	In LSM, a section normal to the axis of the structural element remains on the plane after bending.
In the work stress method, no safety factor is used for the load.	In the limit state method, the design load is obtained by multiplying the load's partial safety factors to the work load.
The working stress method is less economical as it gives thicker parts.	The limit state method is more economical because it gives thin sections.

Analysis and design of singly and doubly reinforced rectangular beams by Limit state method

The Concrete beam whose only tension zone of cross-section area is covered with steel rod is known as a **singly reinforced beam**.

ANALYSIS OF SINGLY REINFORCED BEAMS PROBLMS:

TYPE 1 PROBLEM:

GIVEN DATA: Ast in mm2 or number of bars with diameter, size of beam (b, D), type of concrete (fck), type of steel (fy), if load to be calculated then span is given. REQUIRED: Ultimate moment or factored moment or moment or resistance (Mu) or Mu & w.

Note:

1. Ultimate moment or factored moment (Mu) = 1.5 x working moment = 1.5 x M

2. Ultimate load or factored load (wu) = 1.5 x working load = 1.5 x w

DESIGN STEPS:

STEP 1: Note down the value for Xu,max/d by referring IS: 456-2000

F_y in N/mm^2	$X_{u,max}/d$
250	0.53
415	0.48
500	0.46

STEP 2: Determine depth of neutral axis Xu/d Xu/d = (0.87.fy.Ast)/(0.36.fck.b.d)

Where,

Xu = depth of neutral axis

Fy = characteristic tensile strength of steel in N/mm2 Ast = area of steel in tension in mm2

Fck = characteristic compressive strength of concrete in N/mm2 b = breadth or width of member

d = effective depth in mm

Effective depth (d) = overall depth (D) – effective cover (d') Effective cover (d') = clear cover + diameter of bar/2

Clear cover for beam = 25mm.

STEP 3: Compare Xu/d and Xu,max/d

If Xu/d < Xu,max/d, then section is under reinforced. The moment of resistance is calculated by

Mu = 0.87.fy.Ast.d. [1- (fy.Ast)/(fck.b.d)]

If Xu/d > Xu,max/d, then section is over reinforced. The moment of resistance is calculated by

Mu,lim = 0.149.fck.b.d2 for Fe250 steel.

Mu,lim = 0.138.fck.b.d2 for Fe415 steel.

Mu,lim = 0.133.fck.b.d2 for Fe500 steel.

If the section is balanced that is Xu/d = Xu,max/d then the limiting moment of resistance (Mu,lim) is calculated.

STEP 4: Working moment = M = Mu/1.5

The maximum bending moment for simply supported beam carrying UDL = wl2/8 Now equating maximum bending moment and working moment

$M = wl^2/8$, $w = 8M/ l^2$

Where w = total load = dead load + live load. DL

self-weight of beam= ρ.b.D =25.b.D kN/m

Live load = w-DL in kN/m.

PROBLEM 1.Find the depth of neutral axis of a singly reinforced R.C beam of 230mm width and 450mm effective depth. It is reinforced with 4 bars of 16mm diameter. Use M20 concrete and Fe415 bars. Also comment on the type of beam.

Given data: b=230mm,

d=450mm,

Ast=4-#16,

fck=20 N/mm2,

fy=415 N/mm2

Required: Xu

Solution:

Step1: As per IS: 456-2000 Xu,max/d = 0.48 for Fe415

Step2: Xu/d = (0.87.fy.Ast)/(0.36.fck.b.d)

Ast= no of bars x π(diameter)2/4 = 4x πx(16)2/4

= 504.24 mm^2

Xu/d = (0.87x415x804.24) / (0.36x20x230x450)

= 0.39

Xu = 0.39xd = 0.39x450 = 175.5mm.

Step3: By comparing Xu/d < Xu,max/d

Section is under-reinforced.

PROBLEM 2.A singly reinforced concrete beam 250mm width is reinforced with 4 bars of 25mm diameter at an effective depth 400mm. If M_{20} grade concrete and Fe_{415} bars are used. Compute moment of resistance of the section.

Given data: b=250mm, d=400mm, Ast=4-#25, fck=20 N/mm2, fy=415 N/mm^2 Required: Mu

Solution:

Step1: As per IS: 456-2000 Xu,max/d = 0.48 for Fe_{415}

Step2: Xu/d = (0.87.fy.Ast)/(0.36.fck.b.d)

Ast= no of bars x π(diameter)2/4

= 4x πx(25)2/4 = 1963.75 mm^2

Xu/d = (0.87x415x1963.75) / (0.36x20x250x400)

= 0.98

Step3: By comparing Xu/d > Xu,max/d

Section is Over-reinforced.

Step4:

Mu,lim=0.138.fck.bd^2

=0.138x20x250x400 ^{2}

= 110400000 N-mm = 110.4 kN-m

PROBLEM 3.A simply supported singly reinforced beam having 250mm wide and 500mm effective depth provided with Fe_{415} steel and M_{20} grade of concrete. Determine the ultimate moment of resistance of beam.

Given data: b=250mm,

d=500mm,

fck=20 N/mm2,

fy=415 N/mm^2

Required: Mu,lim

Solution:

$Mu,lim=0.138.fck.bd^2$

$=0.138x20x250x500^2$

= 172500000 N-mm

= 172.5 kN-m

PROBLEM 4.A rectangular section of 230x500mm is used as a simply supported beam for effective span of 6m. The beam consists of tensile reinforcement of 4000 mm2 and center of reinforcement is placed at 35mm from the bottom edge. What maximum total UDL can be allowed on the beam? Given M20 concrete and Fe415 steel.

Given data:

b=230mm,

D=500mm,

simply supported beam, l=6m,

Ast=4000mm2, d'=35mm,

fck=20 N/mm2,

$f_y = 415\ N/mm^2$

Required: w

Solution:

Step1: As per IS: 456-2000

Xu,max/d = 0.48 for Fe415

Step2:

Xu/d = (0.87.fy.Ast)/ (0.36.fck.b.d)

d=D-d'=500-35=465mm

Xu/d = (0.87x415x4000) / (0.36x20x230x465)

= 1.875

Step3: by comparing Xu/d > Xu,max/d

Section is Over-reinforced.

Step4: Mu,lim=0.138.fck.bd^2 =0.138x20x230x465^2

= 137259630 N-mm

= 137.26 kN-m

Step5: M=Mu/1.5

=137.26/1.5=91.506kN-m

Maximum bending moment for simply supported beam with UDL

M=wl2/8,

w=8M/l2

=8.91.506/62=20.33kN-m

ANALYSIS OF DOUBLY REINFORCED BEAM

Definition: the RCC beam section in which steel reinforcement is provided to resist both compression and tension is called doubly reinforced beam.

The circumstances under which doubly reinforced sections are provided:

1. When there are architectural restrictions on the depth of otherwise singly reinforced section.

2. Restriction in the depth at the location of beam at plinth level, along with the provision of ventilator between the ground level and the bottom of plinth beam.

3. In a continuous beam floor system, where the beam acts as a T-beam in the midspan and acts as a rectangular beam at the supports where the B.M may be much greater than at the mid span.

4. Where it is required to increase the stiffness of the beam.

5. It is found that the compression steel increases the rotation capacity and ductility

TYPE I PROBLEMS (Mu):

Given data: Ast, Asc, size of beam, effective cover for compression steel (d'), type of concrete (fck) and steel (fy). If load to be calculated then span is given.

Required: ultimate moment or factored moment or moment of resistance (Mu) and super imposed load (w).

Solution:

Step1: calculate Asc= no of bars x π (ϕc)2/4 mm2, Ast= no of bars x π (ϕt)2/4 mm2 Where, ϕc= diameter of compression steel, ϕt= diameter of tension steel

Step2: Xu,max= 0.46d for Fe500

0.48d for Fe415

0.53d for Fe250

Step3: stress in compression (fsc): from the table F of SP16 by linear interpolation Calculate d'/d

For Fe250 fsc=0.87.fy

Step4: Ast_2= Asc.fsc / (0.87. fy)

Step5: Ast= Ast_1+ Ast_2

Ast1= Ast- Ast_2

Step 6: depth of neutral axis

Xu/d= (0.87.fy.Ast1) / (0.36.fck.b.d) or Xu= 0.87.fy.Ast-fsc.Asc / (0.36.fck.b)

If, Xu < Xu,max section is under reinforced.

Therefore, calculate the moment of resistance by the following expression Mu-Mu,lim=fsc.Asc.(d-d')

Mu= 0.36. fck.b.xu.(d-0.42.xu) + fsc.Asc(d-d') If, Xu > Xu,max section is over reinforced.

Put Xu = Xu,max value and calculate moment of resistance by the following expression, Mu= 0.36. fck.b.xu,max.(d-0.42.xu,max) + fsc.Asc(d-d')

To calculate safe udl of live load: follow same steps as in singly reinforced beams.

Poblem 1 A doubly reinforced beam section is 250mm wide and 450mm deep to center of the tensile reinforcement. It is reinforced with 2 bars of 16mm diameter as compressive reinforcement at an effective cover 50mm and 4 bars of 25mm diameter as tensile steel. Using M15 concrete and Fe250 steel. Calculate the ultimate moment of resistance of the beam.

Given data:

b=250mm,

d=450mm,

fck=15 N/mm2,

fy=250 N/mm2,

d'=50mm,

Asc= 2- #16, Ast=4-#25

Required: Mu Solution:

Step1: calculating Asc= no of bars x π (ϕc)2/4

=2 πx(16)2/4=402.12mm2 Ast

= no of bars x π (ϕt)2/4=4x πx(25)2/4=1963.49mm^2

Step2: Xu,max=0.53d for Fe250

=0.53x450=238.5mm

Step3: stress in compression (fsc)

fsc=0.87.fy=0.87x250=217.5 N/mm2

step4: Ast2=(Asc. fsc)/(0.87. fy)

=(402.12x217.5)/(0.87x250)=402.12mm2

step5: Ast=Ast1+Ast2, Ast1

=Ast-Ast2=1963.49-402.12=1561.37mm2

step6: depth of neutral axis (Xu)

Xu= 0.87.fy.Ast1/ (0.36.fck.b)

= 0.87x250x1561.37/ (0.36x15x250)

=251.5mm

Xu> Xu,max section is over reinforced.

Step7: Mu=0.149.fck.b.d2+fsc.Asc.

(d-d')= 0.149x15x250x4502+217.5x402.12x (450- 50) =148.13x106 N-mm

Mu=148.13 kN-m.

Problem 2 A doubly reinforced beam section is 250mm wide and 500mm deep to the center of the tensile reinforcement. It is reinforced with 2 bars of 18mm diameter as compression reinforcement at an effective cover of 40mm and 4 bars of 25mm diameter as tensile reinforcement using M15 concrete and Fe415 steel. Calculate MR of the section.

Given data:

b=250mm,

d=500mm,

fck=15 N/mm^2,

fy=415 N/mm^2,

d'=40mm,

Asc= 2-#18,

Ast=4-#25

Required: Mu

Solution:

Step1: calculating,

Ast =no's.xπ.(ϕt)^2/4

= 4xπx(25)^2/4= 1963.49mm^2 Asc

= no of bars x π (ϕc)^2/4=2x πx(18)^2/4=508.93mm^2

Step2:

Xu,max=0.48d for Fe415

=0.48x500

=240mm

Step3: stress in compression (fsc) d'/d

=40/500

=0.08

By referring table –F of sp16 d'/d fsc

0.05	355	Y1
0.08	?	Y
0.1	353	Y2

Y $= Y1 + ((Y2-Y1)/(X2-X1))*(X-X1)$

$= 355+((353-355)/(0.1-0.05))*(0.08-0.05)=353.8$ N/mm^2

$F_{sc}= 353.8$ N/mm^2

step4:

$A_{st2} =(A_{sc}.\ f_{sc})/(0.87.\ f_y)$

$=(508.93\text{x}353.8)/(0.87\text{x}415)=498.71$mm^2

step5:

$A_{st} =A_{st1}+A_{st2}$,

$A_{st1} =A_{st}-A_{st2}$

$=1963.49-498.71$

$=1464.78$mm^{2}step6: depth of neutral axis (X_u)

$X_u = (0.87.f_y.A_{st}-f_{sc}.A_{sc})/ (0.36.f_{ck}.b)$

$= (0.87\text{x}415\text{x}1963.49-353.8\text{x}508.93)/(0.36\text{x}15\text{x}250)$

$=391.74$mm

$X_u> X_{u,max}$ section is over reinforced.

Step7: $M_u=0.138.f_{ck}.b.d^2+f_{sc}.A_{sc}.\ (d-d')$

$= 0.138\text{x}15\text{x}250\text{x}500^2+353.8\text{x}508.93\text{x}\ (500-40)$

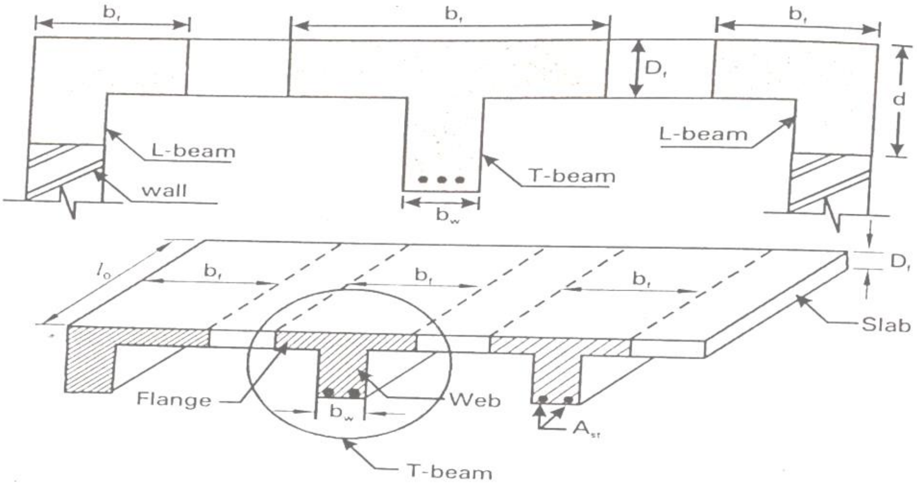
$=212.2\text{x}10^6$ N-mm

$M_u=212.2$ kN-m.

UNIT -II
DESIGN OF BEAMS

2.1 Analysis and design of Flanged beams

In actual practice, T-sections and L-sections are more common than the rectangular section since part of the RC slab, monolithic with the beam and participate with the structural behavior of the beam. For the same load and span T-beam and L- beam carries more moment of resistance than rectangular beams.



When a concrete slab is cast monolithically with and, connected to rectangular beams, a portion of the slab above the beam behaves structurally as a part of the beam in compression. The slab portions are called the flange and beam the web. If the flange projections are on either side of the rectangular web or rib, the resulting cross section resembles the T shape and hence is called a T-beam section. On the other hand, if the flange projects on one side, the resulting cross- section resembles an inverted L and hence is termed as L-beam.

Advantages of T-beam are

1.Beam and slab are casted monolithically hence; casting can be done at a time.

2.Slab and beam combined together to carry more bending moment.

For same section, T-beams have more M.R (flexural strength) than that ofrectangular beam.

EFFECTIVE WIDTH OF FLANGE:

It is that portion of slab which acts integrally with the beam and extends on either side of the beam forming the compression zone. The effective width of flange depends upon the span of the beam, thickness of slab and breadth of the web. It also depends upon the type of loads and support conditions.

As per code (clause 32.1.2 of IS: 456-2000)

Effective flange width for T and L beams are calculated as follows:

a) For T-beams: $b_f = l_0/6 + b_w + 6D_f$
b) For L-beams: $b_f = l_0/12 + b_w + 3D_f$
c) For isolated beams:
   i) For T-beams: $b_f = l_0/[(l_0/b)+4] + b_w$
   ii) For L-beams: $b_f = 0.5l_0/[(l_0/b)+4] + b_w$

Where,

b_f = effective width of the flange.

b_w = breadth of the web

D_f = thickness of the flange,

I = distance between point of zero moment (forcontinuous beam,

I = 0.7x (effective span of beam).

- First segment will be like a rectangular section and steel area A_{st1}.
- Second segment will be like a beam section having concrete section of area [(b_f-b_w)D_f] and steel area of A_{st2}.
- Our consideration in design and analysis for depth of neutral axis $x_u > D_f$ will be ascertain the compressive force taken up by concrete in second segment and its line of action.
- If $x_u \le D_f$, the beam can be thought of as a rectangular section of width b_f. The stress distribution for various values of x_u

STEPS FOR CALCULATING DEPTH OF NEUTRAL AXIS AND MOMENT OF RESISTANCE:

Given: b_f, d, A_{st}, D_f, grade of steel and grade of concrete, span for load calculation.

Required: Factored moment or moment of resistance and load.

Case I: Neutral axis lies within the flange

Steps:1 Calculate depth of neutral axis assuming neutral axis lies within the flange

$X_u/d = (0.87.f_y.A_{st})/(0.36.f_{ck}.b.d)$

Calculate x_u

If $x_u \le D_f$ (Assumption is correct)

Where, D_f = depth of flange or slab

2.Note down the value of $x_{u,max}$ /d from IS:456-2000Calculate

$x_{u,max}$

If $x_u < x_{u,max}$ section is under reinforced, calculate the moment of resistance by the following expression

$$M_u = 0.87.\ f_y.\ A_{st}.d.\ [1-((\ f_y.\ A_{st})/(\ f_{ck}.b.d))]$$

3.If $x_u > x_{u,max}$ section is over reinforced, calculate the moment of resistance by the following expression

$$M_{u.lim} = 0.36.\ f_{ck}.b_f.x_{u,max}.(\ d-0.42.x_{u,max})$$

Case II: Neutral axis lies below the flangeSteps:

Calculate neutral axis assuming neutral axis (NA) lies within flange. If $x_u > D_f$, assumption is wrong. NA lies below the flange.

Recalculate the value of x_u by using following relation $C_1+C_2=T$Where, C_1 = $0.36.f_{ck}.x_u.b_w$

$C_2 = 0.45.f_{ck}.(b_f - b_w).D_f$ $T = 0.87.\ f_y.\ A_{st}$

$0.36.f_{ck}.x_u.b_w + 0.45.f_{ck}.(b_f - b_w).D_f = 0.87.\ f_y.\ A_{st}$ (assume $(D_f / x_u) < 0.43$) andfind x_u

If $x_u > D_f$, assumption is correct, follow step 3.

If $x_u < D_f$, assumption is that $(D_f / x_u) > 0.43$

Then recalculate x_u by using relation $C_1 + C_2$ =

TWhere, $C_1 = 0.36.f_{ck}.x_u.b_w$

$C_2 = 0.45.f_{ck}.(b_f - b_w).y_f$

$T = 0.87.\ f_y.\ A_{st}$

$y_f = (0.15\ x_u + 0.65D_f$

If $x_u \geq x_{u,max}$ section is over reinforced or balanced.

D_f / d ≤ 0.2 use equation G.2.2 page No.96, IS:456-2000 for M_u calculation

$$M_{u.lim} = 0.36.\ f_{ck}.b_w.d^2.(x_{u,max}/d).(\ 1-0.42.(x_{u,max}/d)) + 0.45.f_{ck}.(b_f - b_w).D_f.(d-(D_f/2))$$

D_f / d > 0.2 use equation G.2.2.1 page No.97, IS:456-2000 for M_u calculation

$$M_{u.lim} = 0.36.\ f_{ck}.b_w.\ d^2.(\ (x_{u,max}/d).(\ 1-0.42.(x_{u,max}/d)) + 0.45.f_{ck}.(b_f - b_w).y_f.(d-(y_f/2))$$

Where, $y_f = (0.15\ x_u + 0.65D_f)$, but should not be greater than D_f.

If $x_u < x_{u,max}$ section is under reinforced.

1.D_f / $x_u \leq 0.43$ use equation G.2.2 page No.96, IS:456-2000 for M_u calculation

$$M_u = 0.36.\ f_{ck}.b_w.\ d^2.(\ (x_u/d).(\ 1-0.42.(x_u/d)) + 0.45.f_{ck}.(b_f - b_w).D_f.(d-(D_f/2))$$

2.D_f / $x_u > 0.43$ use equation G.2.2.1 page No.97, IS:456-2000 for M_u calculation

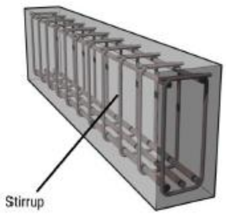
$$M_u = 0.36.\ f_{ck}.b_w.\ d^2.(\ (x_u/d).(\ 1-0.42.(x_u/d)) + 0.45.f_{ck}.(b_f - b_w).y_f.(d-(y_f/2))$$

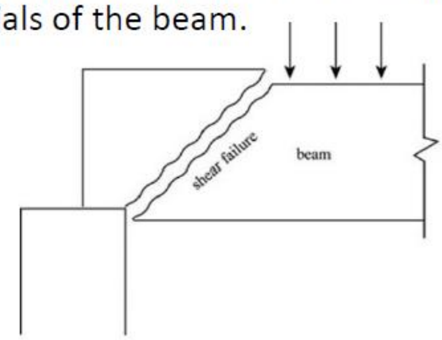
Where, $y_f = (0.15\ x_u + 0.65D_f)$, but should not be greater than D_f.

Design of Shear and torsion

Behaviour of RC members in Shear

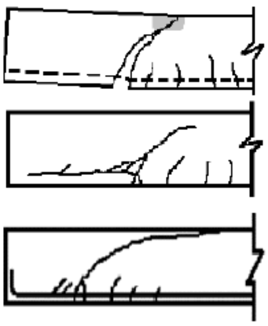
- Shear failure occurs when the beam has **shear resistance lower than flexural strength** and the shear force exceeds the shear capacity of different materials of the beam.





Different types of shear failure in beam

- Shear tension failure
- Shear compression failure
- Diagonal Tension Failure



NOMINAL SHEAR STRESS

$$\tau_v = \frac{V_u}{b\,d}$$

τ_v = nominal shear stress

V_u = shear force due to design load

b = breadth of the member, which for flanged sections shall be taken as the breadth of the web, b_w

d = effective depth

Nominal Shear Stress in Case Beams of Varying Depth

Beams of uniform width and varying depths are commonly used in practice. Cantilever beams continuous beam with haunches at support, footings etc. fall under this category. In case of beams of varying depth the nominal shear stress is calculated by the modified equation given below.

$$\tau_v = \frac{V_u \pm \frac{M_u}{d} \tan\beta}{bd}$$

Where τ_v, V, b and d has same meaning as above

M = bending moment at the section

β = *angle between the top and bottom edges of the beam.*

$$V_c = \tau_c . b . d$$

$$p = \frac{100\,A_s}{bd}$$

Design shear strength of concrete, τ_c in N/mm^2

($100\,A_s/bd$)	Grade of concrete				
	M 20	M 25	M 30	M 35	M40 and above
≤ 0.15	0.28	0.29	0.29	0.29	0.30
0.25	0.36	0.36	0.37	0.37	0.38
0.50	0.48	0.49	0.50	0.50	0.51
0.75	0.56	0.57	0.59	0.59	0.60
1.00	0.62	0.64	0.66	0.67	0.68
1.25	0.67	0.70	0.71	0.73	0.74
1.50	0.72	0.74	0.76	0.78	0.79
1.75	0.75	0.78	0.80	0.82	0.84
2.00	0.79	0.82	0.84	0.86	0.88
2.25	0.81	0.85	0.88	0.90	0.92
2.50	0.82	0.88	0.91	0.93	0.95
2.75	0.82	0.90	0.94	0.96	0.98
≥ 3.00	0.82	0.92	0.96	0.99	1.01

Design of Shear Reinforcement

When τ_v exceeds τ_c given in Table 19, shear reinforcement shall be provided in any of the following forms:

a) Vertical stirrups,

b) Bent-up bars along with stirrups, and

c) Inclined stirrups.

Shear reinforcement shall be provided to carry a shear equal to $V_u - \tau_c bd$ The strength of shear reinforcement V_{us} shall be calculated as below:

a) For vertical stirrups:

$$V_{us} = \frac{0.87 f_y A_{sv} d}{s_v}$$

b) For inclined stirrups or a series of bars bent-up at different cross-sections:

$$V_{us} = \frac{0.8.7 f_y A_{sv} d}{s_v} (\sin\alpha + \cos\alpha)$$

c) For single bar or single group of parallel bars, all bent-up at the same cross-section:

$$V_{us} = 0.87 f_y A_{sv} \sin \alpha$$

The minimum shear reinforcement in the form of stirrups shall be provided such that:

$$\frac{A_{sv}}{b s_v} \geq \frac{0.4}{0.87 f_y}$$

where

A_{sv} = total cross-sectional area of stirrup legs effective in shear,

s_v = stirrup spacing along the length of the member,

b = breadth of the beam or breadth of the web of the web of flanged beam b_w, and

f_y = characteristic strength of the stirrup reinforcement in N/mm^2 which shall not be taken greater than 415 N/mm^2.

Behaviour of rectangular RC beams in shear and torsion

$V_e = V_u + 1.6(T_u/b)$

where V_e = equivalent shear,

V_u = actual shear,

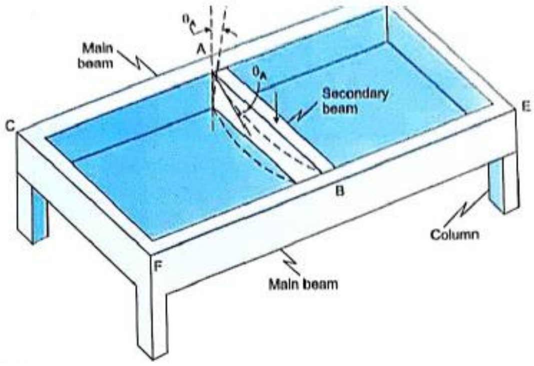
T_u = actual torsional moment,

b = breadth of beam.

(b) The equivalent nominal shear stress τ_{ve} is determined from:

$\tau_{ve} = (V_e / bd)$

, τ_{ve} shall not exceed τ_{cmax} given in Table 20 of IS 456



The longitudinal flexural tension reinforcement shall be determined to resist an equivalent bending moment M_{e1} as given below:

$$M_{e1} = M_u + M_t$$

where M_u = bending moment at the cross-section, and

$M_t = (T_u/1.7)\{1 + (D/b)\}$

where T_u = torsional moment,

D = overall depth of the beam, and

b = breadth of the beam.

The transverse reinforcement consisting of two legged closed loops enclosing the corner longitudinal bars shall be provided having an area of cross-section A_{sv} given below:

$$A_{sv} = \frac{T_u\, s_v}{b_1\, d_1(0.87\, f_y)} + \frac{V_u\, s_v}{2.5\, d_1(0.87\, f_y)}$$

However, the total transverse reinforcement shall not be less than the following:

$$A_{sv} \geq (\tau_{ve} - \tau_c)\, b\, s_v /(0.87\, f_y)$$

where T_u = torsional moment,

V_u = shear force,

s_v = spacing of the stirrup reinforcement,

b_1 = centre to centre distance between corner bars in the direction of the width,

d_1 = centre to centre distance between corner bars,

Design of RC members for combined Bending, Shear and Torsion

Problem: Design a reinforced concrete beam of rectangular cross-section for the following data

b = 300mm   **d = 800mm**
D = 850mm   **f_{ck}= 15 N/mm²**
f_y= 250N/mm²   **M_u= 200 kNm**
V = 100 kN   **T_u=50kN.m**

Step1: Equivalent shear

$$V_e = V_u + 1.6\frac{T_u}{b}$$

$$= 100 + 1.6 \times \frac{50}{0.3} = 366.67\text{kN}$$

$$\tau_{ve} = \frac{366.67 \times 10^3}{300 \times 800} = 1.53\,\text{N/mm}^2$$

For M15 concrete, $\tau_{c,max} = 2.5$ MPa

Since tensile reinforcement is not known at the outset, therefore the minimum % of tension steel is

$$100\frac{A_{st}}{bd} = 100 \times \frac{0.85}{f_y} = 100 \times \frac{0.85}{250} = 0.34\%$$

$$\tau_c = 0.35 + \frac{(0.46 - 0.35)}{(0.5 - 0.25)} \times (0.34 - 0.25) = 0.39\text{MPa} < \tau_{ve}$$

Hence both the longitudinal and transverse reinforcement shall be provided

Equivalent Bending Moment

$$M_{el} = M_u + M_t$$

$$= 200 + T_u \cdot \frac{(1 + D/b)}{1.7}$$

$$= 200 + 50 \times \frac{(1 + 850/300)}{1.7}$$

$$= 200 + 112.75$$

$$= 312.75\text{kNm}$$

Since $M_u > M_t$, *no* longitudinal reinforcement will be required on compression flange.

Longitudinal Reinforcement

$$M_{e1} = 0.87 f_y A_{st} d (1 - \frac{A_{st} f_y}{b d f_{ck}})$$

$$312.75 \times 10^6 = 0.87 \times 250 \times A_{st} \times 800 (1 - \frac{A_{st} \times 250}{300 \times 800 \times 15})$$

$$12.08 A_{st}^2 - 174000 A_{st} + 312.75 \times 10^6 = 0$$

$$A_{st} = 2105.06 \text{mm}^2$$

Provided $4 \phi 28$

$$100 \times \frac{A_{st}}{bd} \% = 100 \times \frac{4 \times \frac{\pi}{4} \times 28^2}{300 \times 800}$$

$$= 1.03\% > 0.34\% \ (A_{st,min})$$

Now revised τ_c is given as

$$\tau_c = 0.6 + \frac{(0.64 - 0.6)}{(1.25 - 1.0)} \times (1.03 - 1.0) = 0.605 \text{MPa}$$

Transverse Reinforcement

$$A_{sv} = \frac{T_u s_v}{b_1 d_1 (0.87 f_y)} + \frac{V_u s_v}{2.5 d_1 (0.87 f_y)}$$

Providing side and top cover of 30mm and $2 \phi 10$ bars at the top

$$b_1 = 300 - 30 - 30 - \frac{28}{2} - \frac{28}{2} = 212 \text{mm}$$

$$d_1 = 800 - 30 - \frac{10}{2} = 765 \text{mm}$$

Assuming ϕ 8 two-legged stirrups

$$A_{s\upsilon} = 2 \times \frac{\pi}{4} \times 8^2 = 100.53 \text{ mm}^2$$

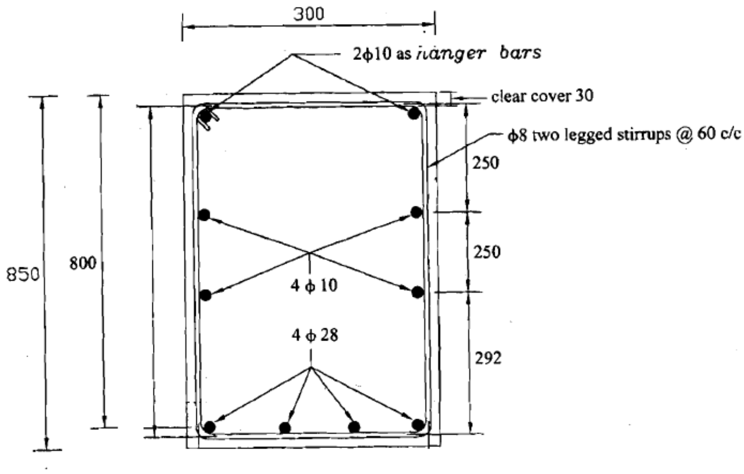
Substituting these values in the above equation

$$100.53 = \frac{50 \times 10^6 s_\upsilon}{212 \times 765 \times (0.87 \times 250)} + \frac{100 \times 1000 s_\upsilon}{2.5 \times 765 \times (0.87 \times 250)}$$

$$s_\upsilon = 60.64 \text{ mm}$$

Provided 2 ϕ 10 on each face.

The arrangement of reinforcements is shown in Figure



ANALYSIS AND DESIGN OF CANTILEVER SLABS

INTRODUCTION

A slab is like a flat plate loaded transversely and supported on its edges. Under the loads, it bends and the directions of its bending depend on its shape and support conditions. A beam bends only in one direction, i.e. in its own plane; where as a slab may have multidirectional bending. Therefore, slabs may have different names depending upon its bending, support conditions and shapes. For example, a slab may be called

(a) One-way simply supported rectangular slab,
(b) Two-way simply supported or restrained rectangular slab,
(c) Cantilever rectangular slab,
(d) Fixed or simply supported circular slab, etc

One-way slab means it bends only in one direction and, therefore, reinforcement for bending (i.e. main reinforcement) is provided only in that direction. A slab supported on all sides bends in all the directions so the main reinforcements provided shall be such that they may be effective in all directions. For ease of analysis and convenience of reinforcement detailing, the bending moments in a slab are calculated in two principal directions only and, therefore, such a slab is called a two-way slab.

A slab is designed as a beam of unit width in the direction of bending. In this unit, only the most commonly used rectangular slabs, with uniformly distributed load is described.

Objectives

After studying this unit, you should be able to

• describe the design and detailing of cantilever slabs,
• design and explain detailing of one-way and two-way simply supported slabs, and
• explain the design and detailing of two-way restrained slabs

GENERAL PRINCIPLES OF DESIGN AND DETAILING OF SLABS

Following are the general principles for design and detailing applicable to all types of slabs

(a) The maximum diameter of reinforcing bars shall not exceed1/8 th of total thickness (D) of the slab.

(b) Normally, shear reinforcement is not provided in slabs. The shear resistance requirements may, then, be complied either by increasing the percentage of tensile reinforcement or by increasing the depth of slab, but the latter is preferred as it is economical. For solid slabs, the design shear strength for concrete slab shall be τ ,c, K, , where K has the values given IS 800.

(c) To take care of temperature and shrinkage stresses, minimum reinforcement in either direction shall not be less than 0.15 percent and 0.12 percent of total cross section area of concrete section for mild steel and high strength deformed bars, respectively.

(d) To meet the requirement for limit state of cracking the following two rules are observed:

(i) The horizontal distance between parallel main reinforcement shall not be more than 3 times the effective depth of slab or 300 mm whichever is smaller.

(ii) The horizontal distance between parallel bars provided against temperature and shrinkage shall not be more than 5 d or 450 mm, whichever is smaller

DESIGN OF SLAB

Definition :

Slab is a thin flexural member used as a floor of structure to support the imposed load

Loads on slab :

Generally in design of horizontal slab two types of loads are considered.

- Dead load
- Imposed load

Dead load :

The dead load in slab comprises of the immovable partitions. Floor finishes weathering courses and primarily its weight .The dead loads are to be determined based on the weight of the materials .

Imposed loads:

Imposed load is the load induced by the intent use or occupancy of the building including the weight of movable partitions load due to impact vibrations.

Basic rules for the design of the slab :

The two main factors to be considered while designing the slab are:

- Strength of the slab against flexure, shear, twisted.
- Stiffness against deflection

One way slab – codal requirements :

When the ratio of the longer span to shorter span is greater than 2, it is called one way slab and bending takes place along one direction. The loads on the slab is transferred to the supports only on the main reinforcement. Hence main reinforcement is provided in the shorter span.

Minimum requirement in slab :

As per clause 26.5.2.1 of IS 456:2000, the reinforcement in either direction ,in slabs shall not be less than 0.12% of the total cross sectional area , when HYSD bars Fe415 are used.

Maximum size of bars in slabs

As per clause 26.5.2.2 of IS 456 :2000 , the reinforcing bars shall not exceed 1/8 of the total thickness of the slab.

DESIGN OF CANTILEVER SLAB

Design a cantilever chajja slab projecting 1m from the support using M20 grade concreteand Fe415 HYSD bars. Adopt a live load of 3kN/m^2.

i. Given

$L = 1$ m

$q = 3$ kN/m^2

$f_{ck} = 20$ N/mm^2

$f_y = 415$ N/mm^2

$\tau_{bd} = 1.2$N/mm^2 for plain bars for

M20grade concrete

ii. Depth of slab

Effective depth d = (span/7)

= 1000/7 = 142.8 mm

Adopt d = 150 mm

D = 175 mm

Adopt maximum depth of 150 mm at support gradually reducing to 100 mm at the free end.

iii. Loads

Self-weight of slab = 0.5 (0.15 + 0.10) 2.5

= 3.125 kN/m

Live load = 3.000

Finishes = 0.875 kN/m

Total working load = 7.000 kN/m

Design ultimate load w_u = (1.5 x 7.00)

= 10.5 kN/m

iv. Ultimate design moments and shear forces

M_u = $0.5\ w_u L^2$

= $0.5 \times 10.5 \times 1^2$

= 5.25 kNm

V_u = $w_u\ l$

= 10.5 x 1

= 10.50 kN

v. Check for depth

$M_{u\ lim}$ = $0.138\ f_{ck}\ bd^2$

= $(0.138 \times 20 \times 10^3 \times 150^2)\ 10^{-6}$

$= 62.10$ kNm

Since $M_u < M_{u\,lim}$,

Section is under – reinforced.

vi. Reinforcements

$$M_u = 0.87 f_y Ast\, d \left(1 - \frac{f_y Ast}{f_{ck} bd}\right)$$

$$5.25 \times 10^6 = 0.87 \times 415 \times Ast \times 150 \left(1 - \frac{140\, Ast}{20 \times 1000 \times 150}\right)$$

Solving Ast $= 105\ mm^2 < Ast_{min}$

Hence provide 10 mm diameter bars at 300 mm centres (Ast = 262 mm^2) in the span direction and the same as distribution reinforcement.

vii. Anchorage length

$$L_d = \frac{0.87 f_y \phi}{4\tau_{bd}}$$

$$= \frac{0.87 \times 415 \times 10}{4 \times 1.2 \times 1.6}$$

$= 470$ mm

viii. Check for deflection control

$$\left(\frac{L}{d}\right)_{max} = \left(\frac{L}{d}\right)_{Basic} \times k_t$$

And $k_c = k_f = 1.00$

$$p_t = \frac{100\, Ast}{bd}$$

$$= \frac{100 \times 262}{10^3 \times 150}$$

$= 0.174$ mm

$k_t = 1.8$

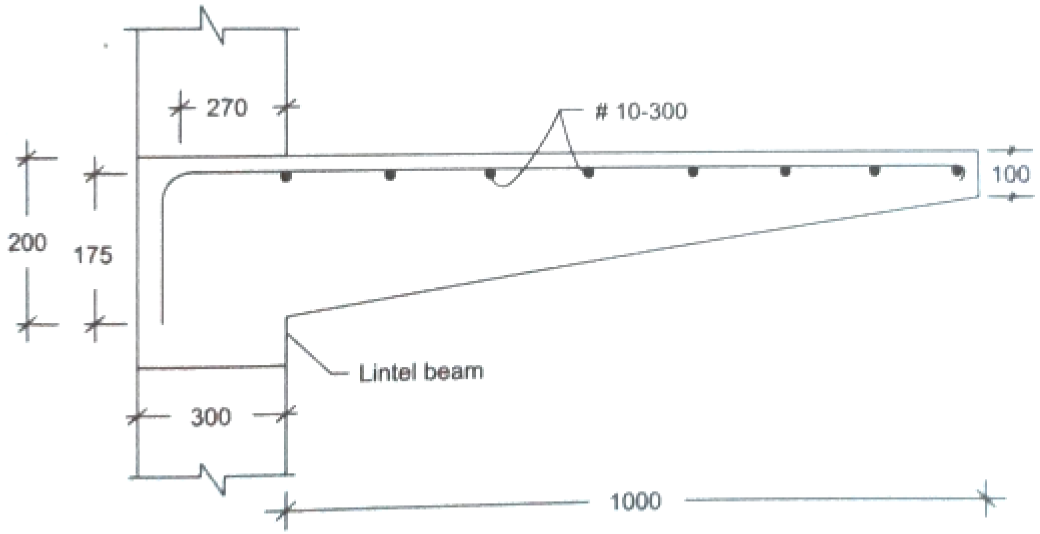
Hence $\left(\frac{L}{d}\right)_{max} = 2.7 \times 1.8 = 12.6$

$$\left(\frac{L}{d}\right)_{Actual} = \frac{1000}{150} = 6.66 < 12.6$$

Hence the slab satisfies the deflection criteria.

ix. Reinforcement details

The reinforcement details in the cantilever slab is shown in fig.



ONE WAY SLAB DESIGN

Design a simply supported one–way slab over a clear span of 3.5 m. It carries a live load of 4kN/m^2 and floor finish of 1.5kN/m^2. The width of supporting wall is 230 mm. Adopt M-20 concrete & Fe-415 steel.

Step: 1 Depth of slab

Assume approximate depth d = L/26

3500/26 = 134 mm

Assume overall depth D = 160 mm

& clear cover 15mm for mild exposured = 160 – 25 = 140mm

Effective span is lesser of the two

i. l = 3.5 + 0.23 (width of support) = 3.73 m

ii. l = 3.5 + 0.14 (effective depth) = 3.64 m

Effective span = 3.64 m

Step: 2 Load on slab

Self-weight of slab = 0.16 x 25 = 4.00

Live load = 4.00

Floor finish = 1.50

Total Load W = 9.5 kN/m^2

Ultimate load Wu = 9.5 x 1.5 = 14.25 kN/m^2

Step 3: Design bending moment and check for depth

$$M_u = \frac{W_u\, l^2}{8}$$

$$= \frac{14.25 \text{ x } 3.64^2}{8}$$

$$M_u = 23.60 \text{ kN/m}$$

Minimum depth required from BM consideration

$$d = \sqrt{\frac{M_u}{0.138\, f_{ck} b}}$$

$$= \sqrt{\frac{23.60 \times 10^6}{0.138 \times 20 \times 1000}}$$

$$d = 92.4 > 140 \quad \text{(OK)}$$

Step: 4 Area of Reinforcement

Area of steel is obtained using the following equation

$$M_u = 0.87\, f_y\, Ast\, d\, (1 - \frac{f_y Ast}{f_{ck} bd})$$

$$23.60 \times 10^6 = 0.87 \times 415 \times Ast \times 140\, (1 - \frac{415\, Ast}{20 \times 1000 \times 140})$$

$$23.60 \times 10^6 = 50547\, Ast - 7.49\, Ast^2$$

$$\text{Solving } Ast = 504 \text{ mm}^2$$

OR

$$Ast = \frac{0.5 f_{ck}}{f_y} \left[1 - \sqrt{1 - \frac{4.6\, M_u}{f_{ck}\, bd^2}} \right] bd$$

$$Ast = \frac{0.5 \times 20}{415} \left[1 - \sqrt{1 - \frac{4.6 \times 23.60 \times 10^6}{20 \times 1000 \times 140^2}}\right] 1000 \times 140$$

$$= 505 \text{ mm}^2$$

$$\text{Spacing of 10mm } S_v = \frac{ast}{Ast} \times 1000$$

$$S_v = \frac{78}{505} \times 1000 = 154\text{mm}$$

Provide 10mm @ 150C/C.

Distribution steel@ 0.12% of the Gross area

$$\frac{0.12}{100} \times 1000 \times 160 = 192 \text{ mm}^2$$

$$\text{Spacing of 10mm } S_v = \frac{50}{192} \times 1000 = 260 \text{ mm}$$

Provide 8mm @ 260mm

Step: 5 Check for shear

$$\text{Design shear } V_u = \frac{W_u\, l}{2}$$

$$= \frac{14.25 \times 3.64}{2}$$

$$= 25.93 \text{ kN}$$

$$\tau_v = \frac{25.93 \times 10^3}{1000 \times 140}$$

$$= 0.18 \text{ N/mm}^2 \qquad (<\tau_{c\,max} = 28 \text{ N/mm}^2)$$

Shear resisted by concrete $\boldsymbol{\tau_c}$ = 0.42 for $\mathbf{p_t}$ = 0.37 (Table 19, IS 456-2000)

$$\boldsymbol{\tau_c > \tau_v}$$

Step: 6 Check for Deflection

$$\left(\frac{l}{d}\right)_{Actual} < \left(\frac{l}{d}\right)_{Allowable}$$

$$\left(\frac{l}{d}\right)_{Allowable} = \left(\frac{l}{d}\right)_{Basic} \text{ x } k_1 \text{ x } k_2 \text{ x } k_3 \text{ x } k_4$$

k_1 - Modification factor for tension steel

k_2 - Modification factor for compression steel

k_3 - Modification factor for T-sections

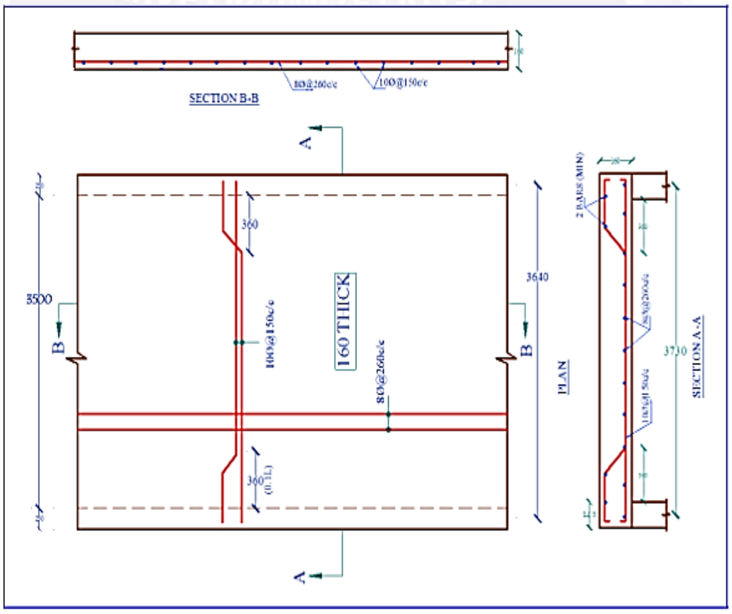
if span exceeds 10 m (10/span)

$\mathbf{k_1}$ = 1.38 for $\mathbf{p_t}$ = 0.37 (Fig. 4, cl.32.2.1)

$$\left(\frac{l}{d}\right)_{Allowable} = 20 \text{ x } 1.38 = 27.6$$

$$\left(\frac{l}{d}\right)_{Actual} = 3630/140 = 25.92$$

$$\left(\frac{l}{d}\right)_{Actual} < \left(\frac{l}{d}\right)_{Allowable} \quad \text{(OK)}$$



DESIGN OF TWO WAY SLAB

Definition :

Slab is a thin flexural member used as a floor of structure to support the imposed load

Loads on slab :

Generally in design of horizontal slab two types of loads are considered.

- Dead load
- Imposed load

Dead load :

The dead load in slab comprises of the immovable partitions. Floor finishes weathering courses and primarily its weight .The dead loads are to be determined based on the weight of the materials .

Imposed loads:

Imposed load is the load induced by the intent use or occupancy of the building including the weight of movable partitions load due to impact vibrations.

Basic rules for the design of the slab :

The two main factors to be considered while designing the slab are:

- Strength of the slab against flexure, shear, twisted.
- Stiffness against deflection

One way slab – codal requirements :

When the ratio of the longer span to shorter span is greater than 2, it is called one way slab and bending takes place along one direction. The loads on the slab is transferred to the supports only on the main reinforcement. Hence main reinforcement is provided in the shorter span.

Minimum requirement in slab :

As per clause 26.5.2.1 of IS 456:2000, the reinforcement in either direction ,in slabs shall not be less than 0.12% of the total cross sectional area , when HYSD bars Fe415 are used.

Maximum size of bars in slabs

As per clause 26.5.2.2 of IS 456 :2000 , the reinforcing bars shall not exceed 1/8 of the total thickness of the slab.

TWO WAY SLAB DESIGN

Design a R.C Slab for a room measuring 6.5mx5m. The slab is cast monolithically over the beams with corners held down. The width of the supporting beam is 230 mm.The slab carries superimposed load of 4.5kN/m^2. Use M-20 concrete and Fe-500 Steel.

Since, the ratio of length to width of slab is less than 2 and slab is resting on beam, the slab is designed as two way restrained slab.

Step: 1 Depth of slab and effective span

Assume approximate depth d = l/30

= 5000/30 = 166mm

Assume D = 180 mm

& clear cover 15 mm for mild exposured d

= 180-20 = 160 mm.

Effective span is lesser of the two

i) ly = 6.5+0.23 = 6.73 m ,

lx = 5.0+0.23 = 5.23 m

ii) ly = 6.5+0.16 = 6.66 m,

lx = 5+0.16 = 5.16 m

ly = 6.66 m

lx = 5.16 m

$$\alpha = \frac{l_y}{l_x} = \frac{6.66}{5.16} = 1.3$$

Step 2: Load Calculation

Self-weight of slab = 0.18X25 = 4.50 kN/m^2

Super imposed load = 4.50

Total load = 9.0 kN/m^2

Ultimate load Wu = 9X1.5 = 13.5 kN/m^2

Step 3: Design bending moment and check for depth

The boundary condition of slab in all four edges discontinuous

(case 9, Table 9.5.2)

$$M_x = \propto_x W_u {l_x}^2$$

$$M_y = \propto_y W_u {l_x}^2$$

For $\frac{l_y}{l_x} = 1.3,$

$$\propto_x = 0.079$$

$$\propto_y = 0.056$$

Positive moment at mid span of short span M_x = 0.079 x 13.5 x 5.16^2

= 28.40 kNm

Positive moment at mid span of longer span M_x = 0.056 x 13.5 x 5.16^2

= 20.13 kNm

Minimum depth required from maximum BM consideration

$$d = \sqrt{\frac{M_u}{0.138\, f_{ck} b}}$$

$$= \sqrt{\frac{28.40 \text{ x } 10^6}{0.138 \text{ x } 20 \text{ x } 1000}}$$

$$d = 103 \text{ mm}$$

However, provide d = 160 mm

Step: 4 Area of Reinforcement

Area of steel is obtained using the following equation.

$$M_u = 0.87\, f_y\, Ast\, d\, (1 - \frac{f_y Ast}{f_{ck} bd})$$

Steel along shorter direction (M_x)

$$28.40 \text{ x } 10^6 = 0.87 \text{ x } 500 \text{x } Ast \text{ x } 160\, (1 - \frac{500\, Ast}{20 \times 1000 \times 160})$$

$28.40 \times 10^6 = 69600\,Ast - 10.875\,Ast^2$

Solving $Ast = 438\ mm^2$

Provide 10 mm @ 175 C/C ($P_t = 0.27\%$)

Steel along shorter direction (M_y)

Since long span bars are placed above short span bars d = 160-10 = 150

$$20.13 \times 10^6 = 0.87 \times 500 \times Ast \times 150\left(1 - \frac{500\ Ast}{20 \times 1000 \times 150}\right)$$

$20.13 \times 10^6 = 65250\,Ast - 10.875\,Ast^2$

Solving, $Ast = 327\ mm^2$

Spacing at 10 mm;

$$\frac{79}{327} \times 100 = 241$$

Provide 10 mm @ 240 mm C/C (<3d = 450)

Step: 5 Check for shear

$$\text{Design shear } V_u = \frac{W_u\, l}{2}$$

$$= \frac{13.5 \times 5.16}{2}$$

$$= 34.83\ kN$$

$$\tau_v = \frac{34.83 \times 10^3}{1000 \times 160}$$

$$= 0.217\ N/mm^2 \qquad (<\tau_{c\,max} = 28\ N/mm^2)$$

Shear resisted by concrete $\boldsymbol{\tau_c}$ = 0.42 for $\mathbf{p_t}$ = 0.37 (Table 19, IS 456-2000)

$$\boldsymbol{\tau_c} > \boldsymbol{\tau_v}$$

Step: 6 Check for Deflection

$$\left(\frac{l}{d}\right)_{Allowable} = \left(\frac{l}{d}\right)_{Basic} \times k_1$$

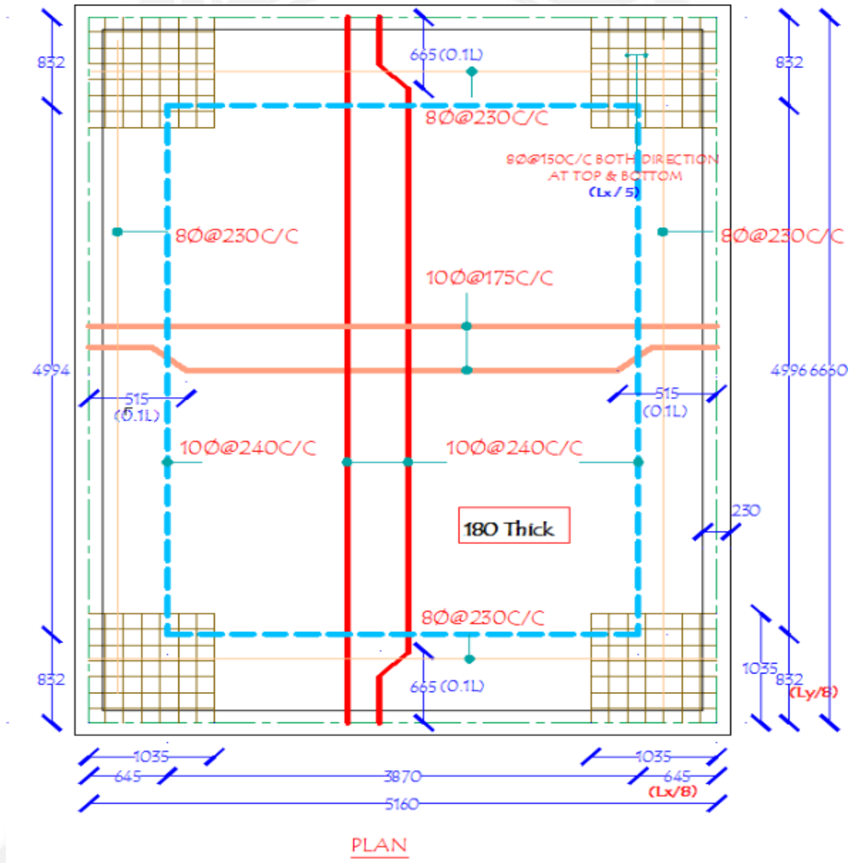
$\mathbf{k_1}$ = 1.5 for $\mathbf{p_t}$ = 0.27% & $f_s = 0.58 \times f_y = 240$

(Fig. 4, cl.32.2.1, IS 456-2000)

$\left(\frac{l}{d}\right)_{\text{Allowable}} = 26 \times 1.5 = 39$

$\left(\frac{l}{d}\right)_{\text{Actual}} = 5.16/0.16 = 32$

$\left(\frac{l}{d}\right)_{\text{Actual}} < \left(\frac{l}{d}\right)_{\text{Allowable}}$ (OK)



Reinforcement Detail of Two way Restrained slab

UNIT –III DESIGN OF SLABS AND STAIRCASE

3.3 DESIGN OF SIMPLY SUPPORTED AND CONTINUOUS SLABS USING IS CODE

DESIGN EXAMPLES

1.A slab has clear dimensions 4 m x 6 m with wall thickness 230 mm the live load on the slab is 5 kN/m^2 and a finishing load of 1kN/m^2 may be assumed. Using M20 concrete and Fe415 steel, design the slab

Given data

Dimension = 4 x 6

Shorter span 1_x = 4m

Longer span 1_y = 6m

$$\frac{l_y}{l_x} = \frac{6}{4}$$

= 1.5 < 2

It is a two way slab.

Width of support = 230 mm

Live load = 5 kN/m^2

Materials ,f_{ck} = 20 N/mm^2

F_y = 415 N/mm^2

Depth of slab:

Effective depth d = $\frac{span}{25}$

= $\frac{4000}{25}$

= 160 mm

Assume cover 20mm, 10mm diameter rod

Overall depth D = 160 + 20 + $\frac{10}{2}$

=185mm

D = 200 mm

Effective span:

1. c/c of supports $l_e = \frac{wall\ thickness}{2} + shorter\ span + \frac{wall\ thickness}{2}$

$$= \frac{0.23}{2} + 4 + \frac{0.23}{2}$$

$$= 4.23 \text{ m}$$

2. clear span + effective depth = 4 + 0.24

$$= 4.24\text{m}$$

Take least value, $1_e = 4.23$ m

Load calculation:

Self weight= B X D X γ

= 1 X 0.2 X 25

= 5 kN/ m

Live load = 5 kN/m

Floor finish = 1 kN/m

Total load = 5 + 5 + 1

= 11 kN/ m

Factor load = 1.5 x 11

= 16.5 kN/ m

Bending moment & shear force:

$$M_X = \alpha_X W_U l_e^2$$

$$M_y = \alpha_y W_U l_e^2$$

From table 26 of IS 456: 2000

$$\frac{ly}{\text{lx}} = 1.5$$

Four edges are discontinuous,

$\alpha_X = 0.089$

$\alpha_y = 0.056$

Bending moment:

$$M_X = 15.59\text{x}4.2^2\text{x}0.089$$

$= 25.01$ kNm

$M_Y = 0.056 \times 15.93 \times 4.2^2$

$= 15.73$ kNm

Shear force :

$$SF = \frac{Wule}{2}$$

$$= \frac{15.93 \times 4.2}{2}$$

$$= 33.45 \text{ KN}$$

Check for Depth :

$M_U = 0.138\, f_{ck} b d^2$

$$d = \sqrt{\frac{25 \times 10^6}{0.138 \times 20 \times 1000}}$$

$$= 95.17 \text{ mm}$$

$d_{prov} > d_{req}$

Hence the design is safe.

Area of reinforcement:

For shorter span:

$$M_U = 0.87\, f_y \times A_{st} \times d\, [1 - \frac{Ast \times fy}{b \times d \times fck}]$$

$$25 \times 10^6 = 0.87 \times 415 \times A_{st} \times 160\, [1 - \frac{Ast \times 415}{1000 \times 160 \times 20}]$$

$$25 \times 10^6 = 57768\, A_{st} - 7.4\, A_{st}^2$$

$$A_{st} = 459.85 \text{ mm}^2$$

$$A_{st\ min} = 0.12\% \times bd$$

$$= \frac{0.12}{100} \times 1000 \times 200$$

$$= 240 \text{ mm}^2$$

Provide 10mm dia bar.

Spacing :

$i.$ $\frac{ast}{Ast} \times 1000 = \frac{\pi/4 \times 10^2}{459.85} \times 1000$

$= 170.79$ mm ≈ 170mm

ii. 3d = 3 x 160 = 480 mm

take the least value = 170 mm

provide 10 mm dia bar 170 mm c/c.

For longer span:

$$M_U = 0.87\, f_y \times A_{st} \times d\, [1 - \frac{Ast \times fy}{b \times d \times fck}]$$

$$15.73 \times 10^6 = 0.87 \times 415 \times A_{st} \times 160\, [1- \frac{Ast \times 415}{1000 \times 160 \times 20}\ ]$$

$$A_{st} = 282.52\ mm^2$$

Spacing :

$$i)\ \frac{ast}{Ast} \times 1000 = \frac{\pi/_4 \times 10^2}{282.52} \times 1000\ = 277.99mm \approx 300\ mm$$

ii) 3d = 3×160

= 480mm

Take the least value for spacing = 300mm,

provide 10mm diameter bar, 300m

Check for shear:

Permissible shear stress, $\tau_v = \frac{Vu}{bd}$

$$= \frac{33.45 \times 10^3}{1000 \times 160} = 0.2 N/mm^2$$

Nominal shear stress = $\tau_c \times K$

To find τ_c ,

Percentage of steel, $p_t = 100 \times \frac{Ast}{b \times d}$

$$= 100 \times \frac{459.85}{1000 \times 160}$$

$$= 0.28\%$$

The value lies between 0.25 and 0.50, use interpolation

X_1	0.25	Y_1	0.36	X	0.28
X_2	0.5	Y2	0.48	Y	?

$$Y = \tau_c = y_1 + \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$$

$$= 0.36 + \frac{0.48 - 0.36}{0.50 - 0.25}(0.28 - 0.25)$$

$$= 0.37 N/mm^2$$

To find K ,

Overall depth, D = 185mm

Refer pg no:73 of IS 456-2000

This value lies between 150 to 175, use interpolation

X_1	150	Y_1	1.3	X	185
X_2	175	Y2	1.25	Y	?

$$Y = K = y_1 + \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$$

$$= 1.3 + \frac{1.25 - 1.3}{175 - 150}(185 - 150)$$

$$= 1.27$$

$\tau_c \times K = 0.38 \times 1.27$

$$= 0.48 N/mm^2$$

$\tau_v < \tau_c \times K$,

Hence the design is safe.

Check for deflection:

$$\frac{l}{d}_{max} = \frac{l}{d}_{basic} \times K_b \times K_c$$

$$= 20 \times 1.4 \times 1 = 30$$

$$\frac{l}{d}_{pro} = \frac{\text{Effective span}}{\text{Effective depth}}$$

$$= \frac{4000}{160} = 26.25mm$$

$$(\frac{l}{d})_{max} > (\frac{l}{d})_{pro}$$

Hence the design is safe for deflection.

Check for crack control:

1. Reinforcement provided must be greater than minimum percentage of reinforcement provided as per IS 456-2000.

$A_{st\,min}$ = 0.12% of cross section area

$= 0.12/100 \times 1000 \times 185$

$= 222\ mm^2$

$A_{st\ pro} > A_{stmin}$,

Hence it is safe.

2. Spacing is not greater than 3d.

$3d = 3 \times 160$

$= 480mm$

Spacing < 3d,

Hence it is safe.

3. Diameter of reinforcement should be less than $\frac{D}{8}$

$d < \frac{D}{8}$

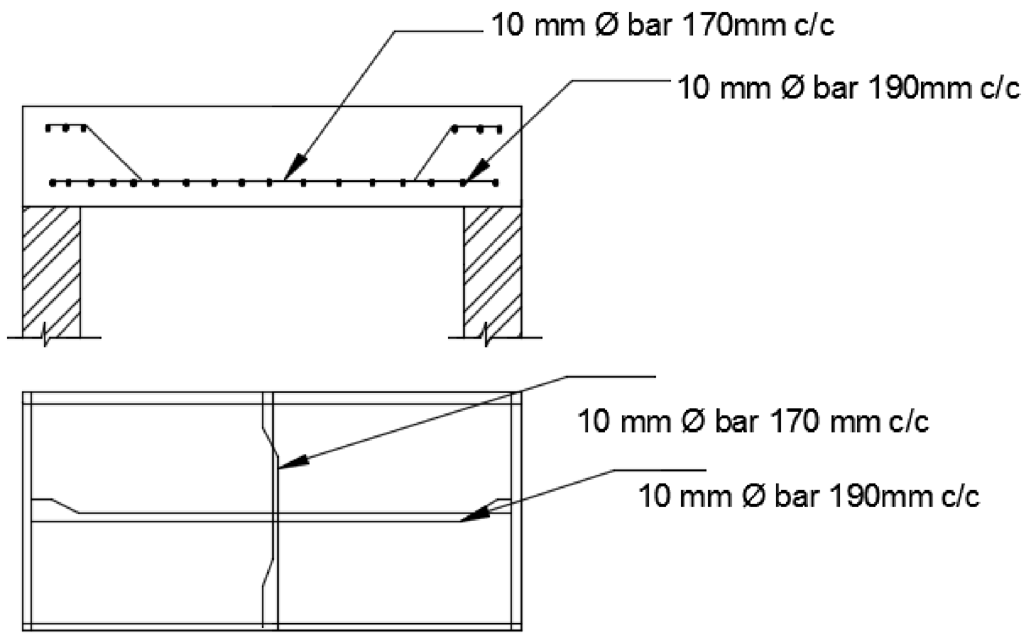
$\frac{D}{8} = \frac{185}{8}$

$= 28.12mm$

$d < \frac{D}{8}$

Hence it is safe.

Reinforcement detailing:



2.A slab has clear dimensions 3.5 m x 6 m with wall thickness 230 mm the live load on the slab is 5 kN/m^2 and a finishing load of 1kN/m^2 may be assumed. Using M20 concrete and Fe415 steel, design the slab

Given data

Dimension = 3.5 x 6

Shorter span 1_x = 3.5

Longer span 1_y = 6

$$\frac{ly}{lx} = \frac{6}{3.5}$$

= 1.7 < 2

It is a two way slab

Width of support = 230 mm

Live load = 5 kN/m^2

Materials ,f_{ck} = 20 N/mm^2

F_y = 415 N/mm^2

Depth of slab,

Effective depth, d $= \frac{span}{25}$

$= \frac{3500}{25}$

Assume cover 20mm, 10mm diameter rod

Overall depth, D = 140 + 20 + 10/2

=165mm

= 125 mm

Effective span:

i. c/c of supports $l_e = \frac{wall\ thickness}{2} + shorter\ span + \frac{wall\ thickness}{2}$

$$= \frac{0.23}{2} + 3.5 + \frac{0.23}{2}$$

= 3.73 m

ii. clear span + effective depth = 3.5 + 0.14

= 3.64

Take least value, l_e = 2.6 m

Load calculation:

Self weight = B X D X γ

= 1 X 0.165 X 25

= 4. 13 KN/ m

Live load = 5 KN/m

Floor finish = 1 KN/m

Total load = 4.13 + 5 + 1

= 10.13 KN/ m

Factor load = 1.5 x 10.13

= 15.2 KN/ m

Bending moment & shear force:

$$M_X = \alpha_X W_U l_e^2$$

$$M_y = \alpha_y W_U l_e^2$$

From table 26 of IS 456: 2000

$$\frac{ly}{lx} = 1.7$$

Four edges are discontinuous,

$$\alpha_X = 0.098$$

$$\alpha_y = 0.056$$

Bending moment:

$$M_X = 0.098 \text{ x } 15.2 \text{ x } 3.64^2$$

$$= 19.74 \text{ KNm}$$

$$M_Y = 0.056 \text{ x } 15.2 \text{ x } 3.64^2$$

$= 11.24$ KNm

Shear force :

$$SF = W_U l_e/2$$

$$= (15.2 \times 3.64)/2$$

$$= 27.66 \text{ KN}$$

Check for Depth :

$$M_U = 0.138 f_{ck} bd^2$$

$$d = \sqrt{\frac{19.74\times10^6}{0.138\times20\times1000}}$$

$$= 84.57 \text{ mm}$$

$$d_{prov} > d_{req}$$

Hence the design is safe

Area of reinforcement:

For shorter span:

$$M_U = 0.87 f_y \times A_{st} \times d\, [1 - \frac{Ast \times fy}{b\times d\times fck}]$$

$$19.74 \times10^6 = 0.87 \times 415 \times A_{st} \times 140\, [1- \frac{Ast \times 415}{1000\times140\times20}]$$

$$19.74 \times 10^6 = 50547 A_{st} - 7.49 A_{st}^2$$

$$A_{st} = 416.19 \text{ mm}^2$$

$$A_{st\,min} = 0.12\% \times bd$$

$$= \frac{0.12}{100} \times 1000\times 165$$

$$= 198 \text{ mm}^2$$

Provide 10mm dia bar

Spacing :

$i\ \frac{ast}{Ast}\times 1000$

$$= \frac{\pi/4 \times10^2}{416.9} \times 1000$$

$$= 188.7 \text{ mm}$$

$$\approx 180\text{mm}$$

ii. 3d $= 3 \times 140$

$= 420$ mm

Take the least value for spacing

provide 10 mm dia bar 180 mm c/c

For longer span:

$$M_U = 0.87\, f_y \times A_{st} \times d\, [1 - \frac{Ast \times fy}{b \times d \times fck}]$$

$$11.24 \times 10^6 = 0.87 \times 415 \times A_{st} \times 140\, [1 - \frac{Ast \times 415}{1000 \times 100 \times 20}\ ]$$

$A_{st} = 230.2 mm^2$

Spacing :

i. $\frac{ast}{Ast} \times 1000$ $= \frac{\pi/_4 \times 10^2}{230.2} \times 1000$

$= 323.72$mm

≈ 300mm

ii. 3d $= 5 \times 140$

$= 800$mm

iii. 300 mm

Take the least value for spacing

provide 10mm diameter bar, 300mm c/c

Check for shear:

Permissible shear stress, $\tau_v = {}^{V_u}/_{b \times d}$

$= \frac{27.66 \times 10^3}{1000 \times 140}$

$= 0.19 N/mm^2$

Nominal shear stress $= \tau_c \times K$

To find τ_c ,

Percentage of steel, $p_t = 100 \times \frac{Ast}{b \times d}$

$= 100 \times \frac{416.69}{1000 \times 140}$

$= 0.29\%$

The value lies between 0.25 and 0.50, use interpolation

X_1	0.25	Y_1	0.36	X	0.29
X_2	0.5	Y2	0.48	Y	?

$$Y = \tau_c = y_1 + \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$$

$$= 0.36 + \frac{0.48 - 0.36}{0.50 - 0.25}(0.29 - 0.25)$$

$$= 0.38 \text{N/mm}^2$$

To find K ,

Overall depth, D = 165mm

This value lies between 150 to 175, use interpolation

X_1	150	Y_1	1.3	X	165
X_2	175	Y2	1.25	Y	?

$$Y = K = y_1 + \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$$

$$= 1.3 + \frac{1.25 - 1.3}{175 - 150}(165 - 150)$$

$$= 1.27$$

$\tau_c \times K$ $= 0.38 \times 1.27$

$$= 0.48 \text{N/mm}^2$$

$$\tau_v < \tau_c \times K,$$

Hence the design is safe.

Check for deflection:

$(l/d)_{max}$ $= (l/d)_{basic} \times K_b \times K_c$

$$= 20 \times 1.5 \times 1$$

$$= 30$$

$(l/d)_{pro}$ $= \frac{\text{Effective span}}{\text{Effective depth}}$

$$= \frac{3.64}{0.14}$$

$= 26mm$

$(l/d)_{max} > (l/d)_{pro}$

Hence the design is safe for deflection.

Check for crack control:

4. Reinforcement provided must be greater than minimum percentage of reinforcement provided as per IS 456-2000.

$A_{st\,\mathbf{min}}$ = 0.12% of cross section area

$= 0.12/100 \times 1000 \times 165$

$= 198\ mm^2$

$A_{st\ pro} > A_{stmin}$,

Hence it is safe.

5. Spacing is not greater than 3d.

$3d = 3 \times 140$

$= 420mm$

Spacing < 3d

Hence it is safe.

6. Diameter of reinforcement should be less than $D/8$

$d < D/8$

$D/8 = 165/8$

$= 20.62mm$

$d < D/8$

Hence it is safe.

Torsion reinforcement in corners:

Area of reinforcement in each corners is,

$A_{st\ torsion} = 0.75 \times 416.19$

$= 312.14\ mm$

Spacing,

Provide 8 mm Ø bar

$\frac{ast}{Ast} \times 1000$ $= \frac{\pi/_4 \times 8^2}{312.14} \times 1000$

$= 161mm$

$\approx 160mm$

Length over which the torsion steel is provided,

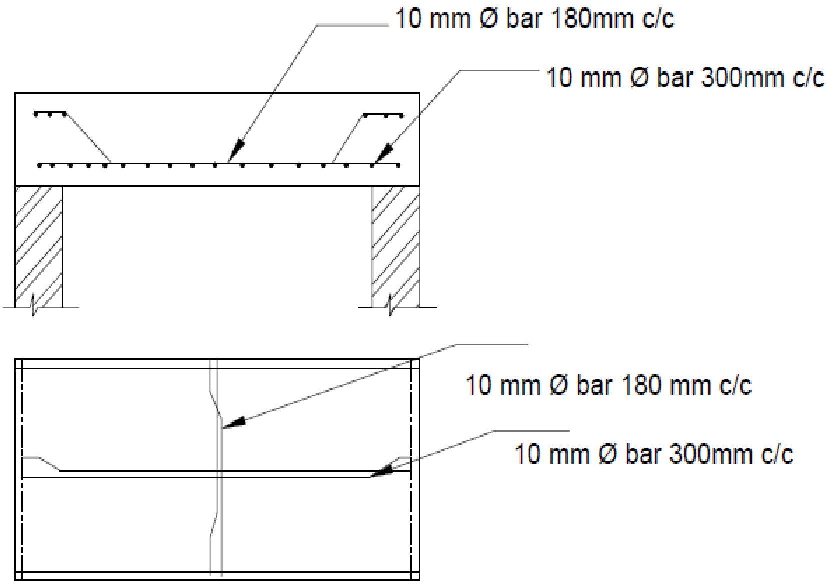
$= \frac{1}{5} \times$shorter span

$= \frac{1}{5} \times 3500$

$= 700$ mm

Provide 8 mm Ø bar 160mm c/c , for the length of 700 mm at the corners

Reinforcement details



CONTINUOUS SLAB DESIGN

Design a one-way slab for an office floor which is continuous over T beams at 3.5m intervals. Assume a live load 4kN/m^2 adopt M_{20} grade concrete and Fe_{415} steel HYSD bars.

Given:

$L = 3.5\ m$

$q = 4\ kN/m^2$

$f_{ck} = 20\ N/mm^2$

$f_y = 415\ N/mm^2$

Step: 1 Depth of slab

Assuming a span/depth ratio of 26 (Clause 23.2.1 of IS 456)

Effective depth $d = (span/26)$

$= 3500/26 = 135\ mm$

Adopt $d = 140\ mm$

$D = 160\ mm$

Step: 2 Load calculation

Self-weight of slab $= 0.165 \times 25 = 4.125\ kN/m^2$

Finishes = $0.875\ kN/m^2$

Total working load (g) = $5.000\ kN/m^2$

Service live load (q) = $4\ kN/m^2$

Step: 3 Bending moment calculation

Referring to Tables 12 and 13, IS 456-2000 code, maximum negative BM at support next to the end support is:

$$M_u\ (-ve) = 1.5\left[\frac{gL^2}{10} + \frac{qL^2}{9}\right]$$

$$= 1.5\left[\frac{5 \times 3.5^2}{10} + \frac{4 \times 3.5^2}{9}\right]$$

$$= 17.35\ kNm$$

Positive BM at centre of span

$$M_u\ (+ve) = 1.5\left[\frac{gL^2}{12} + \frac{qL^2}{10}\right]$$

$$= 1.5\left[\frac{5 \times 3.5^2}{12} + \frac{4 \times 3.5^2}{10}\right]$$

$$= 15\ kNm$$

Step: 4 Shear force calculation

Maximum shear force at the support

$$V_u = 1.5 \times 0.6\ (g + q)\ L$$

$$= (1.5 \times 0.6)\ (5 + 4)\ 3.5$$

$$= 28.35\ kN$$

Step: 5 Check for Depth of the slab

$$M_{u\,lim} = 0.138\ f_{ck}\, bd^2$$

$$= (0.138 \times 20 \times 10^3 \times 140^2)\ 10^{-6}$$

$$= 54.1\ \ kNm$$

Since $M_u < M_{u\,lim}$,

Section is under – reinforced.

Step: 6 Reinforcement details

$$M_u = 0.87\, f_y\, Ast\, d\, (1 - \frac{f_y Ast}{f_{ck} bd})$$

$$17.35 \times 10^6 = 0.87 \times 415 \times Ast \times 140\, (1 - \frac{140\, Ast}{20 \times 1000 \times 140})$$

Solving Ast = 360 mm^2

Provide 10 mm diameter bars at 150 mm centers (Ast = 524 mm^2). The same reinforcement is provided for positive BM at mid-span.

Distribution steel = $0.0012 \times 10^3 \times 165$

= 198 mm^2

Provide 10 mm diameter bars at 300 mm centers (Ast = 262 mm^2).

Step: 7 Check for shear stress

$$\tau_v = \frac{V_u}{bd}$$

$$= \frac{28.35 \times 10^3}{10^3 \times 140}$$

= 0.20 N/mm^2

$$p_t = \frac{100 \times Ast}{bd}$$

$$= \frac{100 \times 262}{10^3 \times 140}$$

= 0.187

Refer to Table 19, IS 456 and readout:

$k\tau_c$ = 1.27 x 0.30 = 0.38 N/mm^2

Since $\boldsymbol{\tau_c} > \boldsymbol{\tau_v}$, the sab is safe against shear stresses.

Step: 8 Check for Deflection

Considering the end and inferior spans

$$(\frac{L}{d})_{max} = (\frac{L}{d})_{Basic} \times k_t \times k_c \times k_f$$

Also $k_c = k_f = 1.00$

$$\mathbf{p_t} = \frac{\mathbf{100 \times 393}}{\mathbf{10^3 \times 140}}$$

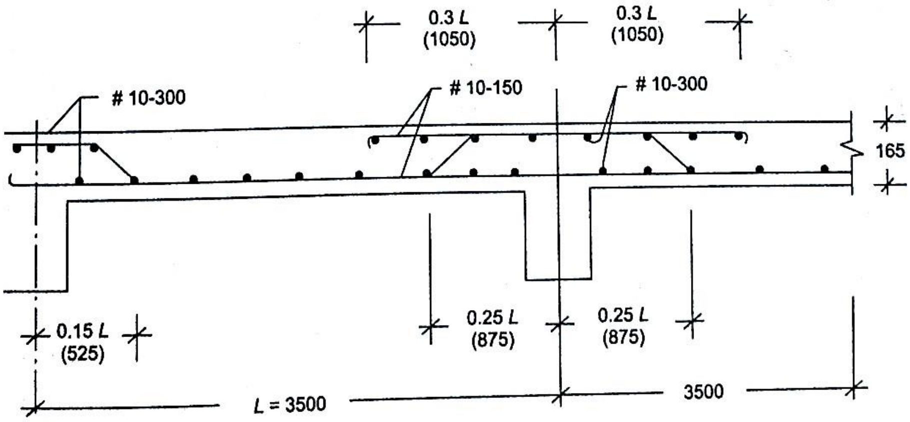
= 0.28

From Fig.8.1, read out k_t = 1.5

$\left(\frac{L}{d}\right)_{max} = \left(\frac{20+26}{2}\right)1.5 = 34.5$

$\left(\frac{L}{d}\right)_{Actual} = \frac{3500}{140} = 25 < 34.5$

Hence the slab is safe against deflection control.



UNIT –III DESIGN OF SLABS AND STAIRCASE

TYPES OF STAIRCASE

General

Staircases are generally provided connecting successive floors of a building and in small buildings. They are only means of access between the floors. The staircase comprises of flight of step generally with one or more intermediate landings provides between the floors level.

Dog-legged staircase is the most common type used in all types of buildings . it comprises of two adjacent flights running parallel with a landing slab at mid height.

Loads on staircases

The various types of loads to be resisted by the staircases are grouped under dead and live load

1. Dead load which includes the self-weight of the stair , tread and risers and self weight of finishes
2. Live load to be considered are specified in IS 875-1987 for residential buildings a uniformly distributed live load of 2 to 3 KN/m^2 depending upon the users and for public buildings, a uniformly distributed load of 5KN/m^2 is specifies in the code

TYPES OF STAIRCASE

- Straight stairs
- Quarter turn stairs
- Half turn stairs
- Spiral stairs
- Curved stairs
- Dog legged stair

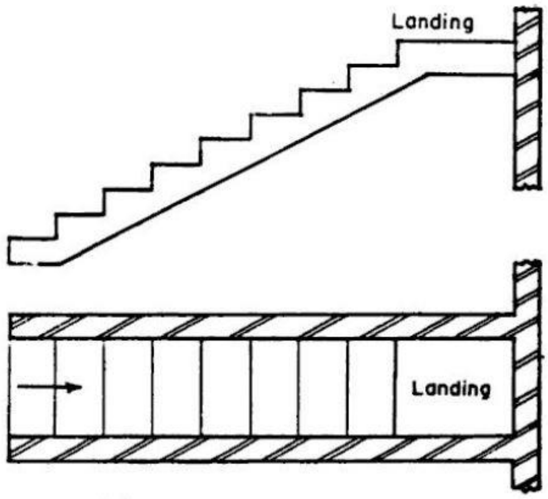
STRAIGHT STAIRS

These are the stairs along which there is no change in direction on any flight between two successive floors. The straight stairs can be of following types.

- Straight run with a single flight between floors
- Straight run with a series of flight without change in direction
- Parallel stairs
- Angle stairs

- Scissors stairs

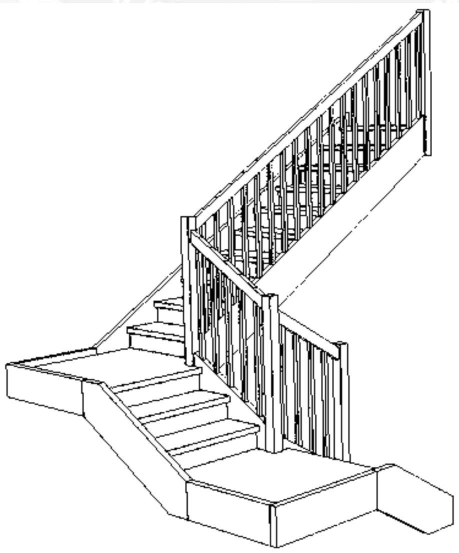
Straight stairs can have a change in direction at an intermediate landing. In case of angle stairs, the successive flights are at an angle to each other. Scissor stairs are comprised of a pair of straight runs in opposite directions and are placed on opposite sides of a fire resistive wall.



Straight Stair with Single Flight

QUARTER TURN STAIRS

They are provided when the direction of flight is to be changed by 90^0. The change in direction can be effected by either introducing a quarter space landing or by providing winders at the junctions.



HALF TURN STAIRS

These stairs change their direction through 180^0. It can be either dog-legged or open newel type. In case of dog legged stairs the flights are in opposite directions and no space is provided between the flights in plan. On the other hand in open newel stairs, there is a well or opening between the flights and it may be used to accommodate a lift. These stairs are used at places where sufficient space is available.

SPIRAL STAIRS

These stairs are similar to circular stairs except that the radius of curvature is small and the stairs may be supported by a center post. Overall diameter of such stairs may range from 1 to 2.5 m.

CURVED STAIRS

These stairs, when viewed from above, appear to follow a curve with two or more centre of curvature, such as ellipse.

DOG LEGGED STAIRCASE:

Dog legged staircase is the simplest type of stairs by which a flight of stairs moves one-half step before 180 degrees and persevering upwards. Due to its appearance in sectional elevation, it is a very common and popular stair consisting of two flights that run in opposite directions separated by a landing in the middle space. These staircases are used when the available space is equal to twice the width of the stairs and stairs lie in their compact layout that has better circulation from a design point of view.

UNIT IV DESIGN OF COLUMNS

Types of columns

Compression members are structural elements primarily subjected to axial compressive forces and hence, their design is guided by considerations of strength and buckling. Examples of compression member pedestal, column, wall and strut.

Effective length: The vertical distance between the points of inflection of the compression member in the buckled configuration in a plane is termed as effective length *le* of that compression member in that plane. The effective length is different from the unsupported length *l* of the member, though it depends on the unsupported length and the type of end restraints. The relation between the effective and unsupported lengths of any compression member is

$$le = k\, l$$

Where *k* is the ratio of effective to the unsupported lengths.

Pedestal: Pedestal is a vertical compression member whose effective length *le* does not exceed three times of its least horizontal dimension *b*. The other horizontal dimension *D* shall not exceed four times of *b*.

Column: Column is a vertical compression member whose unsupported length *l* shall not exceed sixty times of *b* (least lateral dimension), if restrained at the two ends. Further, its unsupported length of a cantilever column shall not exceed $100b^2/D$, where *D* is the larger lateral dimension which is also restricted up to four times of *b*

Wall: Wall is a vertical compression member whose effective height H_{we} to thickness *t* (least lateral dimension) shall not exceed 30. The larger horizontal dimension i.e., the length of the wall *L* is more than 4t.

Types of columns

A column may be classified based on different criteria such as:

1. **Based on shape**
    - Rectangle
    - Square
    - Circular

2. **Based on slenderness ratio or height**

Short column and Long column or Short and Slender Compression Members

A compression member may be considered as short when both the slenderness ratios namely l_{ex}/D and l_{ey}/b are less than 12:
Where
l_{ex}= effective length in respect of the major axis, D= depth in respect of the major axis, l_{ey}= effective length in respect of the minor axis, and b = width of the member.

It shall otherwise be considered as a slender or long compression member.

The great majority of concrete columns are sufficiently stocky (short) that slenderness can be ignored. Such columns are referred to as short columns. Short column generally fails by crushing of concrete due to axial force. If the moments induced by slenderness effects weaken a column appreciably, it is referred to as a slender column or a long column. Long columns generally fail by bending effect than due to axial effect. Long column carry less load compared to long column.

3. **Based on pattern of lateral reinforcement**

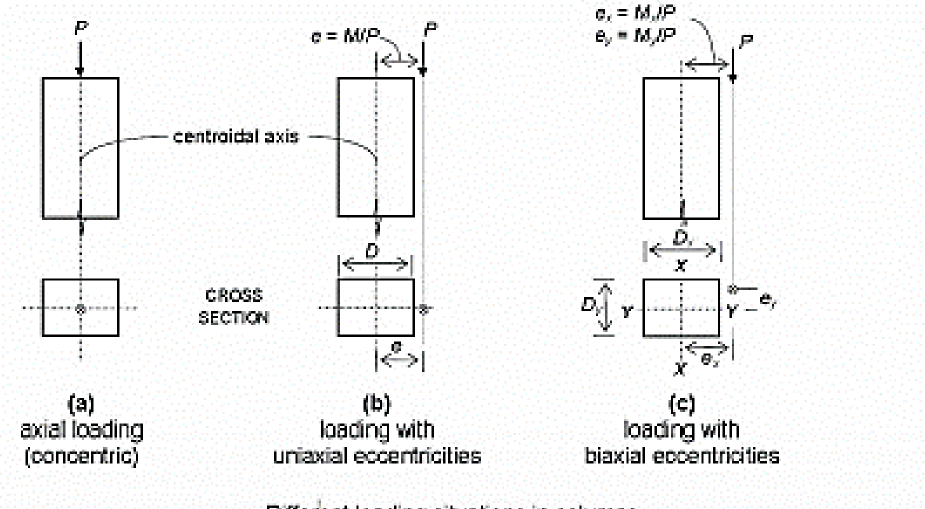
    - Tied columns with ties as laterals
    - Columns with Spiral steel as laterals or spiral columns

Majority of columns in any buildings are tied columns. In a tied column the longitudinal bars are tied together with smaller bars at intervals up the column. Tied columns may be square, rectangular, L-shaped, circular, or any other required shape. Occasionally, when high strength and/or high ductility are required, the bars are placed in a circle, and the ties are replaced by a bar bent into a helix or spiral. Such a column, called a spiral column. Spiral columns are generally circular, although square or polygonal shapes are sometimes used. The spiral acts to restrain the lateral expansion of the column core under high axial loads and, in doing so, delays the failure of the core, making the column more ductile. Spiral columns are used more extensively in seismic regions. If properly designed, spiral column carries 5% extra load at failure compared to similar tied column.

4. **Based on type of loading**

- Axially loaded column or centrally or concentrically loaded column (P_u)
- A column subjected to axial load and uniaxial bending (P_u + $M_{ux)}$ or (P + M_{uy})

- A column subjected to axial load and biaxial bending ($P_u + M_{ux} + M_{uy}$)



Different loading situations in columns

5. Based on materials

- Timber
- Stone
- Masonry
- RCC
- PSC
- Steel
- Aluminium ,
- Composite column

Minimum Eccentricity

In practical construction, columns are rarely truly concentric. Even a theoretical column loaded axially will have accidental eccentricity due to inaccuracy in construction or variation of materials etc. Accordingly, all axially loaded columns should be designed considering the minimum eccentricity

$e_{x\,min} \geq$ greater of ($l/500 + D/30$) or 20 mm

$e_{y\,min} \geq$ greater of ($l/500 + b/30$) or 20 mm

where *l*, *D* and *b* are the unsupported length, larger lateral dimension and least lateral dimension, respectively.

Longitudinal Reinforcement

The longitudinal reinforcing bars carry the compressive loads along with the concrete. stipulates the guidelines regarding the minimum and maximum amount, number of bars, minimum diameter of bars, spacing of bars etc. The following are the salient points:

- The minimum amount of steel should be at least 0.8 per cent of the gross cross-sectional area of the column required if for any reason the provided area is more than the required area.
- The maximum amount of steel should be 4 per cent of the gross cross-sectional area of the column so that it does not exceed 6 per cent when bars from column below have to be lapped with those in the column under consideration.
- Four and six are the minimum number of longitudinal bars in rectangular and circular columns, respectively.
- The diameter of the longitudinal bars should be at least 12 mm.
- Columns having helical reinforcement shall have at least six longitudinal bars within and in contact with the helical reinforcement. The bars shall be placed equidistant around its inner circumference.
- The bars shall be spaced not exceeding 300 mm along the periphery of the column.
- The amount of reinforcement for pedestal shall be at least 0.15 per cent of the cross-sectional area provided.

Pitch and Diameter of Lateral Ties

Pitch: The maximum pitch of transverse reinforcement shall be the least of the following:

(i) the least lateral dimension of the compression members;

(ii) sixteen times the smallest diameter of the longitudinal reinforcement bar to be tied; and

(iii) 300 mm.

(b) Diameter: The diameter of the polygonal links or lateral ties shall be not less than one-fourth of the diameter of the largest longitudinal bar, and in no case less than 6 mm.

UNIT IV DESIGN OF COLUMNS

Design of short column Axially Loaded columns

Assumptions

- The maximum compressive strain in concrete in axial compression is taken as 0.002.
- The maximum compressive strain at the highly compressed extreme fibre in concrete subjected to axial compression and bending and when there is no tension on the section shall be 0.0035 minus 0.75 times the strain at the least compressed extreme fibre.
- Plane sections normal to the axis remain plane after bending.
- The maximum strain in concrete at the outermost compression fibre is taken as 0.0035 in bending.
- The relationship between the compressive stress distribution in concrete and the strain in concrete may be assumed to be rectangle, trapezoid, parabola or any other shape which results in prediction of strength in substantial agreement with the results of test.
- An acceptable stress strain curve is given in IS:456-200. For design purposes, the compressive strength of concrete in the structure shall be assumed to be 0.67 times the characteristic strength. The partial safety factor y of 1.5 shall be applied in addition to this.
  The tensile strength of the concrete is ignored.

Short Axially Loaded Members in Compression

The member shall be designed by considering the assumptions given and the minimum eccentricity. When the minimum eccentricity as per 25.4 does not exceed 0.05 times the lateral dimension, the members may be designed by the following equation:

$$P_u = 0.4\, f_{ck}\, A_c + 0.67\, f_y\, A_{sc}$$

P_u = axial load on the member,

f_{ck} = characteristic compressive strength of the concrete,

A_c = area of concrete,

f_y = characteristic strength of the compression reinforcement, and

A_s = area of longitudinal reinforcement for columns.

P.1.Design the reinforcement in a column of size 400 mm x 600 mm subjected to an axial load of 2000 kN under service dead load and live load. The column has an unsupported length of 4.0 m and effectively held in position and restrained against rotation in both ends. Use M 25 concrete and Fe 415 steel.

Solution:

Step 1: To check if the column is short or slender

Given l = 4000 mm, b = 400 mm and D = 600 mm. Table 28 of IS 456 = $lex = ley = 0.65(l)$ = 2600 mm. So, we have

$lex/D = 2600/600 = 4.33 < 12$

$ley/b = 2600/400 = 6.5 < 12$

Hence, it is a short column.

Step 2: Minimum eccentricity

$ex\ min$ = Greater of ($lex/500 + D/30$) and 20 mm = 25.2 mm

$ey\ min$ = Greater of ($ley/500 + b/30$) and 20 mm = 20 mm

0.05 D = 0.05(600) = 30 mm > 25.2 mm (= $ex\ min$)

0.05 b = 0.05(400) = 20 mm = 20 mm (= $ey\ min$)

Hence, the equation given in cl.39.3 of IS 456 (Eq.(1)) is applicable for the design here.

Step 3: Area of steel

$Pu = 0.4\ fck\ Ac + 0.67\ fy\ Asc$

3000(103) = 0.4(25){(400)(600) – Asc} + 0.67(415) Asc which gives,

Asc = 2238.39 mm2

Provide 6-20 mm diameter and 2-16 mm diameter rods giving 2287 mm2 (> 2238.39 mm^2) and p = 0.953 per cent, which is more than minimum percentage of 0.8 and less than maximum percentage of 4.0. Hence, o.k.

Step 4: Lateral ties

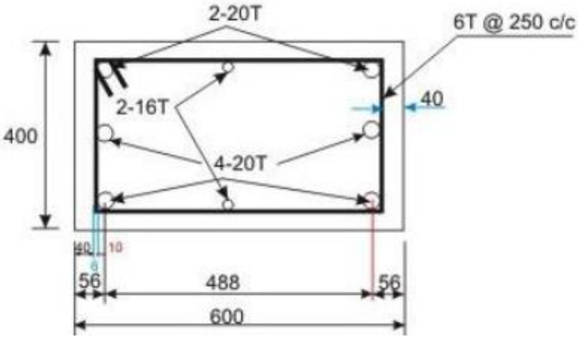
The diameter of transverse reinforcement (lateral ties) is determined from cl.26.5.3.2 C-2 of IS 456 as not less than (i) θ/4 and (ii) 6 mm. Here, θ = largest bar diameter used as longitudinal reinforcement = 20 mm. So, the diameter of bars used as lateral ties = 6 mm.

The pitch of lateral ties should be not more than the least of

(i) the least lateral dimension of the column = 400 mm

(ii) sixteen times the smallest diameter of longitudinal reinforcement bar to be tied = 16(16) = 256 mm

(iii) 300 mm



UNIT IV DESIGN OF COLUMNS

Determine the reinforcement to be provided in a circular column with the following data:

Diameter of column 500 mm
Grade of concrete M20
Factored moment 125 kN.m
Characteristic strength 250 N/mm^2
Factored load 1600 kN

Lateral reinforcement:
(a)Hoop reinforcement
(b) Helical reinforcement
(Assume moment due to minimum eccentricity to be less than the actual moment).
Assuming 25 mm bars with 40 mm cover,

$d^1 = 40 + 12.5 = 52.5$ mm
d^1/D - 52.5/50 = 0.105
Charts for d'/D = 0.10 will be used.
Let b=D

(a) Column *with hoop reinforcement*

$$\frac{P_u}{f_{ck}\,D^2} = \frac{1600\times10^3}{20\times500^2} = 0.32$$

$$\frac{M_u}{f_{ck}\,D^3} = \frac{125\times10^6}{20\times500^3} = 0.05$$

Referring to *Chart 52*, for f_y = 250 N/mm^2

$$\frac{P}{f_{ck}} = 0.87$$

Percentage of reinforcement, p = 0.87 x 20 = 1.74 %

$$A_s = \frac{1.74}{100}\times\frac{\Pi\times500^2}{4} = 3416 mm^2$$

(b) *Column with Helical Reinforcement*

According to 38.4 of the Code, the strength of a compression member with helical reinforcement is 1.05 times the strength of a similar member with lateral ties. Therefore, the, given load and moment should be divided by 1.05 before referring to the chart.

$$\frac{P_u}{f_{ck}\,D^2} = \frac{1600\times10^3}{1.05\times20\times500^2} = 0.31$$

$$\frac{M_u}{f_{ck}\,D^3} = \frac{125\times10^6}{1.05\times20\times500^3} = 0.048$$

Hence, From Chart 52, for f_y = 250 N/mm^2,

$$\frac{P}{f_{ck}} = 0.078$$

p = 0.078 x 20 = 1.56 %

$$A_s = \frac{1.56}{100}\times\frac{\Pi\times500^2}{4} = 3063\,mm^2$$

According to 38.4.1 of the Code the ratio of the volume of helical reinforcement to the volume of the core shall not be less than

$$0.36\left(\frac{A_g}{A_c}-1\right)\times\frac{f_{ck}}{f_y}$$

where A_g is the gross area of the section and A_c is the area of the core measured to the outside diameter of the helix. Assuming 8 mm dia bars for the helix

Core diameter = 500 - 2 (40 - 8) = 436 mm

$$\frac{A_g}{A_c} = \frac{500}{436} = 1.315$$

$$0.36\left(\frac{A_g}{A_c}-1\right)\times\frac{f_{ck}}{f_y} = 0.36\left(\frac{500}{436}-1\right)\times\frac{20}{250} = 0.0091$$

Volume of helical reinforcement / Volume of core

$A_{sh}\ \Pi\times 428/\left(\Pi/4\times 436^{2}\right)s_{h}$

$\Rightarrow 0.9\dfrac{A_{sh}}{S_{h}}$

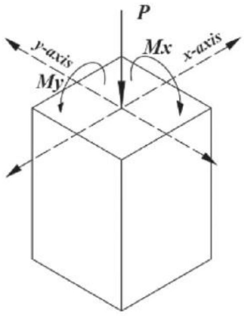
where, A_{sh} is the area of the bar forming the helix and s_h is the pitch of the helix. In order to satisfy the coda1 requirement,

$0.09\ Ash / s_h = 0.0091$
For 8 mm dia bar,
$s_h = 0.09 \times 50 / 0.0091 = 49.7$ mm.
Thus provide 48 mm pitch

UNIT IV DESIGN OF COLUMNS

Types of columns –Axially Loaded columns – Design of short Rectangular Square and circular columns –Design of Slender columns- Design for Uniaxial and Biaxial bending using Column Curves

Design for Biaxial bending using Column Curves



$$b = 400 \text{ mm}$$

$$D = 400 \text{ mm}$$

$$P_u = 1500 \text{ kN}$$

$$M_{ux} = M_{uy} = 50 \text{ kN·m}$$

$$f_{ck} = 20 \text{ N/mm}^2$$
$$f_y = 415 \text{ N/mm}^2$$

Equivalent moment

The reinforcement in section is designed for the axial compressive load P_u and the equivalent moment

$$M_u = 1.15\sqrt{M_{ux}^2 + M_{uy}^2}$$

$$= 1.15\sqrt{50^2 + 50^2}$$

$$= 81.3 \text{ kN} \cdot \text{m}$$

Nondimensional parameters

$$\left(\frac{P_u}{f_{ck}\,bD}\right) = \left(\frac{1500 \times 10^3}{20 \times 400 \times 400}\right) = 0.468$$

Assume $d' = 40$ mm

$(d'/D) = 0.10$

$$\left(\frac{M_u}{f_{ck}\,bD^2}\right) = \left(\frac{81.3 \times 10^6}{20 \times 400 \times 400^2}\right) = 0.063$$

Refer to Chart 44, SP:16

$(p/f_{ck}) = 0.06$

$p = (20 \times 0.06) = 1.2$

$$A_{st} = \left(\frac{pbD}{100}\right)$$

$$= \left(\frac{1.2 \times 400 \times 400}{100}\right)$$

$= 1920\ \text{mm}^2$

Provide 4 bars of 20 mm diameter and 4 bars of 16 mm diameter ($A_{sc} = 2060\ \text{mm}^2$) distributed equally on all faces with 3 bars on each face.

$p = (100 \times 2000)/(400 \times 400) = 1.28$

$(p/f_{ck}) = (1.28/20) = 0.064$

Refer to Chart 44, SP:16 and readout ($M_{ux1}/f_{ck}bD^2$) corresponding to the values of ($P_u/f_{ck}bD$) = 0.468 and (p/f_{ck}) = 0.064.

$$\left(\frac{M_{ux1}}{f_{ck}bD^2}\right) = 0.068$$

$$M_{ux1} = (0.068 \times 20 \times 400 \times 400^2)\ 10^{-6}$$

$$= 87\ \text{kN·m}$$

Due to symmetry $M_{ux1} = M_{uy1} = 87$ kN·m

$$P_{uz} = [0.45 f_{ck}\, A_c + 0.75 f_y\, A_s]$$

$$= (0.45 \times 20)\ [(400 \times 400) - 2060] + 0.75 \times 415 \times 2060$$

$$= 2062 \times 10^3\ \text{m}$$

$$= 2062\ \text{kN}$$

$$\left(\frac{P_u}{P_{uz}}\right) = \left(\frac{1500}{2062}\right) = 0.72$$

$$\alpha_n = 1.8$$

Check for safety under biaxial bending

$$\left[\left(\frac{M_{ux}}{M_{ux1}}\right)^{\alpha_n} + \left(\frac{M_{uy}}{M_{uy1}}\right)^{\alpha_n}\right] \leq 1$$

$$\left[\left(\frac{50}{87}\right)^{1.8} + \left(\frac{50}{87}\right)^{1.8}\right] = 0.736 < 1$$

Hence the section is safe against bending

UNIT V DESIGN OF FOOTINGS

Design of wall footing

Problem

Design a footing for 250mm thick has masonry wall with supports to carry a design load of 200kn/m at service state. Consider unit weight of soil 20KN/m^3. Angle of repose = 30°. Allowable bearing capacity 150KN/m^2, M_{20}, Fe_{415} .

Given Data:

q_0 = $150KN/m^2$

γ = $20\ KN/m^3$

B = 250mm

p_u = 200 KNm

$\emptyset$ = 30°

f_{ck} = $20\ N/mm^2$

f_y = $415\ N/mm^2$

Step 2:

Determination of depth of foundation

h = $\frac{q_0}{\gamma}$ x $[\,1 - sin\emptyset/1 + sin\emptyset\,]^2$

= $\frac{150}{20}$ x $[\,1 - sin30/1 + sin30\,]^2$

= $0.83 \cong 1$m

Step 3:

Find width of footing

B = $\frac{Load}{S.B.C}$

= $\frac{200}{150}$ = 1.35 m

Step 4:

Find total load

Self weight of footing = $(L \times B \times D)\,\gamma$

= $(1 \times 1.35 \times 1)$ 25

$= \quad 33.75 \cong 34$

Total Load $=$ p_u + self weight

$= 200 + 34$

$= 234\ KN/m^2$

Step 5:

Actual width of footing

Actual width $= \frac{234}{150} = 1.56 \cong 1.6$ m

Step 6:

Net upward pressure

$P_o = \frac{Load(given)}{Width \times 1(m\ given)}$

$= \frac{200}{1.6} = 125 KN/m^2\ /m\ length$

Step 7:

a) Depth of Basis of Bending Compression

$M = \frac{P_O}{8}$ x (B – b) x (B - $\frac{b}{4}$)

$= \frac{125}{8}$ x (1.6 – 250 x 10^{-3}) x (1.6 – $\frac{250 \times 10^{-3}}{4}$)

M = 32.43 KNm

Factored Moment = 1.5 x M

= 1.5 x 32.43

M_u = 48.645 KNm

M_u lim $= \frac{0.36\ x_{u\ max}}{d} f_{ck} [\ 1- \frac{0{,}42 x_{u\ max}}{d}\ ]\ bd^2$

$= 0.36 \times 0.48 \times 20\ [1- 0.42 \times 0.48\ ] bd^2$

$= 2.759\ bd^2$

M_u lim $= M_u$

$2.759\ bd^2 = 48.645 \times 10^3$

$2.759 \times 10^{-3} \times 10^3 \times d^2 = 48.645 \times 10^3$

d = 132.78 mm

D = d + cover

= 132.78 + 60

$= 192.78$ mm

$D = 200$ mm

b) Depth on basis of one way shear

Assume, $p_t = 0.3\%$

$\tau_v = \tau_c \times K$

τ_c ref IS456 Pg No: 73 $\quad 0.25 \rightarrow 0.36$

$0.50 \rightarrow 0.48$

$\tau_c = 0.38$

Permissible shear stress, $\tau_v = 0.38$ K

K ref IS 456 Pg No : 73

$K = 1.20$

$\tau_v = 0.38 \times 1.20$

$\tau_v = 0.456$

c)Critical section lies 'd' distances from face of wall

$V_u = 1.5 \, P_o \, a$

$a = \frac{B}{2} - \frac{b}{2}$

$= \frac{1.35}{2} - \frac{250}{2}$

$a = 675$ mm

$a = 0.675$ m

$V_u = 1.5 \times P_o \times a$

$= 1.5 \times 125 \times 0.675 = 126.56 \, N/m^2$

$\tau_v = \frac{V_u}{bd}$

$0.456 = \frac{126.56 \times 10^3}{1000 \times d}$

$d = 277.54$ mm $\cong 280$ mm.

$D = d + d_c$

$= 280 + 60 = 340$ mm

Design Reinforcement

$M_u = 0.87 \, f_y \, Ast \, d \, [1 - \frac{fyAst}{bd \, f_{ck}}]$

$$48.65 \times 10^6 = 0.87 \times 415 \times Ast \times 270 \left[1 - \frac{415\ Ast}{1000 \times 270 \times 20}\right]$$

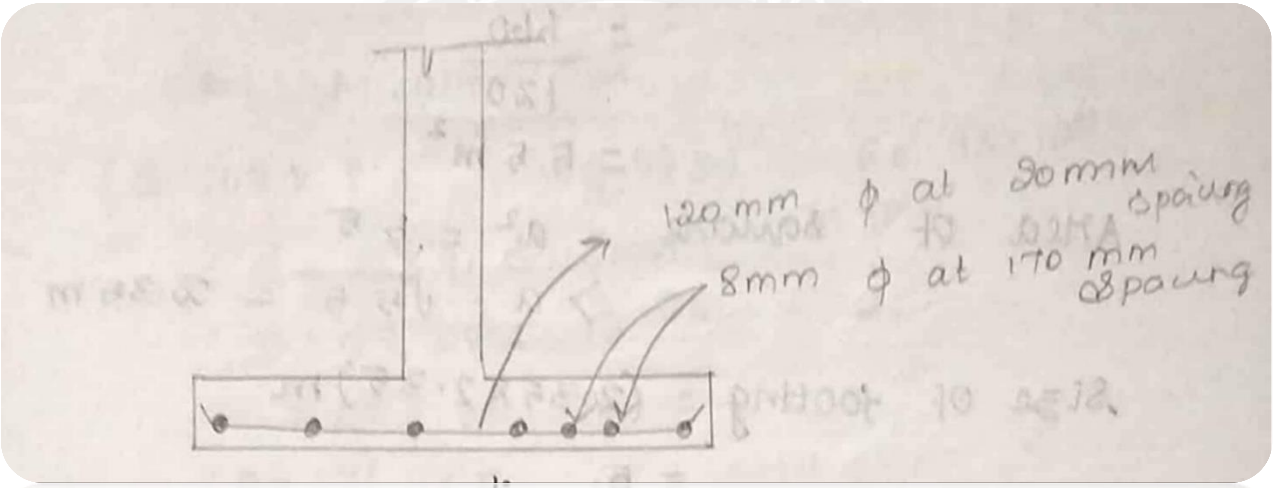
$$Ast = 519.83$$

Provide 12mm Ø at 200mm C/C.

Distribution Reinforcement

$$= \frac{0.12}{100} \times 1000 \times 250$$

$$= 300mm$$



UNIT V DESIGN OF FOOTINGS

RECTANGLE COLUMN FOOTING

Problem

Design a rectangular isolated footing of uniform thickness of R.C column, bearings vertical load of 600 KN, have base size 400 x 600 mm, and have SBC of 120 KN/m^2. Use M$_{20}$ and Fe$_{415}$ grades.

Step 1:

b = 400 mm

d = 600 mm

w = 600 KN

S.B.C = 120 KN/m^2

f_y = 415 N/mm^2

f_{ck} = 20 N/mm^2

Step 2:

Size of footing

W = 600 KN (Weight of column)

Self weight of footing = 10% (column load)

= $\frac{10}{100}$ x 600

= 60 KN

Total load = 600 + 60

= 660 KN

Area of footing = $\frac{Load}{S.BC}$

$\frac{660}{120}$

= 5.5 m^2

A = 5.5

B x L = A = 5.5 m^2

B = $\frac{2}{3}$ L

$\frac{2}{3}$ x L x L = 5.5

L = 2.87 m ≅ 3m

B = 1.91 m ≅ 2m

B = 2m , L = 3m

Step 3: Section Design

a) Depth on basis of Bending compression

Net upward pressure P_o = $\frac{Given\ load}{Area\ of\ footing}$

= $\frac{600}{3\times2}$

= 100 KN/m^2

Along x−x axis

M_x = P_o x B x 1.2 x $\frac{1.2^2}{2}$

= 100 x 2 x 1.2 x $\frac{1.2^2}{2}$

M_x = 172.8

M_{ux} = 1.5 x M = 259.2

Along y−y axis

M_y = P_o x L x 0.8 x $\frac{0.8^2}{2}$

= 100 x 3 x 0.8 x $\frac{0.8^2}{2}$

= 76.8

M_{uy} = 115.2

M_u lim = Take greater one

M_u lim = 259.2 ≅ 260 KNm

b) Depth from $M_u$lim

M_u lim = 0.36 $\frac{M_u max}{d}$ f_{ck} (1− $\frac{0.42\ x_u max}{d}$) bd^2

260 x 10^6 = 0.36 x 0.48 x 20 [1− 0.42(0.48)] bd^2

260 x 10^6 = 3.456 (1− 0.2016) bd^2

bd^2 = 94227807.47

$2 \times 10^3 d^2$ = 94227807.47

d = $217.0 \cong 220$ mm

D = $d + d'$

= 220 + 60

= 280 mm

c) Depth of basis of one way shear

a = 1.2− d

Shear force, V_u = $1.5 \times P_o \times B \times a$

= 1.5 x 100 x 2(1.2− d)

V_u = 360 – 300d

τ_v = $\frac{Vu}{bd}$

τ_v = $\frac{360-300d}{2d}$ ⇢ ①

Assume, P_t = 0.3%

(τ_c ref IS 456 Pg No: 73) 0.25 → 0.36

0.50 → 0.48

τ_c = 0.38 N/mm^2

Permissible shear stress, τ_v = τ_c x K

K = 1.05

D=280mm ⇢ (IS 456 Pg: 72)

275 → 1.05

300 → 1.00

τ_v = K x τ_c

= 0.38 x 1.05

= 0.4 N/mm^2

τ_v = 400 KN/m^2 ⇢ ②

Eqn ①&②

$\frac{360-300d}{2d}$ = 400

$360 - 300d = 800d$

$360 = 1100d$

$d = 0.327\text{ m}$

$d = 3.27 \cong 330\text{ mm}$

$D = 390\text{ mm}$

Depth on basis of 2 way shear

Area of footing, AF = $6m^2$ (3 x 2)

$BC = B + \frac{d}{2} + \frac{d}{2}$

$= 400 + \frac{330}{2} + \frac{330}{2}$

BC = 730mm

$AB = 600 + \frac{330}{2} + \frac{330}{2}$

AB = 930mm

Area = BC x AB

= 730 x 930

= 678900 mm^2

Shear force = P_o [AF – Area of ABCD]

= $100[6 - 678900 \times (10^{-3})^2]$

= 532.11 KN

F_u = 1.5 x 532.11

= 798.17 KN

Length of ABCD = (930 x 2) + (730 x 2)

= 3320 mm

$\tau_v = \frac{F_u}{Length\ of\ ABCD \times d}$

$= \frac{798.17 \times 10^6}{3320 \times 330} = 0.73\text{ N/mm}^2$

Permissible stress = $k_s \times \tau_c$

$k_s = 0.5 + \beta_c$

β_c = $\frac{Short\ side\ of\ column}{Long\ side\ of\ column}$

= $\frac{400}{600}$ = 0.667

k_s = 0.5 + 0.667

= 1.167

But k_s is not greater than one, so k_s = 1

τ_c = $0.25\sqrt{f_{ck}}$

= $0.25 \times \sqrt{20}$

= 1.118 N/mm^2

$k_s\tau_c$ = 1.167 x 1.118

= 1.3 N/mm^2

τ_v < $k_s\tau_c$

Hence safe

Step 4: Design of Reinforcement

Find Ast$_x$:

Mu_x = $0.87\ f_y\ Ast_x\ d\ [1- \frac{f_y Ast_x}{bd\ f_{ck}}]$

260×10^6 = $0.87 \times 415 \times Ast_x \times 330\ [1- \frac{415\ Ast_x}{20 \times 2000 \times 330}]$

Ast_x = 2356.82 mm^2

12mm Ø bar @ 50mm in x- direction C/C spacing.

Mu_y = $0.87\ f_y\ Ast_y\ d\ [1- \frac{f_y Ast_y}{bd\ f_{ck}}]$

120×10^6 = $0.87 \times 415 \times Ast_y \times 330\ [1- \frac{415\ Ast_y}{20 \times 2000 \times 330}]$

Ast_y = 987 mm^2

12mm Ø bar @ 110mm spacing.

Check of development length

i. L_d = 47 x Ø

= 47 x 12 = 564 mm

ii. Length of bar,

$L_o = \frac{1}{2} \times (B - b) - d_c$

$= \frac{1}{2} \times (2000 - 400) - 60$

$= 740mm$

$L_o > L_d$ Hence safe.

$A_1 = 2160 \times 1960$

$A_2 = 600 \times 400$

$\sqrt{\frac{A_1}{A_2}} = \sqrt{\frac{2160 \times 1960}{600 \times 400}}$

$= 4.2$

Adopt values, $\sqrt{\frac{A_1}{A_2}} = 2$

Permissible bearing stress $= 0.45\, f_{ck} \sqrt{\frac{A_1}{A_2}}$

$= 0.45 \times 20 \times 2$

$= 18\ N/mm^2$

Actual bearing pressure $= \frac{Load}{Area}$

$= \frac{600 \times 10^3}{600 \times 400}$

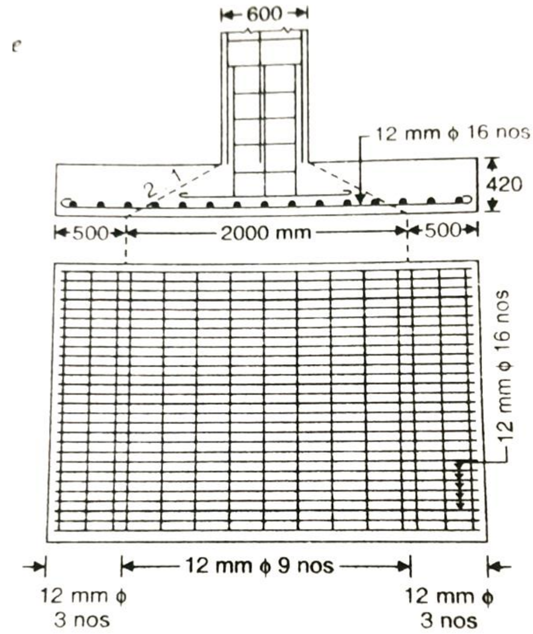
$= 2.5\ N/mm^2$

Actual bearing pressure < Permissible bearing stress

$2.5 < 18\ N/mm^2$

Hence safe.

Reinforcement details



[Source:R.C.C Designs by Dr.B.C.Punmia,page 1091]

DESIGN OF COMBINED RECTANGULAR FOOTING FOR TWO COLUMNS

Whenever two or more columns in a straight line are carried on a single spread footing, it is called a combined footing. Isolated footings for each column are generally the economical. Combined footings are provided only when it is absolutely necessary, as

1.When two columns are close together, causing overlap of adjacent isolated footings

2.Where soil bearing capacity is low, causing overlap of adjacent isolated footings

3. Proximity of building line or existing building or sewer, adjacent to a building column.

Problem

Two interior columns A and B carry 700 kN and 1000 kN loads respectively. Column A is 350 mm x 350 mm and column B is 400 mm X 400 mm in section. The centre to centre spacing between columns is 4.6 m. The soil on which the footing rests is capable of providing resistance of 130 kN/m^2. Design a combined footing by providing a central beam joining the two columns. Use concrete grade M25 and mild steel reinforcement.

Solution: Data

fck = 25 N/mm2,

fy= 250 N/mm2,

fb = l30 kN/m2 (SBC),

Column A = 350 mm x 350 mm,

Column B = 400 mm x 400 mm,

c/c spacing of columns = 4.6 m, PA = 700 kN and PB = 1000 kN

Ultimate loads

P_{ua} = 1.5 x 700 = 1050 kN,

P_{ub} = 1.5 x 1000 = 1500 kN

Working load carried by column A = P_A = 700 kN

Working load carried by column B = P_B = 1000 kN

Self weight of footing 10 % x (P_A + P_B) = 170 kN Total working load = 1870 kN

Required area of footing = A_f = Total load /SBC

$=1870/130 = 14.38\ m^2$

Let the width of the footing = B_f = 2m

Required length of footing = $L_f = A_f / B_f = 14.38/2 = 7.19m$

Provide footing of size 7.2m X 2m, $A_f = 7.2 \times 2 = 14.4\ m^2$

For uniform pressure distribution the C.G. of the footing should coincide with the C.G. of column loads. Let x be the distance of C.G. from the centre line of column A

Then $x = (P_B \times 4.6)/(P_A + P_B) = (1000 \times 4.6)/(1000 + 700)$

= 2.7 m from column A.

If the cantilever projection of footing beyond column A is 'a'

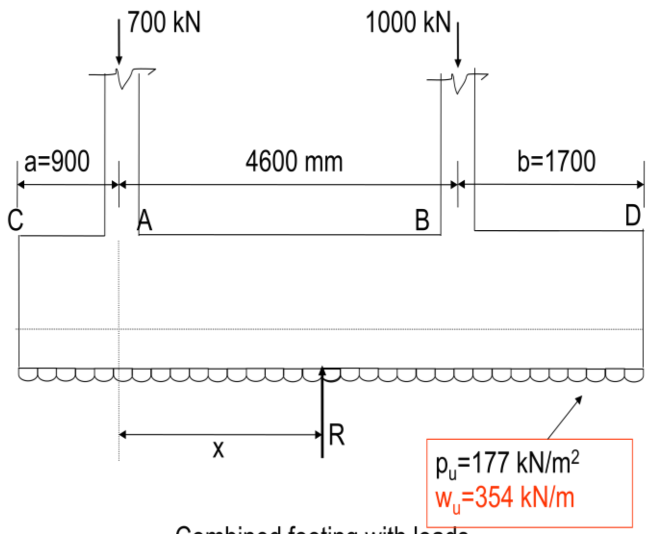
then, $a + 2.7 = L_f/2 = 7.2/2$, Therefore $a = 0.9$ m

Similarly if the cantilever projection of footing beyond B is 'b'

then, $b + (4.6-2.7) = L_f/2 = 3.6$ m,

Therefore $b = 3.6 - 1.9 = 1.7$ m

The details are shown in Figure



Combined footing with loads

Rectangular Footing with Central Beam:-Design of Bottom slab

Total ultimate load from columns = $P_u = 1.5(700 + 1000) = 2550$ kN.

Upward intensity of soil pressure $w_u = P/A_f = 2550/14.4 = 177\ kN/m^2$

Design of slab

Intensity of Upward pressure = w_u =177 kN/m^2

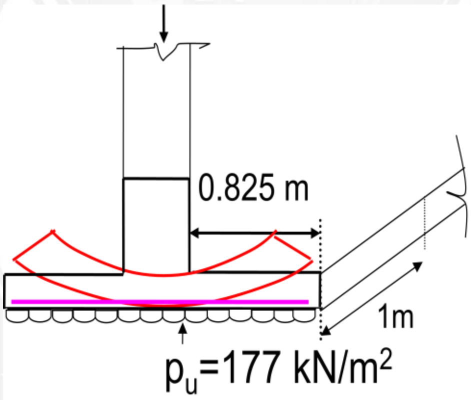
Consider one meter width of the slab (b=1m)

Load per m run of slab at ultimate = 177 x 1 = 177 kN/m

Cantilever projection of the slab (For smaller column)

=1000 - 350/2 = 825 mm

Maximum ultimate moment = 177 x 0.825^2/2 = 60.2 kN-m.



For M25 and Fe 250, $Q_{u\,max}$ = 3.71 N/mm^2

Required effective depth = √ (60.2 x 10^6/(3.71 x 1000)) = 128 mm

Since the slab is in contact with the soil clear cover of 50 mm is assumed.

Using 20 mm diameter bars

Required total depth = 128 + 20/2 + 50 =188 mm say 200 mm

Provided effective depth = d = 200-50-20/2 = 140 mm

Check the depth for one - way

shear considerations- At 'd' from face

Design shear force=V_u=177x(0.825-0.140)=121kN

Nominal shear stress=τ_v=V_u/bd=121000/(1000x140) =0.866MPa

Permissible shear stress

P_t = 100 x 2415 /(1000 x 140) = 1.7 %, τ_{uc} = 0.772 N/mm^2

Value of k for 200 mm thick slab =1.2

Permissible shear stress = 1.2 x 0.772 = 0.926 N/mm^2

$\tau_{uc} > \tau_v$ and hence safe

The depth may be reduced uniformly to 150 mm at the edges.

Check for development length

L_{dt}= [0.87 x 250 / (4 x 1.4)]Φ =39 Φ

= 39 x 20 = 780 mm

Available length of bar=825 - 25 = 800mm

> 780 mm and hence safe.

Transverse reinforcement

Required A_{st}=0.15bD/100

=0.15x1000 x 200/100 = 300mm^2

Using Φ8 mm bars, Spacing=1000x50/300

= 160 mm

Provide distribution steel of Φ8 mm at 160 mm c/c,<300, <5d

Design of Longitudinal Beam

Load from the slab will be transferred to the beam.

As the width of the footing is 2 m, the net upward soil pressure per meter length of the beam

= w_u = 177 x 2 = 354 kN/m

Shear Force and Bending Moment

V_{AC}= 354 x 0.9 =318.6 kN, V_{AB} = 1050-318.6 =731.4 kN

V_{BD}= 354 x 1.7 = 601.8kN, V_{BA} = 1500-601.8 = 898.2 kN

Point of zero shear from left end C

X_1 = 1050/354 = 2.97m from C or

X_2 = 7.2-2.97 = 4.23 m from D

Maximum B.M. occurs at a distance of 4.23 m from D

M_{uE} = 354 x 4.23^2 / 2 - 1500 (4.23 - 1.7) = -628 kN.m

Bending moment under column A= M_{uA}=354x0.9^2 /2 = 143.37 kN.m

Bending moment under column B = M_{uB} = 354 x 1.7^2 = 511.5 kN-m

Let the point of contra flexure be at a distance x from the centre of column A

Then, M_x= I050x - 354 (x + 0.9)^2/ 2 = 0

Therefore x = 0.206 m and 3.92 m from column A

i.e. 0.68 m from B.

Depth of beam from B.M.

The width of beam is kept equal to the maximum width of the column i.e. 400 mm. Determine the depth of the beam where T- beam action is not available.

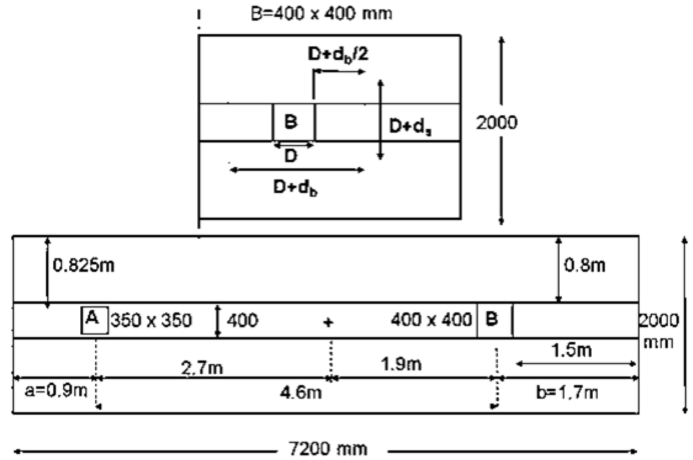
The beam acts as a rectangular section in the cantilever portion, where the maximum positive moment = 511.5 kN/m.

$$d = \sqrt{(511.5 \times 10^6/ (3.73 \times 400))} = 586 \text{ mm}$$

Provide total depth of 750 mm. Assuming two rows of bars with effective cover of 70 mm.

Effective depth provided = d= 750-70 = 680 mm

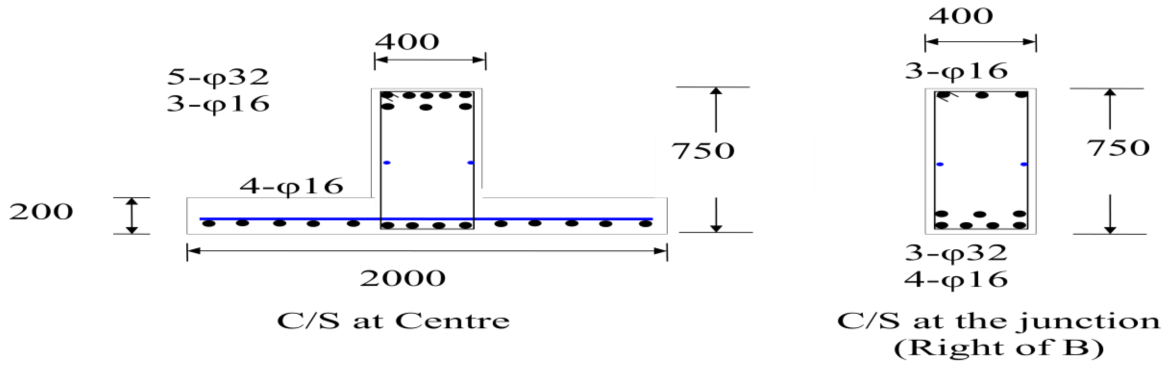
(Less than 750mm and hence no side face steel is needed



In this case b=D=400 mm, d_b=680 mm, d_s=140 mm

Area resisting two - way shear

$$= 2(b \times d_b + d_s \times d_s) + 2 (D + d_b)ds$$



$= 2\ (400 \times 680 + 140 \times 140) + 2(400+680)\ 140 = 885600\ mm^2$

Design shear=P_{ud}= column load – W_u x area at critical section

$= 1500 - 177 \times (b + d_s) \times (D + d_b)$

$= 1500 - 177 \times (0.400+0.140) \times (0.400+ 0.680) \quad = 1377.65 kN$

$\tau_v = P_{ud}/b_o d$

$= 1377.65 \times 1000/885600 = 1.56$ MPa

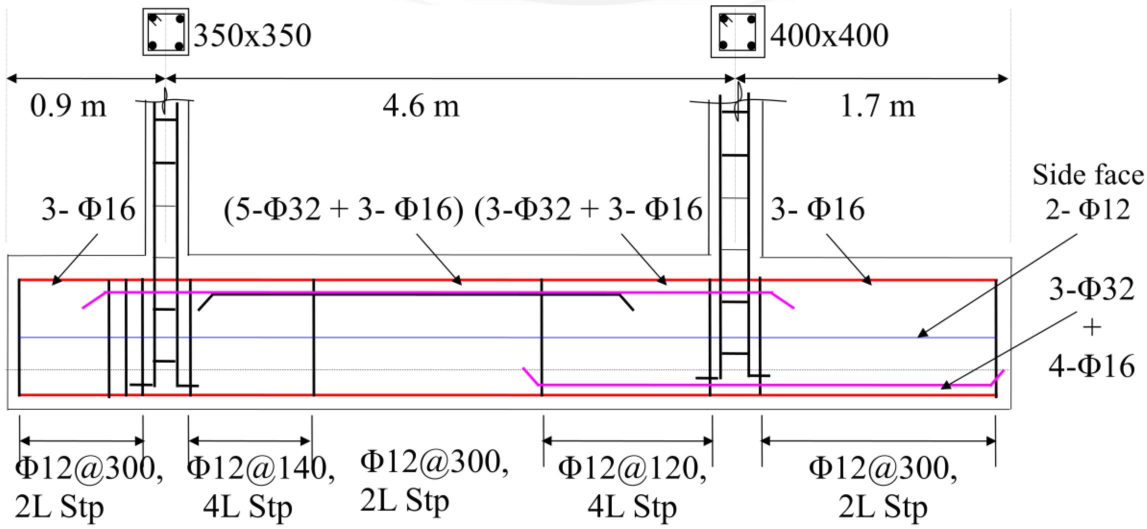
Shear stress resisted by concrete = $\tau_{uc} = \tau_{uc} \times K_s$

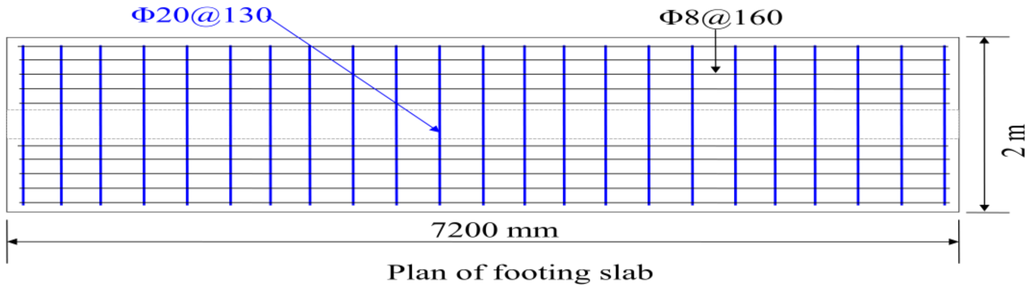
where, $\tau_{uc} = 0.25 \sqrt{f_{ck}} = 0.25\sqrt{25} = 1.25$ N/mm²

$K_s = 0.5 + d / D = 0.5 + 400/400 = 1.5 \leq 1$ Hence $K_s = 1$

$\tau_{uc} = 1 \times 1.25 = 1.25$ N/mm².

Therefore Unsafe





Plan of footing slab