Reg. No. : | E | N | G | G | T | R | E | E | . | C | O | M |

Question Paper Code : 70132

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2022

First Semester

Civil Engineering

MA 3151 – MATRICES AND CALCULUS

(Common to : All Branches (Except Marine Engineering))

(Regulations 2021)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. The eigenvalues and the corresponding eigenvectors of a 2 × 2 matrix is given by $\lambda_1 = 8;\ x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\lambda_2 = 4;\ x_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. Find the corresponding matrix.

2. Determine the nature, index and signature of the quadratic form $x_1^2 + 5x_2^2 + x_3^2 + 2x_2x_3 + 6x_3x_1 + 2x_1x_2$.

3. For what values of the constant c is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx^2 + 2x; & x < 2 \\ x^3 - cx; & x \geq 2 \end{cases}.$$

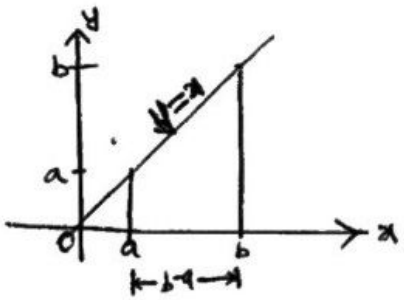
4. Find the slope of the circle $x^2 + y^2 = 25$ at $(3, -4)$.

5. Find $\dfrac{\partial^2 w}{\partial x \partial y}$, if $w = xy + \dfrac{e^y}{y^2 + 1}$.

6. Find $\dfrac{\partial w}{\partial r}$ and $\dfrac{\partial w}{\partial s}$ in terms of r and s if $w = x^2 + y^2$, $x = r - s$ and $y = r + s$.

7. Evaluate $\displaystyle\int \frac{\tan x}{\sec x + \tan x}\,dx$.

8. Find the area of the region shown in the diagram given below, bounded between $x = a$ and $x = b$.



9. Sketch the region of integration in $\int_0^1 \int_x^1 f(x,y)\,dy\,dx$.

10. Change the Cartesian integral $\int_0^6 \int_0^y x\,dx\,dy$ into an equivalent polar integral.

PART B — (5 × 16 = 80 marks)

11. (a) Obtain an orthogonal transformation which will transform the quadratic form $Q = 2x_2x_3 + 2x_3x_1 + 2x_1x_2$ to canonical form.

Or

(b) An elastic membrane in the x_1x_2-plane with boundary circle $x_1^2 + x_2^2 = 1$ is stretched so that a point $P = (x_1, x_2)$ goes over a point $Q = (y_1, y_2)$ given by $y_1 = 5x_1 + 3x_2$ and $y_2 = 3x_1 + 5x_2$. Find the principal directions that is, the directions of the position vector x of P for which the direction of the position vector y of Q is the same or exactly opposite. What shape does the boundary circle take under this deformation?

12. (a) (i) Find y'' if $x^4 + y^4 = 16$. (8)

(ii) Differentiate $y = (2x+1)^5 (x^3 - x + 1)^4$. (8)

Or

(b) Find the intervals on which $f(x) = -x^3 + 12x + 5;\ -3 \le x \le 3$ is increasing and decreasing. Where does the function assume extreme values? What are those values?

13. (a) Find the maximum and minimum values of the function $f(x,y) = 3x + 4y$ on the circle $x^2 + y^2 = 1$.

Or

(b) Find the Taylor series expansion of the function $f(x,y) = \sin x \sin y$ near the origin.

14. (a) (i) Evaluate $\int_0^\infty e^{-ax} \sin bx dx$, for $a > 0$. (8)

(ii) Integrate $\int_0^{\pi/2} \frac{\sin x \cos x}{\cos^2 x + 3\cos x + 2} dx$. (8)

Or

(b) (i) Evaluate $\int \frac{3x^4 + 3x^3 - 5x^2 + x - 1}{x^2 + x - 2} dx$. (8)

(ii) Integrate $\int x\sqrt{1 + x - x^2}\, dx$. (8)

15. (a) (i) Change the order of integration in $\int_0^1 \int_{x^2}^{2-x} xy\, dy\, dx$ and hence evaluate. (8)

(ii) Find the area of the region inside the cardioid $r = a(1 + \cos\theta)$ and outside the circle $r = a$. (8)

Or

(b) Find the volume of the region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$. (16)