Reg. No. :

Question Paper Code : 54014

B.E./B.Tech. DEGREE EXAMINATION, JANUARY 2018

First Semester

Marine Engineering

MA 8101 : MATHEMATICS FOR MARINE ENGINEERING – I

(Regulations 2017)

Time : Three Hours | Maximum : 100 Marks

Answer ALL questions

PART – A **(10×2=20 Marks)**

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1. Write the equation of the tangent plane at (1, 5, 7) to the sphere $(x-2)^2 + (y-3)^2 + (z-4)^2 = 14$.

2. Find the equation of the right circular cone whose vertex is at the origin and axis is the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ having semivertical angle of 45°.

3. If $y = \frac{x+1}{x^2-4}$, then find y_n.

4. Evaluate $\underset{x \to \frac{\pi}{2}}{\text{Lt}} (\sec x - \tan x)$.

5. If $u = xe^y z$, where $y = \sqrt{a^2 - x^2}$ and $z = \sin^2 x$, then find $\frac{dy}{dx}$.

6. State the Euler's theorem for homogeneous function of x, y, z of degree n.

7. Evaluate $\int x e^x \, dx$.

8. Find the area enclosed by the circle $x^2 + y^2 = r^2$, using integration.

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9. Change the order of integration in the integral $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy\, dy\, dx$.

10. Evaluate $\int_0^3 \int_{-1}^2 \int_0^1 xyz^2\, dx\, dy\, dz$.

PART – B **(5×16=80 Marks)**

11. a) i) Find the equation of the sphere having the circle $x^2 + y^2 + z^2 = 9$, $x + y + z = 3$ as a great circle. **(8)**

ii) Find the equation of the right circular cylinder whose guiding circle is $x^2 + y^2 + z^2 = 9$, $x - y + z = 3$. **(8)**

(OR)

b) i) Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ and hence find the point of contact. **(8)**

ii) Find the equation of the right circular cone whose vertex is at the origin and axis is the line $\frac{x}{1} = \frac{y}{2} = \frac{y}{3}$ and which has semivertical angle of 30°. **(8)**

12. a) i) Find n^{th} derivative of $\sin^2 x \cos^3 x$. **(8)**

ii) If $x = \tan(\log y)$, then prove that $(1 + x^2)\, y_{n+1} + (2nx - 1)\, y_n + n(n-1)\, y_{n-1} = 0$. **(8)**

(OR)

b) i) Expand $\log_e x$ in powers of $(x - 1)$ by Taylor's series and hence evaluate $\log_e(1.1)$ correct to four decimal places. **(8)**

ii) Trace the curve $y = x^3 - 12x - 16$. **(8)**

13. a) i) If $v = \left(x^2 + y^2 + z^2\right)^{-1/2}$, then find $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}$. **(6)**

ii) If $u = \sin^{-1}\left(\frac{x + y}{\sqrt{x} + \sqrt{y}}\right)$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$ and hence show that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = (-)\frac{\sin u \cos^2 u}{4\cos^3 u}$. **(10)**

(OR)

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b) i) Discuss the maxima and minima and also find the Saddle points of the function $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$. (10)

ii) If $f(x, y) = 0$, then show that $\frac{d^2y}{dx^2} = -\frac{q^2r - 2pqs + p^2t}{q^3}$, where

$r = \frac{\partial^2 f}{\partial x^2}, s = \frac{\partial^2 f}{\partial x \partial y}, t = \frac{\partial^2 f}{\partial y^2}$. (6)

14. a) i) Evaluate $\int_0^2 e^x dx$ as the limit of a sum. (8)

ii) Evaluate $\int \frac{x}{\sqrt{3 - 2x - x^2}} dx$ by trigonometric substitution. (8)

(OR)

b) i) Find the area common to the cardioids $r = a(1 + \cos\theta)$ and $r = a(1 - \cos\theta)$. (8)

ii) Find the volume formed by the revolution of loop of the curve $y^2(a + x) = x^2(3a - x)$ about the x-axis. (8)

15. a) i) Using the double integral, find the area of the region that lies inside the circle $r = 3\sin\theta$ and outside the cardioid $r = 1 + \sin\theta$. (8)

ii) Evaluate $\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{2} (x^2 + y^2)\, dz dy dx$ by changing into cylindrical coordinates. (8)

(OR)

b) i) Find the volume of a sphere of radius r by using triple integral. (8)

ii) Determine the mass of a hemisphere of radius a with centre at the origin of the density F of its substance at each point (x, y, z) is proportional to the distance of this point from the base. (8)

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